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Design and Realization of Quantum based Digital Integrator

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Abstract

Quantum mechanics is an excellent technique to counter and study the effect of noise in any system. A digital integrator can be designed with the help of quantum mechanics by passing a known signal $x(n)$ to a known digital integrator, with impulse response $g(n)$ and then, the output of this integrator $y(n)$ is given to the input of a known quantum system (specified by its Hamiltonian (H_0), perturbation parameter (ϵ), wave function (ψ) and a potential (V)) gives an output $\psi(n+1)$. Now the same known signal is passed through an unknown FIR filter, which is to be designed and having impulse response $h(n)$, and output of this FIR filter $f(n)$ is again passed through the similar quantum system, which produce the output$\hat{\psi}(n+1)$. Then the coefficients of the unknown filter have been calculated by minimizing the Frobenius norm$\|e(n)\|^2$, where $[error\ signal, e(n) = \psi(n+1) - \hat{\psi}(n+1)]$ and differentiating it with respect to the unknown FIR filter's coefficients and by using adaptive algorithm. Thereafter the magnitude and phase of given and designed integrator and function X_i is plotted, which is defined as $X_i = \|g(n) - h(n)\|$ to justify the validity of the designed digital integrator.

Keywords: Digital integrator, Quantum mechanics, Hamiltonian, Perturbation, Frobenius norm, Adaptive algorithm, Hermition matrix, Cayley transform, Unitary evolution.

1. Introduction

Digital integrators [1, 2, 3, 4, 5] which have been designed till now are not able to provide the ideal phase and frequency response simultaneously at higher frequencies. Quantum mechanics [7, 8] is an excellent tool for microscopic level study such as

noise analysis etc, and hence it can be helpful to overcome the shortcomings of previously designed digital integrators.
Digital integrator in its simplest form is defined as follows-

$$H(z) = \frac{1}{1-Z^{-1}}$$

This is Rectangular rule [9, 10] to define transfer function of integrator. Similarly there are other methods for higher order Integrators, such as Simpson's rule [1], Trapezoidal method [2], Al-Alaoui Integrator [3], Upadhyay and Singh [5] but all of them are incapable of providing the ideal phase and frequency response simultaneously [6]. In the proposed paper quantum mechanics is being used to design the digital integrator.

Quantum mechanics defined by wave function (ψ), which depends on the total energy present in the signal (let say $y(n)$) and whenever there is any change or perturbation takes place in the state then the wave function ($\psi(n+1)$) in discrete time domain is defined by using Cayley transform as follows-

$$\psi(n+1) = \frac{I - i\frac{\Delta}{2}(H_0 + \varepsilon * y(n) * V)}{I + i\frac{\Delta}{2}(H_0 + \varepsilon * y(n) * V)} * \psi(n)$$

Where,
I = Identity matrix,
H_0= Hamiltonian,
ε = Perturbation parameter
V = Potential,
$\psi(n)$ = Wave function,
$y(n)$ = Input signal to quantum system.
This equation can also be written as follows-

$$\psi(n+1) = \left(I - i\frac{\Delta}{2}(H_0 + \varepsilon * Y(n) * V)\right) * \left(I + i\frac{\Delta}{2}(H_0 + \varepsilon * Y(n) * V)\right)^{-1} * \psi(n) \tag{1}$$

2. Quantum Approach

Quantum mechanics has been used in the proposed paper in order to visualize the microscopic study of integrator to overcome the existing problems in the designed integrators.

The basic idea used to design the digital integrator in the proposed paper can be understand by the following block diagram-

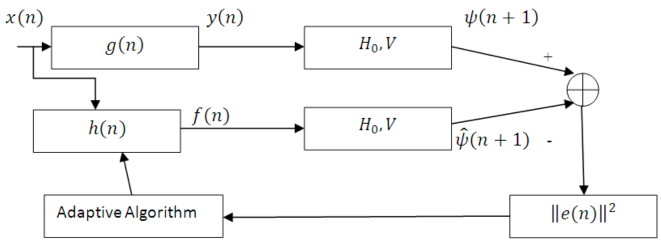

Fig. 1: Block Diagram Representation.

In this system output $y(n)$is nothing but the convolution of input $x(n)$ and impulse response $g(n)$. Output of quantum system is given by eq. (1). Similarly output $f(n)$ is nothing but the convolution of input $x(n)$ and impulse response $h(n)$.

This output $f(n) = \sum_{k=1}^{p} h_k(n) * x(n-k), n \geq p+1$ (2)

Output of quantum system can be written as follows-

$$\hat{\psi}(n+1) = \left(I - i\frac{\Delta}{2}(H_0 + \varepsilon * f(n) * V)\right) * \left(I + i\frac{\Delta}{2}(H_0 + \varepsilon * f(n) * V)\right)^{-1} * \hat{\psi}(n)$$

Now consider the following,

$$\left(I + i\frac{\Delta}{2}(H_0 + \varepsilon * f(n) * V)\right) = A$$

And hence $\left(I - i\frac{\Delta}{2}(H_0 + \varepsilon * f(n) * V)\right) = A'$

And $\hat{\psi}(n+1) = A^{-1} * A' * \hat{\psi}(n)$ (3)

Error signal, $e(n) = \psi(n+1) - \hat{\psi}(n+1)$ (4)

Applying adaptive algorithm on this error signal will produce the following equation-

$$\frac{\partial \|e(n)\|^2}{\partial h_k(n)} = \frac{\partial \left\|\psi(n+1) - \hat{\psi}(n+1)\right\|^2}{\partial h_k(n)}$$

As, $\frac{\partial \left\|\psi(n+1) - \hat{\psi}(n+1)\right\|^2}{\partial h_k(n)} \frac{\partial \langle \psi(n+1) - \hat{\psi}(n+1), \psi(n+1) - \hat{\psi}(n+1)\rangle}{\partial h_k(n)}$

Here, $\hat{\psi}(n+1) = A^{-1} * A' * \hat{\psi}(n)$ and

$$\psi(n+1) = \left(I - i\frac{\Delta}{2}(H_0 + \varepsilon * Y(n) * V)\right) * \left(I + i\frac{\Delta}{2}(H_0 + \varepsilon * Y(n) * V)\right)^{-1} * \psi(n)$$

Also, as

$$\frac{d\langle x,x\rangle}{d\theta} = \langle \frac{dx}{d\theta}, x\rangle + \langle x, \frac{dx}{d\theta}\rangle$$

Hence,

$$\frac{d\langle x,x\rangle}{d\theta} = 2 * Re\,\langle x, \frac{dx}{d\theta}\rangle$$

So,

$$\frac{\partial \|e(n)\|^2}{\partial h_k(n)} = 2 * Re\,\langle \psi(n+1) - \hat{\psi}(n+1), \frac{\partial\left(A^{-1} * A' * \hat{\psi}(n)\right)}{\partial h_k(n)}\rangle \quad (5)$$

In this way the coefficients have been calculated of the unknown filter.

Once the coefficients are calculated, the function X_i is plotted, which shows the difference between the norm of the two filters i.e. one which is given and second is designed.

3. Conclusion

After obtaining the coefficient of unknown filter by applying the algorithm, magnitude and phase of both given and designed integrator are plotted which are as shown below-

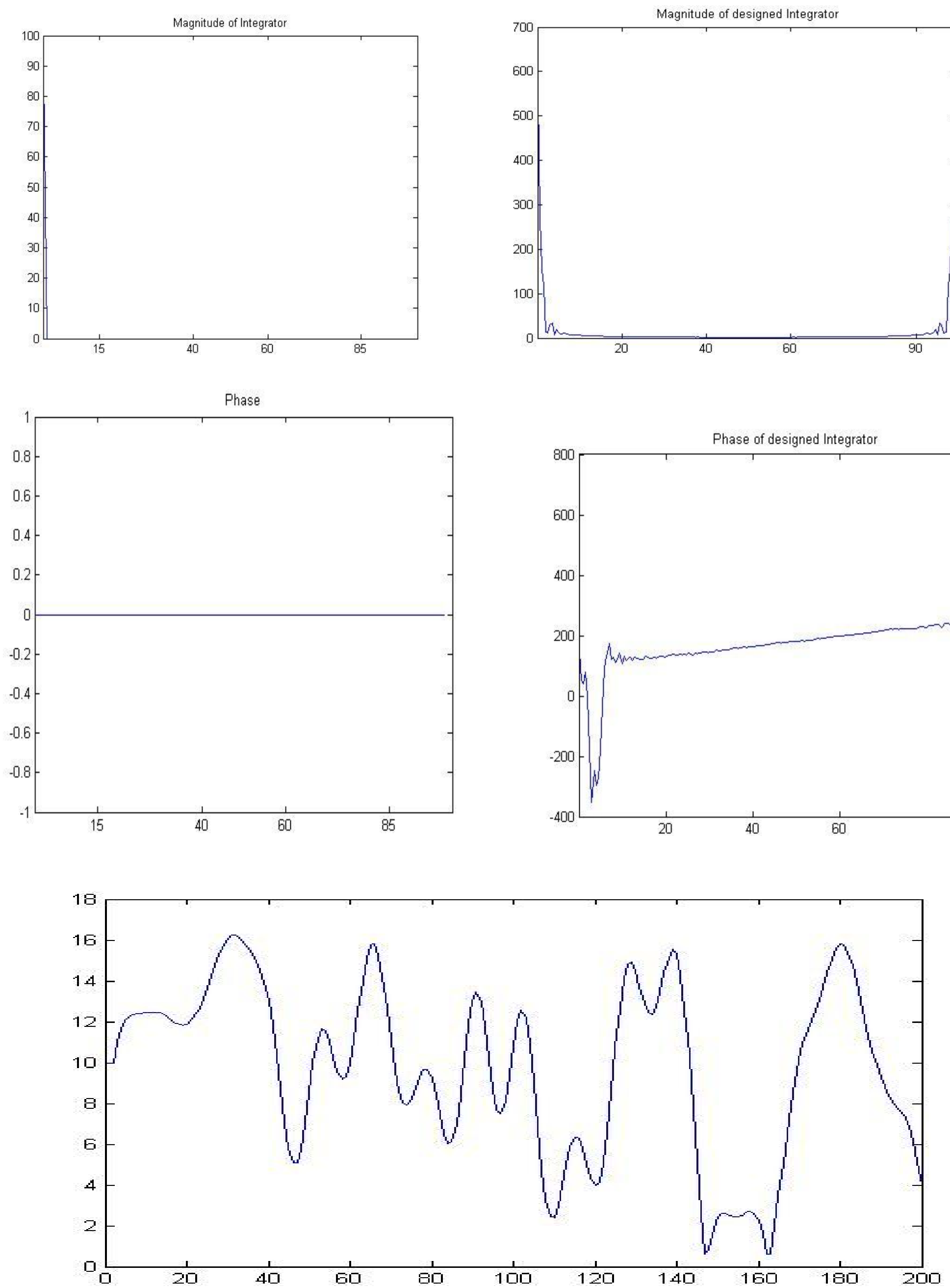

Fig. 2: Magnitude, Phase and Error plot/ Function X_i plot.

These plots are showing that the error between the given filter and designed filter is well within the permissible range and also as we are moving towards the higher iteration the difference is further decreasing.

Hence in this way the proposed method is successfully able to design the unknown filter close to the given filter with the help of quantum mechanics. This means that this technique can be used to design the replica of various systems once one knows the system to be designed.

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