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Piezoelectric Materials Subjected to a Moving Heat Source under Fractional Order Equation of Motion Associated With Two Relaxation Times

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Abstract

In this work, we study the thermoelastic properties of an isotropic and homogeneous one dimensional semi-infinite piezoelectric material excited by a moving heat source and subjected to a sudden thermal loading in the light of the generalized thermoelasticity theory with fractional order strain. Laplace transformation method is implemented to solve the proposed system of equations. Thermally induced temperature, stress and strain distribution functions are determined in the Laplace domain. We used a complex inversion formula of Laplace transform based

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on a Fourier expansion to obtain the different inverse field functions numerically. The effects of different parameters; such as the speed, the strength of the moving heat source, fractional order and time on the field functions; thermodynamical temperature, stress and on the strain distribution, are studied and presented graphically.

AMS Subject Classification:
Keywords: Fractional order strain; moving heat source; fractional order equation of motion; piezoelectric materials; two relaxation times.

1. Introduction

Lord and Schulman [1] introduced the igirst generalized thermoelasticity theory of Biot [2] and Nowacki [3] theories. This generalization contains one relaxation time capable for describing the thermoelastic properties of isotropic materials and guarantee finite speeds of thermal waves. The second generalization is known as thermoelasticity theory with two relaxation times or thermoelasticity with temperature rate - dependent. In this second generalization; Green and Lindsay [4] introduced two parameters act as relaxation times. Thermoelasticity with two relaxation times also predicts finite speeds of thermal signals. Many reviews and research works appeared during the last decades to generalize and investigate deeply the applications of generalized thermoelasticity theories [5].

The concept of derivative and integral have been generalized to a non integer order and studied by many researchers. Various physical process and models have been implemented through application of fractional order derivatives [6]–[7].

The fractional order differential operator is non-local. This leads to the dependency of the current state of a system on its previous state.

Comparison between the very famous definition of fractional order due to Riemann-Liouville given by [6]:

$$ {}_{RL}D_t^{\beta} f(t) = \frac{d^n}{dt^n}\left[\frac{1}{\Gamma(n-\beta)}\int_0^t (t-\tau)^{n-\beta-1} f(\tau)d\tau\right] \; n-1<\beta<n, \qquad (1.1)$$

and the definition of Caputo [8]

$$ {}_{C}D_t^{\beta} f(t) = \frac{1}{\Gamma(n-\beta)}\int_0^t (t-\tau)^{n-\beta-1} \frac{d^n f(\tau)}{d\tau^n} d\tau \; n-1<\beta<n, \qquad (1.2)$$

where $\Gamma(\beta)$ is the gamma function; show that, Caputo fractional derivative has two major advantages. It allows traditional initial and boundary conditions to be included in the formulation of the problem and in addition the derivative of a constant is zero. However, if $f(0)=0$ then, the fractional derivative defined by Caputo and Riemann–Liouville are the same.

We will use the following form of the fractional order time derivative of Caputo:

$$L_C D_t^{\beta} f(t) = s^{(\beta - n)} L\{f^n(t)\},\ n - 1 < \beta < n, \tag{1.3}$$

where s is a complex parameter related to Laplace transform.

Using the fractional order derivative in the present work permits to the differential equations of the system to take into consideration the effects of the intermediate as well as the previous states to express the present and the next states of the medium.

Thus, the present model is a new model as it uses a new fractional order equation of motion together with equation (1.3) of the fractional order time derivative of Caputo to investigate the thermoelastic properties of an isotropic and homogeneous one dimensional semi-infinite piezoelectric ceramic material such as Barium Titanate with the molecular formula $BaTiO_3$ subjected to a thermal shock in the presence of an inner moving heat source.

The properties of $BaTiO_3$ have been reported Vijatovi et al [9] and references there on. Laplace transformation method is implemented to solve the proposed thermoelasticity model in one dimension. The effects of different parameters such as the speed v, the strength Qo of the moving heat source and the fractional order β on the field functions θ, σ and on e are presented graphically and investigated. According to the numerical results, some comparisons have been shown in figures to show the effect of these parameters on all variable fields and discussion has been established for the piezoelectric Barium Titanate material.

2. Formulation of The Problem

Consider an infinite homogeneous isotropic thermoelastic piezoelectric solid occupying half space $x \geq 0$. The medium is subjected to a moving source of heat of intensity Q(x, t). Following Ezzat et al [10] and Youssef [11], the generalized thermopiezoelasticity with two relaxation times is described by the following system; the constitutive equation:

$$\sigma_{ij} = 2\mu(1 + \tau^{\beta} D_t^{\beta})e_{ij} + \lambda(1 + \tau^{\beta} D_t^{\beta})e_{kk}\delta_{ij} - \gamma\hat{\theta}\delta_{ij} - h_{ijk}B_k,\ \beta > 0, \tag{2.1}$$

with

$$\hat{\theta} = \theta - \nu_o \frac{\partial \theta}{\partial t} \tag{2.2}$$

where λ and μ are the known Lamé constants and ν_o is the second relaxation time, h_{ijk} is the piezoelectric coefficients, B_k is the component of the electric displacement. In the context of the theory of generalized thermoelasticity with fractional order, the equations of motion in the absence of body forces has the form:

$$\mu(1 + \tau^{\beta} D_t^{\beta})u_{i,jj} + (\lambda + \mu)u_{j,ji} - \gamma\theta = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{2.3}$$

If the thermal conductivity K is constant, the heat conduction equation takes the form;

$$K\ \theta(x,t)_{,}ii = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)(\rho C_E \theta(x,t) + \gamma T_o(1+\tau^\beta D_t^\beta)e(x,t)_{ii}) - \rho\left(1+\tau_o \frac{\partial}{\partial t} Q\right), \tag{2.4}$$

where τ_o is the relaxation time, ρ is the density, C_E is the specific heat at constant strain, T_o is the references temperature, $\gamma = \alpha_T(3\lambda + 2\mu)$, α_T is the thermal linear expansion, $\theta = T - T_o$ is the temperature increment such that $\theta/T_o \ll 1$, e is the cubic dilatation and Q is the moving heat source.

It is known in industrial applications of laser, in general, that the size of the laser spot at the work piece is small and the absorption depth of the work piece is considerably smaller than the thickness of the work piece. Consequently, one-dimensional heating situation may become appropriate to formulate the heating problem. So we consider a half-space$x \geq 0$ with the x-axis pointing into the medium with initial temperature distribution T_o. The moving heat source with constant strength starts to evolve its energy continuously while moving along the axial direction with a constant velocity v. The thermal changes of the material is assumed to take place along x-direction. Thus, without any lose of generality, we can assume that the temperature θ, the stress σ, the displacement u and the strain e are functions of time and the axial coordinate x. We assume that there is no body forces affecting the medium and all the state functions initially are equal to zero. For the present problem, the generalized thermoelastic governing differential equations, in the absence of body force, and free charge, for the piezoelectric materials can be considered in the following one dimensional linearized system of equations; the displacement distribution;

$$u_x = u\,(x,t)\,,\ u_y = u_z = 0,\ \theta = \theta(x,t), \tag{2.5}$$

where u_x is the component of the displacement vector, the constitutive equation carrying the second relaxation time ν_o becomes;

$$\sigma(x,t) = (1+\tau^\beta D_t^\beta)(\lambda + 2\mu)e(x,t) - \gamma\left(\theta(x,t) - \nu_o \frac{\partial \theta(x,t)}{\partial t}\right) - hB, \tag{2.6}$$

the equation of motion;

$$\rho \frac{\partial^2 e(x,t)}{\partial t^2} = (\lambda + 2\mu)\,(1+\tau^\beta D_t^\beta)\frac{\partial^2 e(x,t)}{\partial x^2} - \gamma \frac{\partial^2 \theta(x,t)}{\partial x^2}, \tag{2.7}$$

the heat equation;

$$\frac{\partial^2 \theta(x,t)}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_E\ \theta(x,t)}{K} + \frac{T_o \gamma}{K}(1+\tau^\beta D_t^\beta)e(x,t)\right) - \frac{\rho}{K}\left(1+\tau_o \frac{\partial}{\partial t}\right)\ Q, \tag{2.8}$$

and the strain - displacement relation;

$$e(x,t) = \frac{\partial u(x,t)}{\partial x}, \tag{2.9}$$

where λ, μ are Lamé constants and σ is the principal stress component.

For writing the governing equations(2.6)-(2.9) in a dimensionless form, we introduce the following nondimensional variables,

$$\begin{aligned}
u' &= c_o\eta u,\ x' = c_o\eta x,\ \sigma' = \frac{\sigma}{\lambda+2\mu}, \\
\theta' &= \frac{\gamma\ \theta}{\lambda+2\mu},\ t' = c_o^2\eta t,\ B' = \frac{h}{\lambda+2\mu}B \\
Q' &= \frac{Q}{KT_o c_o^2\eta^2},\ \tau' = c_o^2\eta\tau, \\
\tau'^{\beta} &= c_o^2\eta\tau^{\beta},\ \tau_o' = c_o^2\eta\tau_o,\ \nu_o' = c_o^2\eta\nu_o,
\end{aligned} \tag{2.10}$$

where $c_o = \sqrt{(\lambda+2\mu)/\rho}$ is the longitudinal wave speed; and $\eta = \rho C_E/\kappa$ is the thermal viscosity. Using (2.10) and dropping the prime, for convenience, in the equations (2.6)-(2.7), the constitutive equation (2.6) becomes;

$$\sigma(x,t) = (1+\tau^{\beta}D_t^{\beta})e(x,t) - \omega\left(\theta(x,t) - \nu_o\frac{\partial\theta(x,t)}{\partial t}\right) - B, \tag{2.11}$$

the non-dimensional equation of motion becomes;

$$\frac{\partial^2 e(x,t)}{\partial t^2} = (1+\tau^{\beta}D_t^{\beta})\frac{\partial^2 e(x,t)}{\partial x^2} - \omega\frac{\partial^2\theta(x,t)}{\partial x^2}, \tag{2.12}$$

while the heat equation takes the form;

$$\frac{\partial^2\theta(x,t)}{\partial x^2} = \left[\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right](\theta(x,t)\ + \xi(1+\tau^{\beta}D_t^{\beta})e(x,t)) - \left(1+\tau_o\frac{\partial}{\partial t}\right)\ Q, \tag{2.13}$$

and the non dimensional strain - displacement relation (2.9) is

$$e(x,t) = \frac{\partial u(x,t)}{\partial x}, \tag{2.14}$$

where $\xi = \gamma/\rho C_E$ and $\omega = \gamma\ T_o/(\lambda+2\mu)$, are non dimensional constants.

The moving heat source Q in non-dimensional form is

$$Q = Q_o\delta(x - vt), \tag{2.15}$$

where Q_o and v are the strength and the speed of the heat source respectively and $\delta(.)$ is the known Dirac delta function.

The non dimensional system of equations (2.11)-(2.14) can be considered as a new fractional order thermoelastic model with two relaxation times and fractional order equation of motion being used for studying the thermoelastic behaviour of an isotropic and homogeneous piezoelectric medium subjected to a moving heat source given by the equation (2.15). The procedure for the solution is as follows; first we solve the nondimensional system of equations (2.11)-(2.13) analytically to obtain their solution in the domain of Laplace, then we find the inverse field functions numerically. So; we start with applying the Laplace transform together with the form (1.3) of the fractional order derivative to these equations, we get the following generalized thermoelasticity system of equations with two relaxation times in the domain of Laplace based on the fractional order strain equation of generalized thermoelasticity:

The transformed constitutive equation takes the forms;

$$\overline{\sigma}(x,s) = (1 + s^{\beta}\tau^{\beta})\,\overline{e}(x,s) - \omega\,(1 + \nu_o s)\overline{\theta}(x,s) - \frac{B}{s}. \tag{2.16}$$

Next, the equation of motion in the domain of Laplace becomes;

$$\frac{\partial^2 \overline{e}(x,s)}{\partial x^2} = \frac{1}{1+s^{\beta}}\left[s^2\,\overline{e}(x,s) - \omega\,\frac{\partial^2 \overline{\theta}(x,s)}{\partial x^2}\right], \tag{2.17}$$

while the transformed energy equation assumes the form;

$$\frac{\partial^2 \overline{\theta}(x,s)}{\partial x^2} = s(s\tau_o + 1)[(\varepsilon_1 \xi(\tau^{\beta}s^{\beta} + 1)\,\overline{e}(x,s)\; + \overline{\theta}(x,s)] - \Omega\, e^{-sx/v}, \tag{2.18}$$

while the strain - displacement constitutive relation in the domain of Laplace becomes;

$$\overline{e}(x,s) = \frac{\partial \overline{u}\,(x,s)}{\partial x}. \tag{2.19}$$

Combining equations (2.12) and (2.11) and applying the Laplace transform leads to the following stress - strain relation in the domain of Laplace;

$$\frac{\partial^2 \overline{\sigma}(x,s)}{\partial x^2} = s^2 \overline{e}(x,s), \tag{2.20}$$

where $\Omega = (1 + s\;\tau_o)Q_o/v$ and $\varepsilon_1 = \dfrac{\gamma^2 K}{\rho C_E(\lambda + 2\mu)}$.

3. Solution in the Laplace Domain

Eliminating $\overline{e}$ between the (2.17) and (2.18), we get the following fourth order non homogeneous differential equation;

$$ce^{-sx/v} + b\overline{\theta} - a\frac{\partial^2 \overline{\theta}}{\partial x^2} + \frac{\partial^4 \overline{\theta}}{\partial x^4} = 0, \tag{3.1}$$

where

$$
\begin{aligned}
a &= s\left[\frac{s}{1+s^{\beta}\tau^{\beta}}+(1+s\tau_o)(1+\varepsilon_1\xi\omega)\right]\\
b &= \frac{s^3(1+s\tau_o)}{1+s^{\beta}\tau^{\beta}}\\
c &= \frac{\Omega s^2(1+s^{\beta}\tau^{\beta}-v^2)}{v^2(1+s^{\beta}\tau^{\beta})}
\end{aligned}
$$

According to the present formulation of the problem the most general solution of (3.1) satisfies the boundary conditions (4.1) and (4.2) takes the form;

$$\overline{\theta}(x,s)=\sum_{i=1}^{2}\theta_i e^{-k_i x}+\psi e^{-sx/v}, \tag{3.2}$$

where

$$\psi=-cv^4/(s^4-as^2v^2+bv^4),$$

θ_i are parameters, depending on s, to be determined from the the boundary conditions, $\pm k_1$ and $\pm k_2$ are the roots of the characteristic equation;

$$b-ak^2+k^4=0.$$

Hence the general solutions of the equations (2.17) and (2.16)are found respectively as;

$$\overline{e}=\sum_{i=1}^{2}e_i\, e^{-k_i x}+f_e(s)\, e^{-sx/v}, \tag{3.3}$$

and

$$\overline{\sigma}=\sum_{i=1}^{2}\sigma_i\, e^{-k_i x}+f_\sigma(s)\, e^{-sx/v}, \tag{3.4}$$

where e_i, σ_i, $f_e(s)$ and $f_\sigma(s)$ are parameters, depending on s, to be determined by using the boundary conditions. The equations (3.2) and (3.3)-(3.4) represent the complete solution of the system (2.16)-(2.18) in the Laplace transform domain.

4. Determination of the Parameters

To determine the previous parameters, we assume that the medium is initially at rest and has reference temperature T_o so that the initial conditions are given by;

$$
\begin{aligned}
\theta(x,0) &= 0,\ \ e(x,0)=0,\ \ \sigma(x,0)=0,\\
\partial\theta(x,0)/\partial t &= 0, \partial e(x,0)/\partial t=0,\ \ \partial\sigma(x,0)/\partial t=0,
\end{aligned}
$$

and the medium is subjected to a thermal loading with general function F(t), so that the boundary conditions at the near end $x = 0$ is given by;

$$\theta(0,t) = F(t),\ \sigma(0,t) = 0, \tag{4.1}$$

while the boundary conditions at $x = \infty$ are,

$$\theta(\infty,t) = 0, \quad , \sigma(\infty,t) = 0, \qquad 0 < t < \infty, \tag{4.2}$$

We consider that the half space $x \geq 0$ is subjected to a thermal shock at $x = 0$, so that the boundary condition in (4.1) becomes;

$$F(t) = F_o\ H(t), \tag{4.3}$$

where F_o being constant represents the strength of the thermal shock on the boundary at $x = 0$ and $H(t)$ is the Heavyside unit step function. Using the dimensionless variables (2.10) and applying the Laplace transform to the boundary conditions (4.2) and (4.3) we obtain;

$$\begin{aligned} \overline{\theta}(0,s) &= \overline{F}(s) = \frac{F_o}{s}, \qquad \overline{\sigma}(0,s) = 0, \\ \overline{\theta}(\infty,s) &= 0,\ \overline{\sigma}(\infty,s) = 0, \end{aligned} \tag{4.4}$$

Similarly the dimensionless initial conditions in the domain of Laplace can be obtained. By applying these conditions to (3.2), (3.3) and (3.4), the parameters $\theta_i,\ e_i,\ \sigma_i,\ f_e(s)$ and $f_\sigma(s)$ are obtained as given below;

$$\theta_1 = \frac{d - Q_o v(1 + s\tau_o) + k_2^2 v^2(\psi - \varphi_o) + s(M - s\psi)}{v^2(k_1^2 - k_2^2)}, \tag{4.5}$$

$$\theta_2 = \frac{d - Q_o v(1 + s\tau_o) + k_1^2 v^2(\psi - \varphi_o) + s(M - s\psi)}{v^2(k_1^2 - k_2^2)}, \tag{4.6}$$

$$\begin{aligned} e_i = \frac{\theta_i(k_i^2 - s\ (1 + s\tau_o))}{s\varepsilon_1\xi(1 + s\tau_o)(1 + s^\beta\tau^\beta)},\ \sigma_i &= \frac{\theta_i[k_i^2 - s\ L]}{s\ \varepsilon_1\xi(1 + s\tau_o)}, \\ i &= 1, 2, \end{aligned} \tag{4.7}$$

$$f_e(s) = \frac{Q_o v((1 + s\tau_o) - s(v^2 + s(-1 + v^2\tau_o)))}{s v^2\varepsilon_1\xi(1 + s\tau_o)(1 + s^\beta\tau^\beta)},$$

and

$$f_\sigma(s) = \frac{Q_o v(1 + s\tau_o) + s\psi(s - v^2(1 + \nu_o)\xi\omega)}{s v^2\varepsilon_1\xi(1 + s\tau_o)} \tag{4.8}$$

where

$$\begin{aligned} M &= v^2(1+s\tau_o)\varphi_o L,\ L = (1+\varepsilon_1(1+s\nu_o))\xi\omega, \\ d &= Bv^2\varepsilon_1\xi(1+s\tau_o),\ \varphi_o = F_o/s. \end{aligned}$$

After substituting the parameters given by (4.5) - (4.8) into (3.2), (3.3) and (3.4), we obtain the complete solution in the in the Laplace domain of the non-dimensional field functions; temperature, stress and strain respectively.

5. Numerical Inversion of the Laplace Transform

To obtain the solutions of the non-dimensional field functions; temperature, stress and strain the equations (3.2), (3.3) and (3.4) must be inverted back to the time domain. Therefore, we compute numerically the inverse of these field functions by a method based on Fourier expansion technique. In this technique, any function $\overline{f}(s)$ is inverted back to the original function $f(t)$ in the time domain as given below;

$$f(t) = \frac{\exp(ct)}{t_1}\left[\frac{1}{2}\overline{f}(c) + \Re\left(\sum_1^N\right)\overline{f}\left(c + \frac{ik\pi}{t_1}\right)\exp\left(\frac{ik\pi}{t_1}\right)\right],\ 0 < t_1 < 2t, \tag{5.1}$$

where $\Re$ is the real part, i is imaginary number unit and N is a sufficiently large integer representing the number of terms in the truncated Fourier series chosen such that;

$$\exp(ct)\Re\left[\overline{f}\left(c + \frac{iN\pi}{t_1}\right)\exp\left(\frac{iN\pi t}{t_1}\right)\right] \leq \epsilon_1, \tag{5.2}$$

where ϵ_1 is prescribed small positive number that corresponds to the degree of accuracy required. The parameter c is a positive free parameter that must be greater than the real part of all the singularities of $\overline{f}(s)$. The optimal choice of c was obtained according to the criteria described in Honig and Hirdes [13]. Details about the analysis of the formula (5.1) can be found in [14]

6. Numerical Results and Discussion

For numerical computations, we use the following physical constants for Barium Titanate materials extracted from [12]

$k = 386 N/K sec$, $B = 10^{-4}$, $\gamma = 2.44(10)^6 N K^{-1} m^{-2}$, $C_E = 188\ m^2/K$, $\mu = 4.4(10)^{10}\ N/m^2$, $\lambda = 6.6(10)^{10}\ N/m^2$, $\rho = 4200\ kg/m^3$, $\tau_0 = 0.002\ sec$, $\nu_o = 0.003$, $T_o = 293\ K$.

In all figures, the dotted black lines represent the case when the parameter takes the minimum value, the blue dash dotted lines represent the case when the parameter takes the maximum value, the solid black and the red dashed lines represent the case when the value of the parameter lie between the minimum and the maximum value.

We investigate the distributions of the thermodynamical temperature θ, the normal stress component σ and the strain distribution e for different values of the parameters such as the speed v and the strength of the heat source Qo and fractional order β. The results were presented in four groups of figures, each group presents the effect of one of the parameters v, Qo and β.

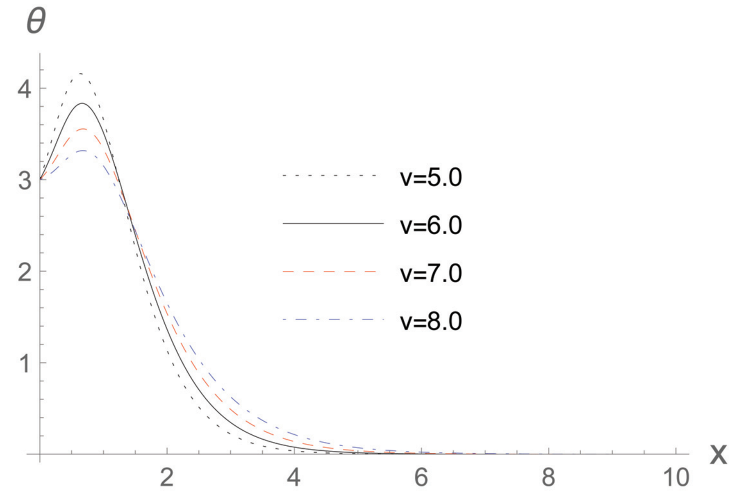
Figures (1) show the dependence of the thermodynamical temperature θ, the stress σ and the strain e on the speed v of the moving heat source.

Figure 1(a) illustrates that the increases in the speed of the moving heat source decreases the magnitude of the amplitude of the thermodynamical temperature especially near the point of application of the thermal load at $x = 0$. It is also noticed that the peak point of the temperature distribution curve occur at the same point. Near the point $x = 1.8$ it is noticed that the amplitude of the temperature starts increasing with increasing the speed of the moving heat source. It can also be seen from figure 1(a) that the temperature decay inside the material starts gradually and it decays exponentially towards the far end of the material, which is associated with the changes in the internal energy due to the irradiated field in the region.

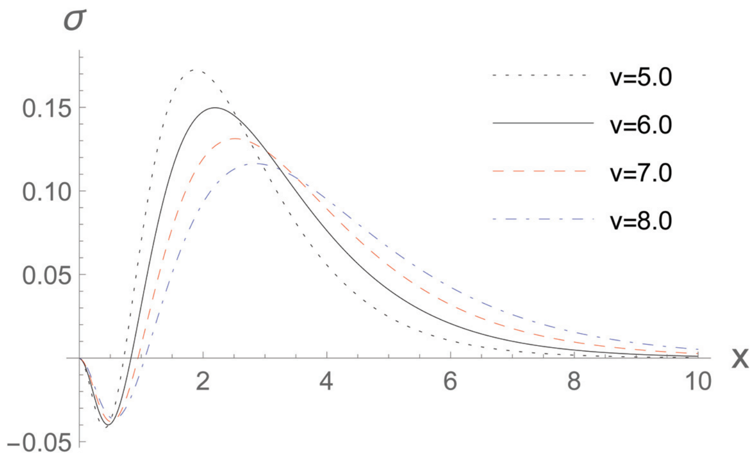
Figure 1(b) shows that the stress component initially decreases rapidly near the point $x = 0$ where the shock wave is applied on the medium and starts to change its type at the point $x = 0.5$. For the values $0.5 \leq x \leq 3$ the amplitude of the stress distribution decreases with increasing the speed of the heat source until the medium starts to change its type and decays beyond the point $x \simeq 0.3$ along the x- axis. We notice that as the amplitude of the temperature θ starts to decrease with increases the speed v the amplitude of the stress distribution function starts also to decreases. The peak points of the stress distribution curve are shifted away from the point of application the thermal load. The stress reaches the equilibrium state at the same point; $x \geq 10$ for any value of the speed v. Figure 1 (b) shows also that the numerical solution of the stress function satisfies the boundary conditions.

Figure 1(c) shows that the strain distribution curve resembles the same behaviour of the stress distribution function except at the point of application of the thermal load at $x = 0$. Comparison between figure 1(a), figure 1(b) and figure 1(c) shows that the temperature function attain its equilibrium state faster than the strain and stress distribution functions.

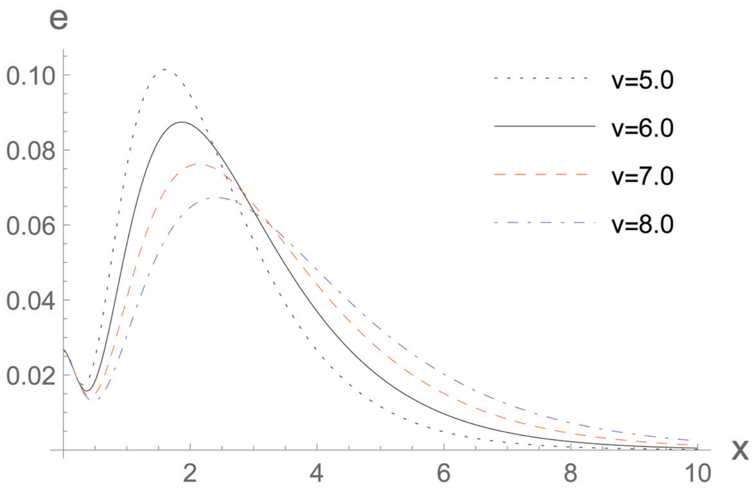
Figures 2 show that effect of the strength of the heat source Qo on the field functions; θ, σ and e. Figure 2(a) reflects the profile of the thermodynamical temperature along x - axis. It is noticed that the amplitude of the temperature distribution function increases with increasing the value of the strength of the heat source along the x-axis. We further noticed that the peak attain its maximum at the same point. It was also found that the profile of the temperature distribution function depends strongly on Qo. Figure 2(b) illustrate the behaviour of the stress function on the strength of the heat source. The stress distribution function vary proportionally with the value of Qo along all value of the x-axis until it reaches the equilibrium state. It is also noticed that the peak occurs



Thermodynamical temperature

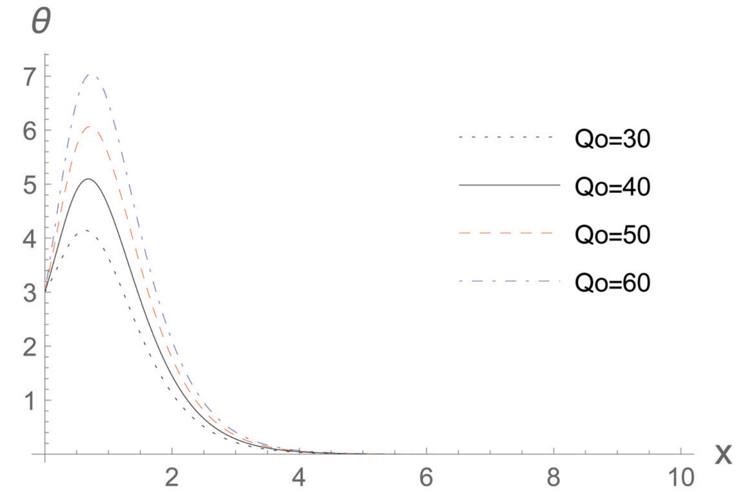


Stress distribution



Strain distribution

Figure 1: Effect of speed v on θ, σ and e for $t = 0.3,\ \beta = 0.5,\ \tau_o = 0.002\ and\ \nu_o = 0.003$



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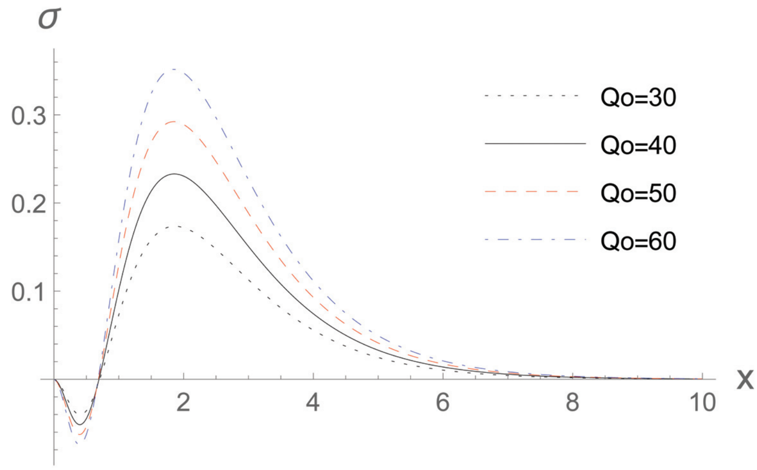
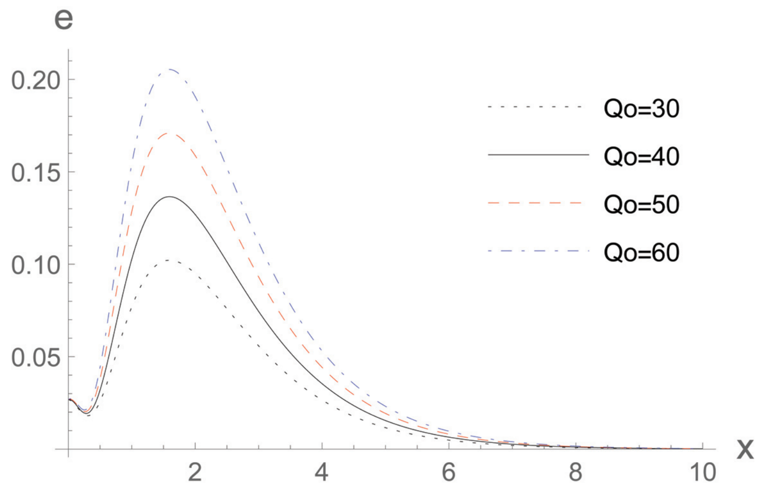


Stress distribution



Strain distribution

Figure 2: Effect of Strength of Heat Source Qo on θ, σ and e for $t = 0.3, \beta = 0.5,\ v = 5,\ \tau_o = 0.002\ and\ v_o = 0.003$

at the same point for all values of Qo. Comparison with the figure 2(c) shows that the strain distribution function resembles the same behaviour of the stress function except at $x = 0$.

Figures 3 show the effect of the fractional order parameter β on the field functions; θ, σ and e along the x-axis. Comparisons with figure 1 and figure 2 show that the fractional order parameter has less significant effect on the temperature θ and on the stress σ as the parameters v and Qo but figure 3(c) shows that the strain distribution function depends strongly on the fractional order parameter β. It is also noticed that increasing the value of the fractional order parameter β leads to shift the peaks points of the strain amplitude to higher value.

Figures 4 show the represent the effect of time t on the field functions; θ, σ and e. Figure 4(a) reflects the profile of the thermodynamical temperature along x - axis. It was found that the profile of the temperature distribution function depends strongly on t. It is also noticed that increasing the time t does not shift the peaks points of the temperature profile. It is also noticed that the effect of time on the temperature takes place after the temperature reaches its maximum value. It can also be seen that after the temperature attains its maximum the amplitude of the temperature increases with increasing t.

Figure 4(b) illustrates the effect of the time t on the stress function. The stress behaves in two different manner before and after it reaches the peak points. that is , while the amplitude decreases with increasing the time before the peak points, it increases with increasing the value of t after this points. We further noticed that it reaches the equilibrium state at $x = 10$ while the temperature reaches faster to its equilibrium state at $x = 5$. Figure 4(c) shows that the strain resembles the same behaviour of the stress. It is also noticed that increasing the value of time t results in shifting the peak points of the strain function.

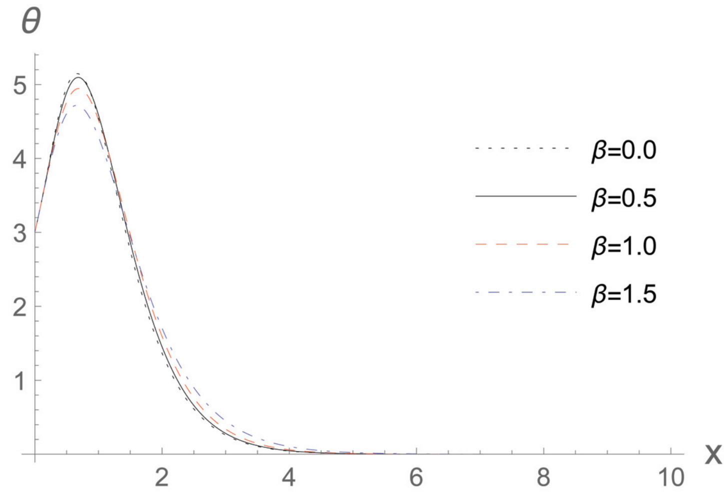
7. Conclusion

The effect of the parameters v, Qo and β on the behaviour of the field functions θ, σ and e are studied and it is noticed that:

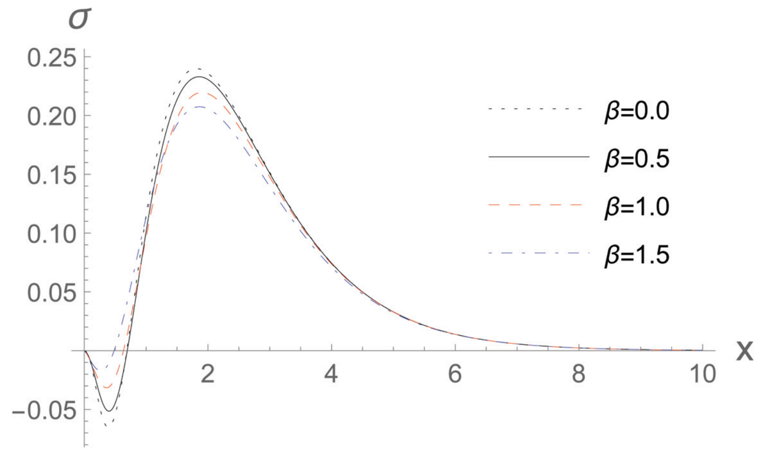
The strain distribution function resembles the same behaviour of the stress under the variation of all parameters. It depends strongly on the fractional order parameter β. The profile of the thermodynamical temperature changes slightly with the variation of the fractional order parameter β. The stress distribution function behaves in two different manners before and after the peak points only with the parameters v.

The increase of the value of the speed v of the moving heat source results in decreasing the absolute value of the amplitude of these field functions.

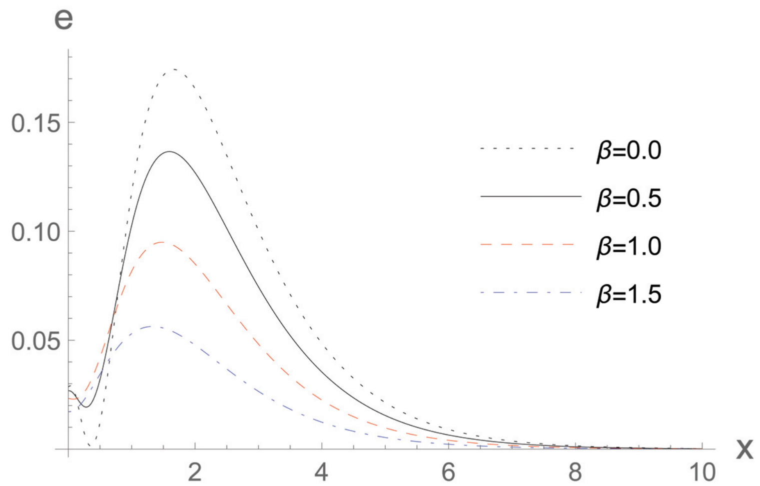
The temperature attains its equilibrium state faster than the other field function with the variation of all parameters. Each of the field functions; the temperature θ, the stress σ and the strain e attains its equilibrium state at the same point under the variation of any parameter.



Thermodynamical temperature

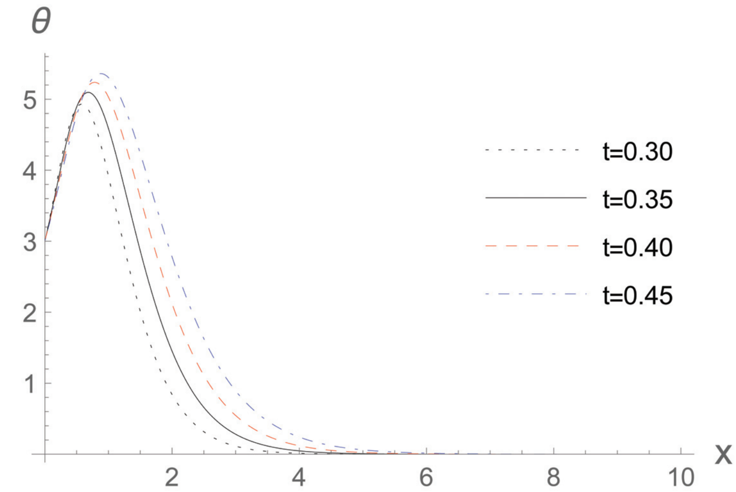


Stress distribution

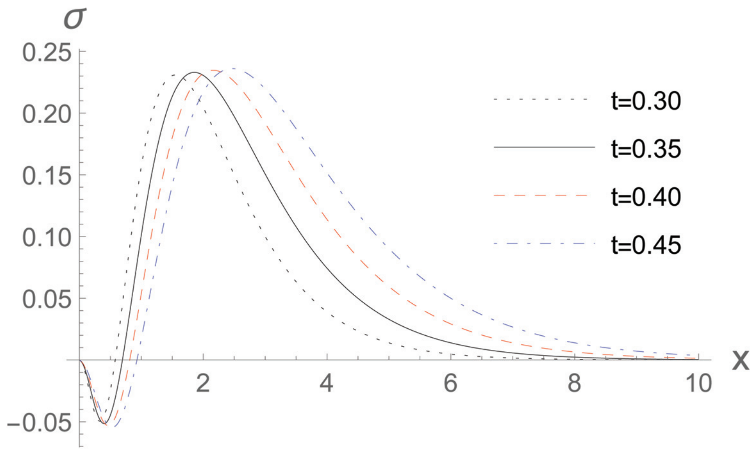


Strain distribution

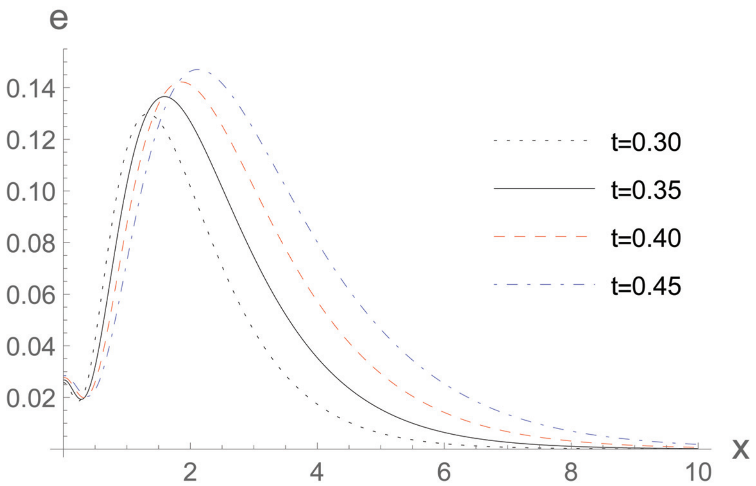
Figure 3: Effect of fractional order parameter β on θ, σ and e for $t = 0.t,\ v = 5,\ \tau_o = 0.002\ and v_o = 0.003$



Thermodynamical temperature



Stress distribution



Strain distribution

Figure 4: Effect of time t on θ, σ and e for $\beta = 0.5,\ \nu = 5,\ \tau_o = 0.002\ and\ \nu_o = 0.003$

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