Statistical Analysis of Voltage and Current Fluctuations in a Transmission Line with Distributed Parameters Varying Randomly with Space and Time

Rohit Kumar Singh[1], Harish Parthasarathy[2], Dharmendra Upadhyay[3]

[1,2,3]*Division of Electronics and Communication Engineering*
Netaji Subhas Institute of Technology, University of Delhi
New Delhi, India

ABSTRACT

In this paper, we have taken into consideration the effect of fluctuations on the distributed parameters in a transmission line due to physical conditions like temperature, humidity, atmospheric dust, wind etc. The crucial idea is to carry out the calculation of fluctuations in the distributed parameters by transforming continuous time Fourier series into discrete time Fourier series with the help of appropriate sampling in space and time. The DFT equations enable us to write MATLAB programmes for solving the Perturbative matrix linear equations. The distributed parameters of a transmission line [R, L, G, C] are assumed to be constants plus small random functions of space and time. The line voltage v (t, z) and line current i(t,z) then satisfy linear first order partial differential equation in (t,z) whose coefficient are random t-z functions. We apply here first order perturbation theory to a Bi-variate Fourier series of [⬚R,⬚L,⬚G,⬚C], so that we obtain linear matrix equation for Fourier series coefficients of ⬚V& ⬚I in terms of Fourier series coefficient of [⬚R,⬚L,⬚G,⬚C]. The source and load conditions gives us up to first order values of v(t,0) and i(t,0). Assuming the space time Fourier series coefficients of [⬚R, ⬚L, ⬚G, ⬚C] to be uncorrelated random variables, we obtain expressions for the voltage and current correlations along the length of transmission line. We use these correlations to estimate the variances in the Fourier series coefficient for [⬚R, ⬚L, ⬚G, C] based on correlation matching (Method of Moments).

KEYWORDS: Distributed parameters, First order perturbation theory, Bi-variate Fourier series, unperturbed voltage and current, uncorrelated random variable, Method of Moments, DFT.

INTRODUCTION

The study of Transmission line is based on the calculation of distributed parameters R, L, G, C [1, 2] from the basic linear partial first order differential equation. Distributed parameters are calculated from the above method doesn't take into account the random fluctuations occurring inside the parameters.

Thus we have taken into account the effect of disturbance on [R, L, G, C] parameters with space and time. With the use of first order perturbation theory, we have obtained linear matrix equations for the Fourier series coefficient of V and ⸮I in terms of [⸮R, ⸮L, ⸮G, ⸮C] and unperturbed voltage and current. v(t, 0) and i(t, 0) gives us the source and load boundary conditions. The end product is an expression for v(t, z) and i(t, z) as the sum of non-random plus linear function of the Fourier series coefficients of ⸮R(t, z), ⸮L(t, z), ⸮C(t, z), ⸮G(t, z). We finally suggest a technique to estimate the variances in the Fourier series for ⸮R, ⸮L, ⸮G, ⸮C based on correlation matching.

Mathematical Equations: As we have extracted R, L, G, C parameters with FTDT method and other numerical tools for 2-D transmission lines [3, 4], the introduction of random fields to R, L, G, C parameters, supported by first order perturbation theory approach, transforms the distributed parameters into-

$$R(t,z) = R_0 + \delta.R_1(t,z)$$
$$L(t,z) = L_0 + \delta.L_1(t,z)$$
$$G(t,z) = G_0 + \delta.G_1(t,z)$$
$$C(t,z) = C_0 + \delta.C_1(t,z)$$
$$-V_{,z}(t,z) = R(t,z) * i(t,z) + (L(t,z) * i(t,z))_{,z}$$
$$-i_{,z}(t,z) = G(t,z) * v(t,z) + (C(t,z) * v(t,z))_{,z}$$

$$R_1(t,z) = \sum_{n,m=1}^{N} R(n,m)\frac{2}{\sqrt{dT}}\sin\left(\frac{n\pi z}{d}\right)\sin\frac{m\pi t}{T}$$

$$C_1(t,z) = \sum_{n,m=1}^{N} C(n,m)\frac{2}{\sqrt{dT}}\sin\left(\frac{n\pi z}{d}\right)\sin\frac{m\pi t}{T}$$

$$G_1(t,z) = \sum_{n,m=1}^{N} G(n,m)\frac{2}{\sqrt{dT}}\sin\left(\frac{n\pi z}{d}\right)\sin\frac{m\pi t}{T}$$

$$L_1(t,z) = \sum_{n,m=1}^{N} L(n,m)\frac{2}{\sqrt{dT}}\sin\left(\frac{n\pi z}{d}\right)\sin\frac{m\pi t}{T}$$

$$\frac{\partial v(t,z)}{\partial z} = \left(R_0 + \delta R_1(t,z)\right) * i(t,z) + \frac{\partial}{\partial t}\left(\left(L_0 + \delta L_1(t,z)\right)\right) * i(t,z)$$
$$\frac{\partial i(t,z)}{\partial z} = \left(G_0 + \delta G_1(t,z)\right) * v(t,z) + \frac{\partial}{\partial t}\left(\left(C_0 + \delta C_1(t,z)\right)\right) * v(t,z)$$

Thus $i(t,z)$ and $v(t,z)$ can be expressed in terms of their Fourier series expansions-

$$i(t,z) = \delta \sum i(m,n) * u_{m,n}(t,z) + i_0(t,z)$$

$$v(t, z) = \delta \sum v(m, n) * u_{m,n}(t, z) + v_0(t, z)$$

$$-\frac{\partial v_0}{\partial z} = R_0 i_0 + L_0 \frac{\partial i_0}{\partial z}$$

$$-\frac{\partial i_0}{\partial z} = G_0 v_0 + C_0 \frac{\partial v_0}{\partial z}$$

$$\frac{\partial v_1}{\partial z} + R_0 i_1 + L_0 \frac{\partial i_1}{\partial z} = \varphi_1(t, z)$$

$$\frac{\partial i_1}{\partial z} + G_0 v_1 + C_0 \frac{\partial v_1}{\partial z} = \varphi_2(t, z)$$

Where,

$$\varphi_1(t, z) = -R_1(t, z) i_0(t, z) - \frac{\partial L_1(t, z)}{\partial t} i_0(t, z)$$

$$\varphi_2(t, z) = -G_1(t, z) v_0(t, z) - \frac{\partial C_1(t, z)}{\partial t} v_0(t, z)$$

$$\gamma_0 = \sqrt{(R_0 + j\omega L_0) * (G_0 + j\omega C_0)}$$

Hence transforming time domain linear partial differential voltage and current equations in frequency domain we get-

$$V(z, \omega) = V_+ e^{-\gamma_0 z} + V_- e^{-\gamma_0 z}$$

$$I(z, \omega) = \frac{V_+ e^{-\gamma_0 z} - V_- e^{-\gamma_0 z}}{R_0}$$

Thus representing frequency dependent voltage and current equations in matrix form-

$$\frac{\partial}{\partial z} \begin{bmatrix} \hat{v}_1 \\ \hat{\imath}_1 \end{bmatrix} = A_0 \begin{bmatrix} \hat{v}_1(\omega, z) \\ \hat{\imath}_1(\omega, z) \end{bmatrix} + \begin{bmatrix} \hat{\varphi}_1(\omega, z) \\ \hat{\varphi}_2(\omega, z) \end{bmatrix}$$

Where,

$$A_0 = \begin{bmatrix} 0 & R_0 + j\omega L_0 \\ G_0 + j\omega C_0 & 0 \end{bmatrix}$$

Now calculations are made for source conditions i.e. z=0, to obtain source voltage and current-

$$V_s(\omega) = V_+(\omega) + V_-(\omega)$$

Similarly,

$$I_s(\omega) = \frac{V_+(\omega) - V_-(\omega)}{R_0}$$

From above values of voltage and current (in frequency domain), parameters like Reflection coefficient is being calculated and compared with the one calculated using time dependent terminations [5], and hence gives us values of voltage and current at a specific distance, d from the load.

Rohit Kumar Singh, Harish Parthasarathy, Dharmendra Upadhyay
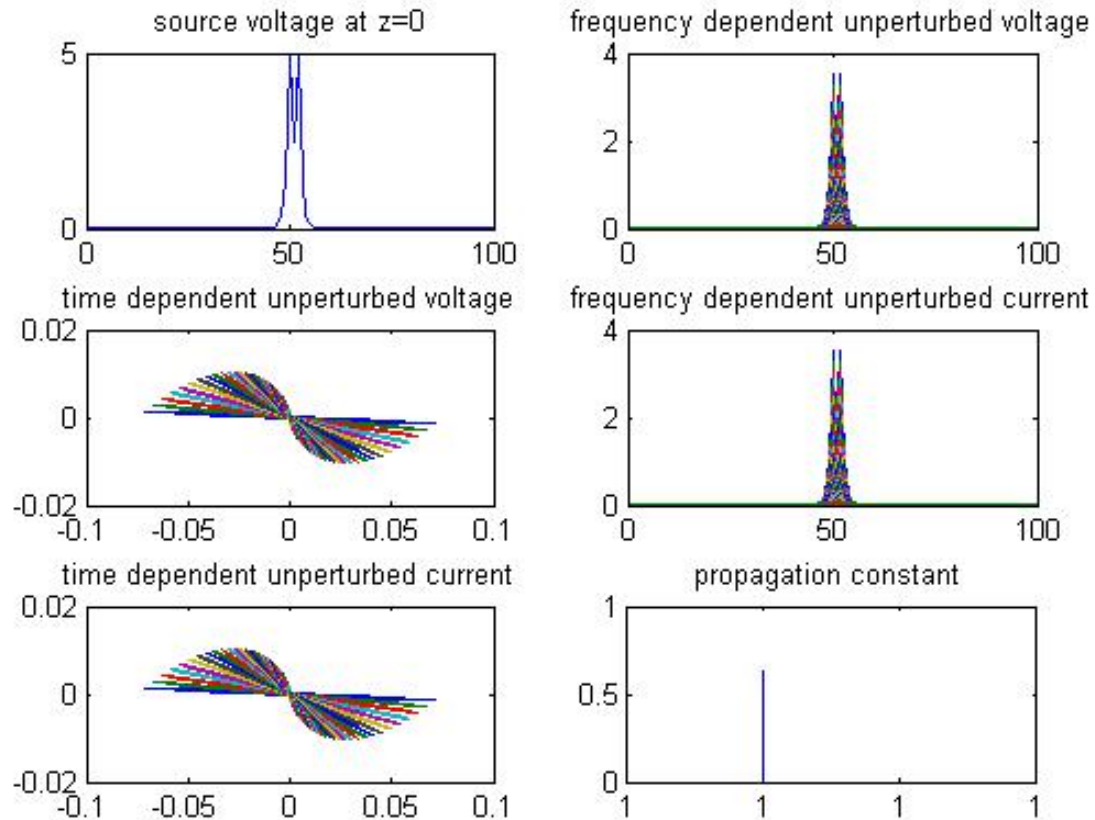
RESULTS:

Figure (1) Time dependent and frequency dependent unperturbed voltage and current

CONCLUSION:

As we have chosen different values of N, results in varying propagation constant, leading to different plots for voltage and current in time and frequency domain. Higher value of N leads to clearly defined peaks. However in the existing techniques we get fixed plots for voltage and current which is being determined by fixed value of frequency and propagation constant

REFERENCES:

[1] Zhan Min, Zhou Su-quan, Li Bi-jun, Li Lei, Lin Gui-hua and Yi Jia-liang, "Parameter Estimation of Transmission Line Based on GATS", *IEEE Conference on Digital Object Identifier, Computational Intelligence and Design (ISCID),* 2010, P: 179-182

[2] Yuan Liao, "Some algorithms for transmission line parameter estimation", *IEEE Conference on System Theory and Digital Object Identifier,* 2009.P: 127- 132

[3]   Afrooz, K. Abdipour and A. Martín, "Time domain analysis of one-dimensional linear and non-linear composite right/left-handed transmission lines using finite-difference time-domain method", *IET Journals & Magazines on Microwaves*, Antennas & Propagation, Volume: 6, Issue: 3, 2012

[4]   Charlet, F. Bermond, C. Putot, S. Le Carval and G. Flechet, "Extraction of (R,L,C,G) interconnect parameters in 2D transmission lines using fast and efficient numerical tools and Simulation of Semiconductor Processes and Devices", *IEEE Conference on Digital Object Identifier.*2000, P: 87- 89

[5]   Cheldavi A., "Transient analysis of transmission lines with time-dependent terminations and general input; Mathematical Methods in Electromagnetic Theory", *IEEE Conference on Digital Object Identifier.*1998. P: 324- 326.