

## Implementation of Model Predictive Control for Cascaded CSTR Model Using Lab View

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### Abstract

Over the past three decades, Model Predictive Control (MPC) has emerged as one of the most powerful and widely used control algorithm. Model predictive control uses the explicit process model to predict the future behaviour of a plant. This algorithm also takes into consideration the various constraints in input and output while designing the controller. This paper explores the capability of model predictive control algorithm in controlling the temperature parameter of a non linear cascaded Continuous Stirred Tank Reactor (CSTR) process model. The model predictive control algorithm is implemented in LabVIEW using the control and simulation toolkit and the reference tracking capability of the system is verified. The simulated performance of the system widens the option of using LabVIEW platform in designing MPC for a non-linear multi input- multi output (MIMO) process.

**Keywords:** CSTR, LabVIEW, Model predictive control.

### 1. Introduction

The change in control action in case of traditional feedback controllers like PID takes place in response to the change in output set point of a system. Contrary to that, model predictive control is a technique that tries to create controllers which can change its control action even before there is an actual change in the output set point. This is made possible by the long range predictive ability of model predictive control which distinguishes it from other control algorithms. The predictive ability of MPC is usually used in addition with the traditional feedback operation so that the control action is

smoother and the output tracks the set point easily. The five major parts of MPC are the process model, the cost function, process constraints, prediction and control horizons.

*Model-* For MPC algorithm it is important to have a model of the process which is under control. The linearized discrete state space model, as in (1), is used in this work for prediction purpose.

$$\begin{aligned} \mathbf{x}(\mathbf{k} + 1) &= A\mathbf{x}(\mathbf{k}) + B\mathbf{u}(\mathbf{k}) \\ \mathbf{y}(\mathbf{k}) &= C\mathbf{x}(\mathbf{k}) + D\mathbf{u}(\mathbf{k}) \end{aligned} \quad (1)$$

*Prediction horizon-* It is defined as the number of samples in future for which the MPC controller predicts the plant output.

*Control horizon-* It is defined as the number of samples within prediction horizon during which the MPC controller affects the control action

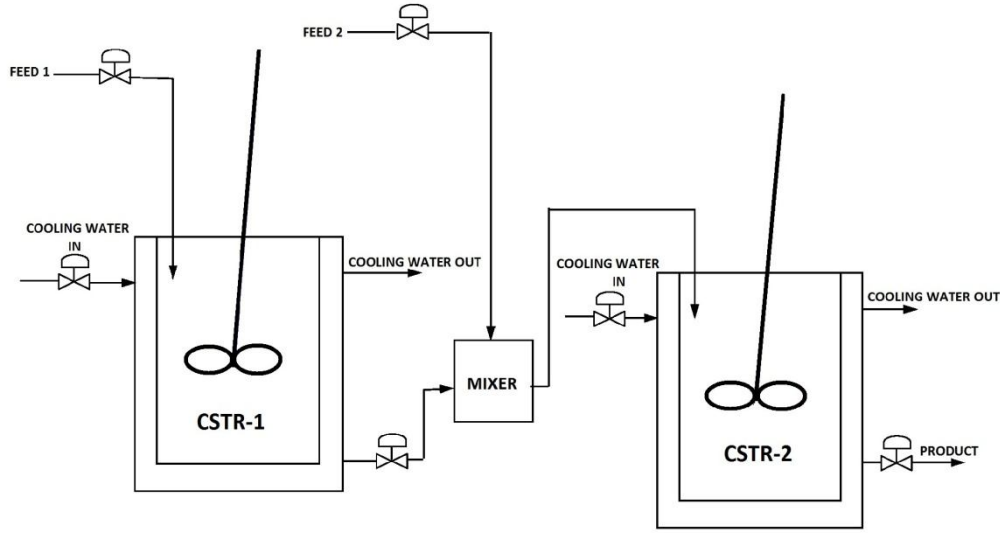
*Cost function-* The main objective of MPC controller is to calculate a set of control action values such that the cost function is minimized as much as possible. The cost could depend on the reference tracking error, deviation of the controller output and the unbounded movement of the controller output

*Constraints-* There could be many physical parameters that may affect the input or output operation which are known as constraints. Model predictive control tries to prevent the violations of these constraints in accordance with whether they are hard or soft constraints. A hard constraint should not be violated under any circumstances whereas the soft constraint could be violated when some other constraint has a higher priority.

The designed MPC was validated on a cascaded continuous stirred tank reactor process, which are commonly used in many food and chemical industries. For achieving qualitative product it is important to maintain the component balance of chemicals. This could be achieved by controlling the temperature of the process medium. In this work a cascaded CSTR process was reckoned and temperature of both reactors' medium was chosen as parameter to be controlled. The cooling water flow rates in both reactors were selected as manipulated variables, which will be computed by the MPC scheme.

## 2. Cascaded CSTR process

To implement the MPC algorithm, a two CSTR process is chosen where an irreversible, exothermic, first order reaction converts a reactant A to product B. An intermediate mixer is used to introduce a second feed in the system. As the reaction is exothermic in nature, a jacket with a cooling water flow in it is used to maintain the temperature inside the CSTRs. The objective is to control both tank temperatures by changing manipulated variables which are cooling water flow rate of two tanks in this case.



**Figure 1:** Cascaded CSTR process.

The above mentioned non-linear process was modelled with the help of Ordinary Differential Equation (ODE) model. This ODE model, as in (2), was simulated in LabVIEW platform to validate the MPC impact.

$$\begin{aligned}
 \frac{dx_1}{dt} &= -k_1 x_1 + \frac{Q_{I1} C_{I1}}{V_1} - \frac{K_{V1} x_1}{\sqrt{V_1}} \\
 \frac{dx_2}{dt} &= \Delta H K_1 x_1 + \frac{Q_{I1} T_{I1}}{V_1} - \frac{K_{V1} x_2}{\sqrt{V_1}} - U_{a1} (x_2 - x_3) \\
 V_{I1} \frac{dx_3}{dt} &= Q_{CW1} (T_{CW1} - x_3) + U_{a1} (x_2 - x_3) \\
 \frac{dx_4}{dt} &= -K_2 x_4 + \frac{K_{V1} x_1 \sqrt{V_1}}{V_2} + \frac{Q_{I2} C_{I2}}{V_2} - \frac{K_{V2} x_4}{\sqrt{V_2}} \\
 \frac{dx_5}{dt} &= \Delta H K_2 x_4 + \frac{K_{V1} x_2 \sqrt{V_1}}{V_2} + \frac{Q_{I2} T_{I2}}{V_2} - \frac{K_{V2} x_5}{\sqrt{V_2}} - \frac{U_{a2} (x_5 - x_6)}{V_2} \\
 V_{I2} \frac{dx_6}{dt} &= Q_{CW2} (T_{CW2} - x_6) + U_{a2} (x_5 - x_6)
 \end{aligned} \tag{2}$$

The six state variables here are,

- $x_1$ : Outlet concentration of 1<sup>st</sup> CSTR ( $C_{O1}$ )
- $x_2$ : Outlet temperature of 1<sup>st</sup> CSTR ( $T_{O1}$ )
- $x_3$ : Outlet cooling water temperature of 1<sup>st</sup> CSTR ( $T_{CW1}$ )
- $x_4$ : Outlet concentration of 2<sup>nd</sup> CSTR ( $C_{O2}$ )
- $x_5$ : Outlet temperature of 2<sup>nd</sup> CSTR ( $T_{O2}$ )
- $x_6$ : Outlet cooling water temperature of 2<sup>nd</sup> CSTR ( $T_{CW2}$ )

### 3. Implementation

This section discusses the preparation of MPC components and implementing them in a LabVIEW platform in detail.

*Linearized Model:* It can be seen that the process equations (2) are nonlinear in nature. This ODE model was linearized around the operating points mentioned in Table.1. The physical and process constants mentioned in Table. 2. The linearization results in following matrices of discrete state space model. This linearized model, as in (3) was used for prediction purpose.

$$\dot{\mathbf{X}} = \begin{bmatrix} 0.1555 & -13.7665 & -0.0604 & 0 & 0 & 0 \\ 0.0010 & 1.0008 & 0.0068 & 0 & 0 & 0 \\ 0 & 0.0374 & 0.9232 & 0 & 0 & 0 \\ 0.0015 & -0.1024 & -0.0003 & 0.1587 & -13.6705 & -0.0506 \\ 0 & 0.0061 & 0 & 0.0006 & 0.9929 & 0.0057 \\ 0 & 0.0001 & 0 & 0 & 0.0366 & 0.9398 \end{bmatrix} \begin{bmatrix} C_{O1} \\ T_{O1} \\ T_{CWO1} \\ C_{O2} \\ T_{O2} \\ T_{CWO2} \end{bmatrix} + \begin{bmatrix} 0.0001 & 0 \\ 0 & 0 \\ -0.0036 & 0 \\ 0 & 0.0001 \\ 0 & 0 \\ 0 & -0.0028 \end{bmatrix} \begin{bmatrix} Q_{CW1} \\ Q_{CW2} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 0 & 362.995 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 362.995 & 0 \end{bmatrix} \begin{bmatrix} C_{O1} \\ T_{O1} \\ T_{CWO1} \\ C_{O2} \\ T_{O2} \\ T_{CWO2} \end{bmatrix} \quad (3)$$

**Table 1:** Operating points.

Variable	Value
Outlet concentration of 1 <sup>st</sup> CSTR ( $C_{O1}$ )	0.084 mol/m <sup>3</sup>
Outlet temperature of 1 <sup>st</sup> CSTR ( $T_{O1}$ )	364.995° K or 91.845° C
Outlet cooling water temperature of 1 <sup>st</sup> CSTR ( $T_{CWO1}$ )	327.560° K
Outlet concentration of 2 <sup>nd</sup> CSTR ( $C_{O2}$ )	0.053 mol/m <sup>3</sup>
Outlet temperature of 2 <sup>nd</sup> CSTR ( $T_{O2}$ )	364.995° K or 91.845° C
Outlet cooling water temperature of 2 <sup>nd</sup> CSTR ( $T_{CWO2}$ )	335.447° K
Feed flow rate in 1 <sup>st</sup> CSTR ( $Q_{I1}$ )	0.339 m <sup>3</sup> /s
Inlet temperature of i <sup>th</sup> CSTR ( $T_{Ii}$ )	300° K
Cooling water temperature of i <sup>th</sup> CSTR ( $T_{CWi}$ )	300° K
Inlet flow rate in 2 <sup>nd</sup> CSTR ( $Q_{I2}$ )	0.261 m <sup>3</sup> /s
Inlet concentration of i <sup>th</sup> CSTR ( $C_{Ii}$ )	20 mol/m <sup>3</sup>
Cooling water flow rate in 1 <sup>st</sup> CSTR ( $Q_{CW1}$ )	0.45 m <sup>3</sup> /s
Cooling water flow rate in 2 <sup>nd</sup> CSTR ( $Q_{CW2}$ )	0.272 m <sup>3</sup> /s
Reaction rate in 1 <sup>st</sup> CSTR ( $K_1$ )	$K_0 \exp(-E/R T_{O1})$
Reaction rate in 2 <sup>nd</sup> CSTR ( $K_2$ )	$K_0 \exp(-E/R T_{O2})$

**Table 2:** Physical and process constants.

Variable	Value
Outlet valve constant of 1 <sup>st</sup> CSTR ( $K_{V1}$ )	$0.16 \text{ m}^{3/2} / \text{s}$
Outlet valve constant of 2 <sup>nd</sup> CSTR ( $K_{V2}$ )	$0.256 \text{ m}^{3/2} / \text{s}$
Heat transfer coefficient multiplied by the heat transfer area of $i^{\text{th}}$ CSTR ( $U_{ai}$ )	$0.35 \text{ m}^3/\text{s}$
Activation energy ( $E/R$ )	$6000^\circ \text{ K}$
Reaction heat coefficient ( $\Delta H$ )	$5 \text{ m}^3 \text{ K/mol}$
Arrhenius constant ( $K_0$ )	$2.7 \times 10^8 \text{ s}^{-1}$
Cooling jacket volume of both reactors ( $V_{11}, V_{12}$ )	$1 \text{ m}^3$

### 3.1 Prediction and Control horizon

The MPC was designed to predict  $N_P$  number of samples from the  $N_w^{\text{th}}$  sample. Based on this prediction horizon the MPC will generate a control horizon of  $N_C$  number of samples.

### 3.2 Cost Function

The cost function used in this work is given in (4)

$$J(k) = \sum_{i=N_w}^{N_P} \left[ \hat{y}(k+i|k) - r(k+i|k) \right]^T \cdot Q \cdot \left[ \hat{y}(k+i|k) - r(k+i|k) \right] + \sum_{i=0}^{N_C-1} \left[ \Delta u^T(k+i|k) \cdot R \cdot \Delta u(k+i|k) \right] \quad (4)$$

Where

$Q$  is the output error weight matrix

$R$  is the rate of change in control action weight matrix

$\hat{y}(k+i|k)$  - predicted plant output at  $k+i$ , given all measurements upto and including those at  $k$

$r(k+i|k)$  - output set point profile at  $k+i$ , given all measurements upto and including those at  $k$

$\Delta u(k+i|k)$  - predicted rate of change in control action at  $k+i$ , given all measurements upto and including those at  $k$

### 3.3 Constraints

In this the constraints (5) are imposed on the controller outputs, which are cooling water flow rate in both CSTRs.

$$0.05 \text{ m}^3 / \text{s} \leq Q_{cwi} \leq 0.8 \text{ m}^3 / \text{s} \quad (5)$$

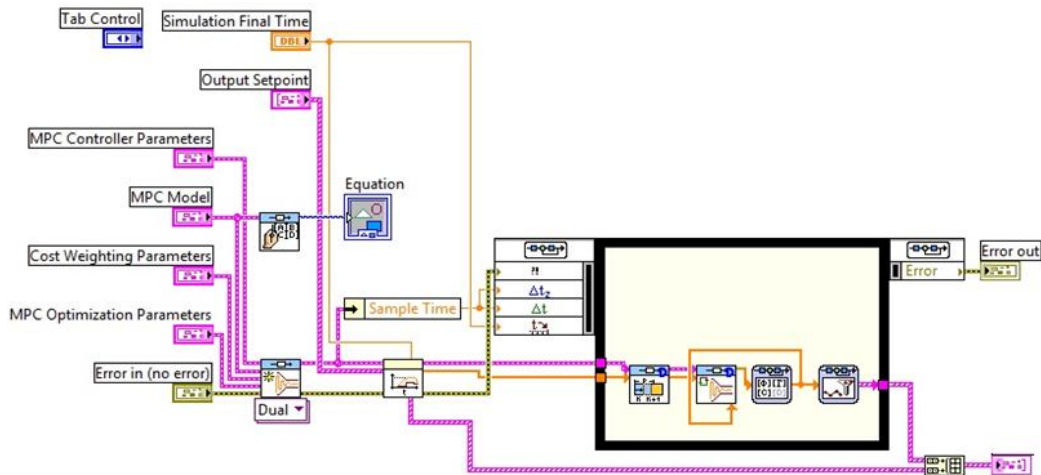
The MPC algorithm is implemented in LabVIEW using the control and simulation toolkit which has inbuilt MPC blocks for creating and implementing MPC controller.

The *CD create MPC controller* is used to create the MPC controller by giving inputs such as MPC controller parameters like prediction horizon and control horizon. Apart from that, a state space model of the process, MPC cost weights and constraints are also provided to *CD create MPC controller* block.

The *CD step forward MPC window* VI is used to calculate the appropriate portion, or window, of the set point and disturbance profile. This VI also moves the control and prediction horizon forward.

The *CD implement MPC controller* is used to calculate the control action to be applied to the plant which in this case is a discrete state space model of the plant.

The complete MPC implementation in LabVIEW is shown in figure.2. This VI can be viewed as two stages. The first stage prepares the system for MPC by collecting all MPC parameters from the user. The second stage enforces the control action, generated by the MPC scheme, on the non linear ODE model of the plant.



**Figure 2:** Block diagram.

#### 4. Result

The front panel was created using LabVIEW where all the MPC parameters are to be entered as per design. Linearized state space model given by (3) is inserted to the Virtual Instrument (VI) as shown in figure.3. This MPC was designed to have a prediction horizon of 25 samples and a control horizon of 3 samples. Entry of controller output constraints and cost function parameters are shown in figure.4. For the cost function the output error weight matrix ( $Q$ ) and the rate of change in control action weight matrix ( $R$ ) are chosen as identity matrices.

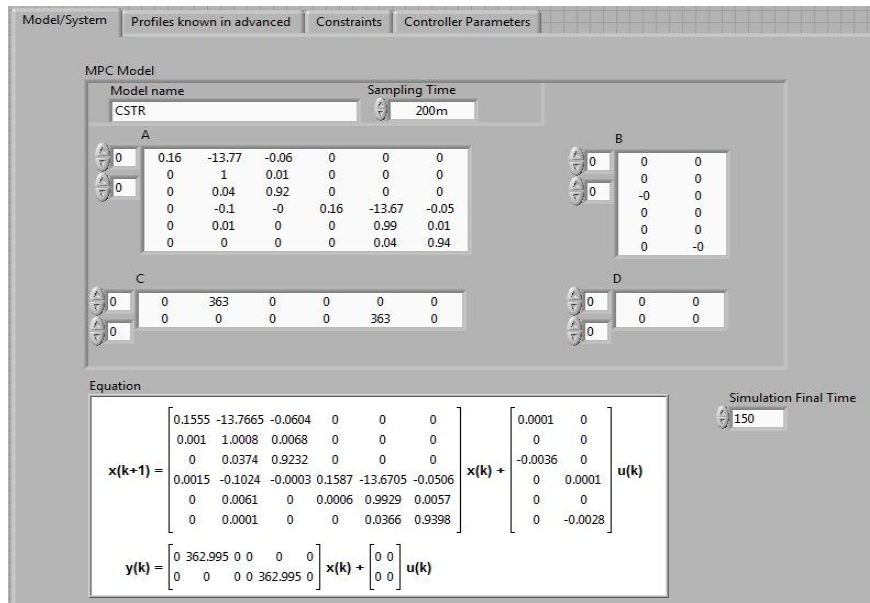


Figure 3: Front panel of VI for collection process model.

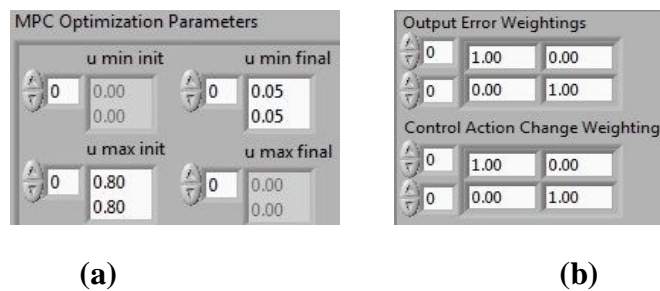


Figure 4: Numeric inputs from user for giving (a) Constraints and (b) Cost function parameters.

The MPC action on the cascaded CSTR’s ODE model was simulated for 150 seconds duration. During this period the reference signal for process output, which is inside temperature of both reactors in this case, was generated and given to MPC scheme as in figure.5. To test the robustness of the MPC scheme, both positive and negative step changes are incorporated.

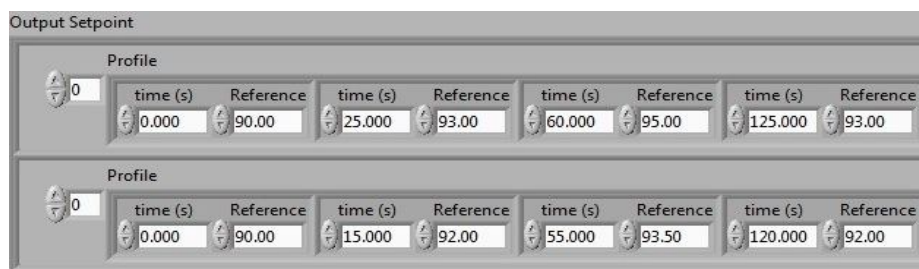
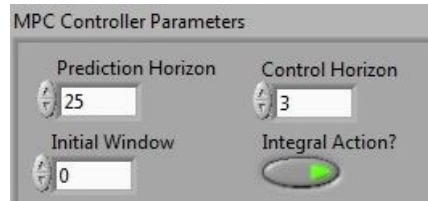
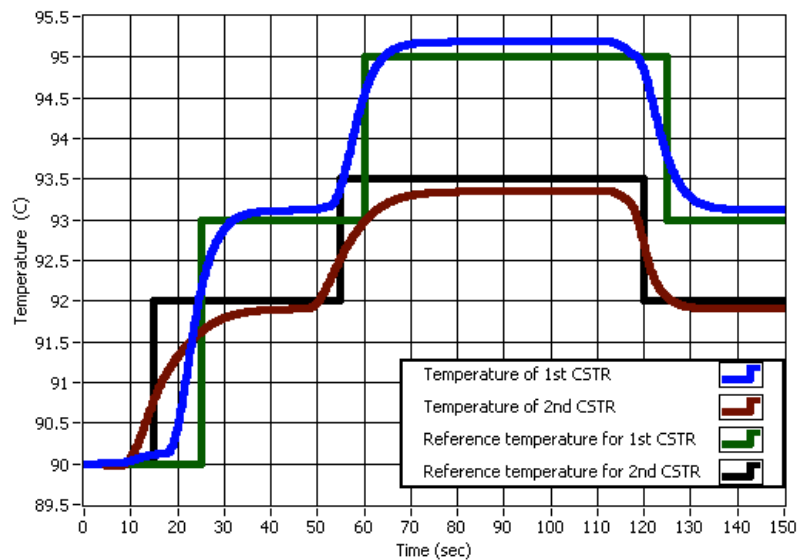


Figure 5: Set point profile to be tracked by the plant.



**Figure 6:** Prediction and control horizon.

Running the MPC implementation, shown in figure.2 for a 150 sec simulation period with a sampling interval of 0.2 sec produces the response as shown in figure.7. The plot shows both reference signal and process output, which are temperatures in both reactors.



**Figure 7:** Closed loop response of MPC scheme.

It can be clearly seen from the response graph that the response starts to change before there is an actual change in the set point profile which is due to the long range predictive ability of the MPC controller.

## Conclusion

This paper investigated the impact of model predictive control algorithm on the temperature parameter of a cascaded CSTR process. The response showed that the output changed even before there was an actual change in the set point profile, which is the distinguishing feature of model predictive control algorithm. This work can be continued by adding known disturbance in it during the MPC controller creation. Further, this implementation on LabVIEW platform eases the problem of handling real time control problems.



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