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Independent Attributes for m-Concepts in a Soft Context Induced by a Soft Set

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Abstract

For the purpose of studying more effective ways of finding the reduction in a formal context, we have combined the formal contexts with the soft sets to form so-called soft contexts, and proposed the notion of soft concepts And to study the structure of soft contexts, we introduced a new type of soft concept (called m-concept or object oriented soft concept) based on soft sets and the set of all m-concepts. In this paper, we introduce and study the notion of m-dependent and m-independent attributes in a given soft context. And, we show that every m-dependent attribute is generated by some m-independent attributes and the family of all m-independent attributes generates all m-concepts in a given soft context. Finally, we show that a reduction of a soft concept lattice is obtained by the family of all m-independent attributes.

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1. Introduction

Wille introduced the formal concept analysis in [18], which is an important theory for the research of information structures induced by a binary relation between the set of attributes and objects attributes. The basic notions of formal concept analysis are formal context, formal concept, and concept lattice. A formal context is a kind of information system, which is a tabular form of an object-attribute value relationship [3,4, 6, 7]. A formal concept is a pair of a set of objects as called the extent and a set of attributes as called the intent. The set of all formal concepts together with the order relation forms a complete lattice called the concept lattice [6,17]. Formal concept lattice is the core data structure and a kind of a formal knowledge representation.

Molodtsov introduced the notion of soft set in 1999 [15], which is to deal complicated problems and uncertainties. Maji et al. introduced the operations for soft set theory in [12]. In [1], Ali et al. proposed new operations modified some concepts introduced by Maji. Until recently, researches combining soft sets with other mathematical concepts have been extensively studied. [2,4,5,11,13,16]

In [14], we have formed a soft context by combining the concepts of the formal context and the soft set defined by the set-valued mapping. And we introduced and studied the new concepts named soft concepts and soft concepts lattices. Furthermore, in [8], we introduced some operations on a parameter set of a soft set, and studied some properties of such notions. In [9], for a soft set over a universe set, we investigated a special operation induced by two operations defined in [8], and studied some related properties and several characterizations. And also, by using the two operation, we investigated the new concept of m-concepts related closely the object oriented concept in formal context, and showed that the family of all the m-concepts in a soft context is a supra topology but not a topology. Moreover, we studied the notion of independent and dependent m-concept. In particular, we showed that the set of all independent m-concepts completely determines every m-concept in a soft context and the smallest base for the set of all soft concepts as a supratopological structure.

In this paper, we introduce and study the notion of m-dependent and m-independent attributes in a given soft context (Definition 3.1). And, we show that every m-dependent attribute is generated by some m-independent attributes (Theorem 3.9) and the family of all m-independent attributes generates all m-concepts in a given soft context (Theorem 3.13). Finally, we show that a reduction of a soft concept lattice is obtained by the family of all m-independent attributes (Theorem 3.16).

2. Preliminaries

A formal context is a triplet (U, V, I), where U is a non-empty finite set of objects, V is a nonempty finite set of attributes, and I is a relation between U and V. Let (U, V, I) be a formal context. For a pair of elements $x \in U$ and $y \in V$, if $(x, y) \in I$, then it means that object x has attribute y and we write xIy. The set of all attributes with a given object $x \in U$ and the set of all objects with a a given attribute $y \in V$ are denoted as the following [17,18]:

$$x^* = \{y \in V | xIy\}; \; y^* = \{x \in U | xIy\}.$$

And, the operations for the subsets $X \subseteq U$ and $Y \subseteq V$ are defined as:

$$X^* = \{y \in V | \textit{for all } x \in X, xIy\}; \;\; Y^* = \{x \in U | \textit{for all } y \in Y, xIy\}.$$

In a formal context (U, V, I), a pair (X, Y) of two sets $X \subseteq U$ and $Y \subseteq V$ is called a *formal concept* of (U, V, I) if $X = Y^*$ and $X = Y^*$, where X and Y are called the *extent* and the *intent* of the formal concept, respectively.

Let U be a universe set and E be a collection of properties of objects in U. We will call E *the set of parameters* with respect to U.

A pair (F, E) is called a *soft set* [15] over U if F is a set-valued mapping of E into the set $P(U)$ of all subsets of the set U, i.e.,

$$F : E \to P(U).$$

In other words, for $a \in E$, every set $F(a)$ may be considered as the set of a-elements of the soft set (F, E).

Let $U = \{z_1, z_2, \ldots, z_m\}$ be a non-empty finite set of *objects*, $E = \{e_1, e_2, \ldots, e_n\}$ a non-empty finite set of *attributes*, and $F : E \to P(U)$ a soft set. Then the triple (U, E, F) is called *a soft context* [14].

And, in a soft context (U, E, F), we introduced the following mappings:

For each $Z \in P(U)$ and $Y \in P(E)$,

(1) $\mathbf{F}^+ : P(E) \to P(U)$ *is a mapping defined as* $\mathbf{F}^+(Y) = \cap_{y \in Y} F(y)$;

(2) $\mathbf{F}^- : P(U) \to P(E)$ is a mapping defined as $\mathbf{F}^-(Z) = \{a \in E : Z \subseteq F(a)\}$;

(3) $\Psi : P(U) \to P(U)$ is an operation defined as $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z)$.

Then Z is called a *soft concept* [14] in (U, E, F) if $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z) = Z$. The set of all soft concepts is denoted by $sC(U, E, F)$.

In [10], we introduced the notion of m-concepts which is independent of the notion of soft concepts to each other as the following: For each $X \in P(U)$,

$\mathfrak{F} : P(U) \to P(U)$ is an operation defined by $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X)$,

where two operators $\mathbb{F} : P(A) \to P(U)$ and $\overleftarrow{\mathbb{F}} : P(U) \to P(A)$ are defined by :

$\mathbb{F}(C) = \cup_{c \in C} F(c)$; $\overleftarrow{\mathbb{F}}(X) = \{c \in A : F(c) \subseteq X\}$.

Then for $X \in P(U)$, X is called an *m-concept* (or *object oriented soft concept*) in (U, A, F) if $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = X$.

The set of all m-concepts is denoted by $m(U, A, F)$.

Theorem 2.1 ([10]) *Let (U, A, F) be a soft context. Then we have:*

(1) $\mathfrak{F}(\emptyset) = \emptyset$.

(2) $\mathfrak{F}(X)$ is an m-concept.

(3) For $B \subseteq A$, $\mathbb{F}(B)$ is an m-concept.

(4) For $a \in A$, $F(a)$ is an m-concept.

(5) X is an m-concept if and only if there is some $B \subseteq A$ such that $X = \mathbb{F}(B)$.

In [10], we introduced the notion of independent and dependent soft concepts: Let (U, A, F) be a soft context. Then for $Z \in m(U, A, F)$,

(1) Z is said to be *dependent* on $m(U, A, F)$ if there exist $Z_1, \cdots, Z_n \in m(U, A, F)$ satisfying $Z_i \subsetneq Z$ and $Z = \cup Z_i$, $i = 1, \cdots, n$.

(2) Z is said to be *independent* of $m(U, A, F)$ if Z is not dependent.

We will denote:

$mD = \{Z \in m(U, A, F) \mid X \text{ is dependent on } m(U, A, F)\}$;

$mI = \{Z \in m(U, A, F) \mid X \text{ is independent of } m(U, A, F)\}$.

Theorem 2.2 ([10]) *Let (U, A, F) be a soft context. Then*

(1) $mD \cap mI = \emptyset$; $mD \cup mI = m(U, A, F)$.

(2) For each $X \in mD$, there is a family $\mathcal{B} \subseteq mI$ satisfying $X = \cup\mathcal{B}$.

(3) For $Z \in mI$, there is $c \in A$ satisfying $F(c) = Z$.

3. Main Results

First, we study the notion of m-dependent and m-independent attributes in a given soft context. And, we show that the family of all m-independent attributes is a base for the set of all m-concepts in a given soft context. Finally, we show that a reduction of a soft concept lattice $mL(U, A, F)$ is obtained by the family of all m-independent attributes.

Definition 3.1 *Let (U, A, F) be a soft context. Put $M_a = \{g \in A \mid F(a) \supsetneq F(g)\}$. Then for $d \in A$, d is said to be m-dependent on A if there exists $M_d \neq \emptyset$ satisfying $F(d) = \mathbb{F}(M_d) = \cup_{a \in M_d} F(a)$.*

Otherwise, d is said to be m-independent on A.

We denote: $M_D = \{a \in A \mid a \text{ is } m\text{-dependent on } A\}$;

$M_I = \{a \in A \mid a \text{ is } m\text{-independent on } A\}$.

Example 3.2 *Let $U = \{1,2,3,4,5\}$ and $A = \{a,b,c,d,e,f,g\}$. Consider a soft context (U,A,F) as Table 1.*

Table 1:A soft context

-	a	b	c	d	e	f	g
1	1	1	0	1	1	1	1
2	1	0	1	0	0	0	0
3	0	1	0	1	0	1	0
4	1	0	1	0	0	0	0
5	0	1	0	0	1	1	0

Then, the set-valued mapping $F : A \to P(U)$ is defined as follows:

$F(a) = \{1,2,4\};\ F(b) = F(f) = \{1,3,5\};\ F(c) = \{2,4\};\ F(d) = \{1,3\};$

$F(e) = \{1,5\};\ F(g) = \{1\}.$

So,

$M_a(A) = \{c,g\};\ M_b(A) = M_f(A) = \{d,e,g\}; M_c(A) = \emptyset;$

$M_d(A) = M_e(A) = \{g\};\ \ M_g(A) = \emptyset.$

For $a,b,f \in A$,

$F(a) = \mathbb{F}(M_a) = F(c) \cup F(g)$;

$F(b) = F(f) = \mathbb{F}(M_b) = \mathbb{F}(M_f) = F(d) \cup F(e) \cup F(f)$.

So, a,b and f are m-dependent. But since $F(d) \neq \mathbb{F}(M_d) = F(g)$ and $F(e) \neq \mathbb{F}(M_e) = F(g)$, d and e are not m-dependent.

Then, we have:

$$M_D = \{a,b,f\};\ \ M_I = \{c,d,e,g\}.$$

Theorem 3.3 *Let (U,A,F) be a soft context. Then*

(1) $M_D \cap M_I = \emptyset$; $M_D \cup M_I = A$.

(2) a is m-independent if and only if either $M_a = \emptyset$ or if $M_a \neq \emptyset$, then $\mathbb{F}(M_a) = \cup_{g \in M_a} F(g) \neq F(a)$.

(3) For $a \in A$, $a \in M_D$ if and only if $F(a) \in mD$.

(4) For $a \in A$, $a \in M_I$ if and only if $F(a) \in mI$.

Proof.

(1) and (2) Obvious.

(3) Let $a \in M_D$. Then $M_a(A) = \{g \in A \mid F(a) \supsetneq F(g)\} \neq \emptyset$ and $\mathbb{F}(M_a) = \cup_{g \in M_a} F(g) = F(a)$. Hence, by definition of dependency of soft concepts, $F(a) \in mD$.

For the converse, let $F(a) \in mD$ for $a \in A$. Then, by (5) of Theorem 2.1, there exists $B \in P(A)$ such that $\mathbb{F}(B) = F(a)$. It implies that $B \subseteq M_a = \{g \in A : F(a) \supsetneq F(g)\}$. And from $\mathbb{F}(B) \subseteq \mathbb{F}(M_a)$, it follows $F(a) \supseteq \mathbb{F}(M_a) \supseteq \mathbb{F}(B) = F(a)$. Consequently, there is nonempty set M_a satisfying $\mathbb{F}(M_a) = F(a)$. So, $a \in M_D$.

(4) For $a \in M_I$, suppose $F(a) \notin mI$. Then from $mD \cap mI = \emptyset$ and $mD \cup mI = m(U, A, F)$, $F(a) \in mD$. Then by (1), $a \in M_D$ and $a \notin M_I$, which is a contradiction. Hence, $F(a) \in mI$.

In the same way, the converse is obviously showed. ■

Theorem 3.4 *Let (U, A, F) be a soft context. If $\varphi : M_I \to mI$ is a mapping as defined by $\varphi(a) = F(a)$ for $a \in M_I$, then φ is surjective.*

Proof. Let $a \in M_I$. Then $F(a) \in mI$ and $\varphi(a) = F(a) \in mI$. Thus, the mapping φ is well-defined. For the surjection, let $X \in mI$. Then by (3) of Theorem 2.2, there exists an element $a \in A$ such that $F(a) = X$. From (4) of Theorem 3.3, $a \in M_I$ and $X = F(a)$. Thus, φ is surjective. ■

Definition 3.5 *Let (U, A, F) be a soft context. For $a \in A$, we say that an element a is $generated$ by finitely many elements if $F(a) = \cup_{b \in B} F(b)$ for $B = \{b_1, b_2, \cdots, b_n\} \subseteq A$, and $b \in B$ is called $generator$ for a.*

Lemma 3.6 *Let (U, A, F) be a soft context. For $d \in A_D$, $M_d = \{g \in A \mid F(d) \supsetneq F(g)\}$ is a set of generators for d.*

Proof. Obvious. ■

Example 3.7 *In Example 3.2, for $b \in A$, b is generated by $\{d, e\}$ and $M_b(A) = \{d, e, g\}$, respectively. d, e, and g are generators of b.*

Theorem 3.8 ([10]) *Let (U, A, F) be a soft context. Then for each $X \in mD$, there is a family $\mathcal{B} \subseteq mI$ satisfying $X = \cup\mathcal{B}$.*

Theorem 3.9 *Let (U, A, F) be a soft context. For each $d \in M_D$, there exists $B \subseteq M_I$ such that $\mathbb{F}(B) = \cup_{b\in B}F(b) = F(d)$.*

Proof. Let $d \in M_D$. Then $F(d) \in mD$ and since $F(d)$ is a dependent soft concept, there exist $Z_1, \cdots, Z_n \in m(U, A, F)$ such that $F(d) \supsetneq Z_i$ and $F(d) = \cup Z_i$, $i = 1, \cdots, n (n \geq 2)$. And, since mI is a base for $m(U, A, F)$, for each Z_i, there exists $\mathbb{T}_i \subseteq mI$ such that $\cup\mathbb{T}_i = Z_i$ for $i = 1, \cdots, n$.

And, for each $T_{i_j} \in \mathbb{T}_i \subseteq mI$ $(j = 1, \ldots, l)$, by (3) of Theorem 2.2, there is an $m_{i_j} \in A$ such that $F(m_{i_j}) = T_{i_j}$. Then for each $F(m_{i_j}) = T_{i_j}$, from $F(m_{i_j}) = T_{i_j} \in mI$ and (4) of Theorem 3.4, $m_{i_j} \in M_I$. Put $B_i = \{m_{i_j} \in M_I \mid F(m_{i_j}) = T_{i_j} \textit{ for } T_{i_j} \in \mathbb{T}_i\}$ $(i = 1, \cdots, n)$.

Then for $i = 1, \cdots, n$, $B = \cup B_i \subseteq M_I$ and $\mathbb{F}(B) = \cup_{b\in B}F(b) = \cup(\cup_{m_{i_j}\in B_i}F(m_{i_j})) = \cup(\cup\mathbb{T}_i) = \cup Z_i = F(d)$. So, the proof is completed. ■

Let (U, A, F) be a soft context. Then a family $\mathcal{S}$ of subsets of $m(U, A, F)$ is called a *base* for (U, A, F) if it satisfies the following two conditions:

(1) $\mathcal{S} \subseteq m(U, A, F)$.

(2) For each $X \in m(U, A, F)$, there exists $\mathcal{S}' \subseteq \mathcal{S}$ such that $X = \cup\mathcal{S}'$.

In [10], we obtained the properties of base for $m(U, A, F)$ as the following:

Theorem 3.10 ([10]) *Let (U, A, F) be a soft context. Then:*

(1) The family $\mathcal{F}_A = \{F(a) \mid a \in A\}$ is a base:

(2) mI is the smallest base for $m(U, A, F)$:

(3) For $B \subseteq A$, if a set-valued mapping $\varphi : B \to mI$ defined by $\varphi(b) = F(b)$ for $b \in B$ is surjective, then $\varphi(B) = \{F(b) \mid b \in B\}$ is a base for $m(U, A, F)$.

Theorem 3.11 *Let (U, A, F) be a soft context. Then $\mathcal{M} = \{F(a) \mid a \in M_I\}$ is a base for $m(U, A, F)$.*

Proof. From Theorem 3.4, a set-valued mapping $\varphi : M_I \to mI$ defined by $\varphi(a) = F(a)$ for $a \in M_I$ is surjective, and by (3) of Theorem 3.10, $\varphi(M_I) = \{F(a) \mid a \in M_I\} = \mathcal{M}$ is a base for $m(U, A, F)$. ■

Corollary 3.12 *Let (U, A, F) be a soft context. Then $\cup_{a\in M_I}F(a) = U$.*

Proof. It follows from Theorem 3.11. ■

Finally, using Theorem 3.11, we have the following theorem:

Theorem 3.13 *Let (U, A, F) be a soft context and $\mathcal{F}_{M_I} = \{F(a) \mid a \in M_I\}$. Then*

$$m(U, A, F) = \{\cup\mathcal{S} | \mathcal{S} \subseteq \mathcal{F}_{M_I}\}.$$

Example 3.14 *For $U = \{1, 2, 3, 4, 5\}$ and $A = \{a, b, c, d, e, f, g\}$, let us consider a soft context (U, A, F) as in Example 3.2. In the example, we showed that:*

$$M_D = \{a, b, f\}; \ \ M_I = \{c, d, e, g\}.$$

For $F(c) = \{2, 4\}, \ F(d) = \{1, 3\}, \ F(e) = \{1, 5\}$, and $F(g) = \{1\}$,

$\mathcal{F}_{M_I} = \{\{1\}, \{1, 3\}, \{1, 5\}, \{2, 4\}\}.$

So,

$m(U, A, F)$

$= \{\cup\mathcal{S} | \mathcal{S} \subseteq \mathcal{F}_{M_I}\}$

$= \{\emptyset, \{1\}, \{1, 3\}, \{1, 5\}, \{2, 4\}, \{1, 2, 4\}, \{1, 3, 5\}, \{1, 2, 3, 4\},$

$\{1, 2, 4, 5\}, U\}.$

Now, we recall the notion of order on $m(U, A, F)$ defined in [10] as the following: For $X, Y \in m(U, A, F)$,

$$X \preceq Y \text{ if and only if } X \subseteq Y.$$

X is called a *sub-m-concept* of Y, and Y is called a *super-m-concept* of X.

For the ordered set $(m(U, A, F), \preceq)$, the infimum $\wedge$ and supremum $\vee$ are defined by:

$$X \wedge Y = \mathfrak{F}(X \cap Y); \quad X \vee Y = X \cup Y.$$

Then $(m(U, A, F), \preceq, \wedge, \vee)$ is complete lattice.

The complete lattice $(m(U, A, F), \preceq, \wedge, \vee)$ is called *m-concept lattice* (or *object oriented soft concept lattice*) and simply will be denoted by $mL(U, A, F)$.

Let $mL(U, B, F)$ and $mL(U, C, G)$ be two m-concept lattices. $mL(U, B, F)$ is said to be finer than $mL(U, C, G)$, which is denoted by

$$mL(U, B, F) \leq mL(U, C, G) \Leftrightarrow m(U, C, G) \subseteq m(U, B, F)$$

If $mL(U, B, F) \leq mL(U, C, G)$ and $mL(U, C, G) \leq mL(U, B, F)$, then two m-concept lattices are said to be isomorphic to each other, and denoted by

$$mL(U, B, F) \cong mL(U, C, G).$$

Theorem 3.15 ([10]) *Let (U, A, F) be a soft context and $C \subseteq A$. Then $mL(U, A, F) \cong mL(U, C, F_C)$ if and only if $\mathbf{Im}(\mathbb{F}) = \mathbf{Im}(\mathbb{F}_C)$.*

Theorem 3.16 *Let (U, A, F) be a soft context. Then $mL(U, A, F) \cong mL(U, M_I, F_{M_I})$.*

Proof. From Theorem 3.11, $\mathbf{Im}(\mathbb{F}) = \mathbf{Im}(\mathbb{F}_{M_I})$. So, $mL(U, A, F) \cong mL(U, M_I, F_{M_I})$. ■

Finally, by using the family of all m-independent attributes, we show a reduction process of a soft context concept lattice $mL(U, A, F)$:

Remark. Let us consider a soft context (U, A, F) as shown in Table 2, where $U = \{1, 2, 3, 4, 5\}, A = \{a, b, c, d, e, f, g\}$.

Table 2:A formal context

-	a	b	c	d	e	f	g
1	1	1	0	1	1	1	1
2	1	0	1	1	1	1	1
3	0	1	0	1	0	1	1
4	0	0	0	0	0	0	1
5	0	0	1	0	1	0	0

Then (F, A) is a soft set as follows:

$F(a) = \{1, 2\}$; $F(b) = \{1, 3\}$; $F(c) = \{2, 5\}$; $F(d) = F(f) = \{1, 2, 3\}$;

$F(e) = \{1, 2, 5\}$; $F(g) = \{1, 2, 3, 4\}$.

And,

$M_D = \{d, e, f\}$; $M_I = \{a, b, c, g\}$.

$$m(U, A, F) = \{\emptyset, \{1, 2\}, \{1, 3\}, \{2, 5\}, \{1, 2, 3\}, \{1, 2, 5\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, U\}.$$

Hence, $mL(U, A, F)$ is obtained as shown in the below diagram:

$$A = \{a, b, c, d, e, f, g\}$$

$$\begin{array}{ccc}
 & U & \\
 & \nearrow \quad \nwarrow & \\
\{1,2,3,5\} & & \{1,2,3,4\} \\
\uparrow & \nwarrow \nearrow & \\
\{1,2,5\} & & \{1,2,3\} \\
\uparrow & \nwarrow \nearrow & \uparrow \\
\{2,5\} & \{1,2\} & \{1,3\} \\
\nwarrow & \uparrow & \nearrow \\
 & \emptyset &
\end{array}$$

$$mL(U, A, F)$$

Finally, for $M_I = \{a, b, c, g\}$, by Theorem 3.16, we have $mL(U, A, F) \cong mL(U, M_I, F_{M_I})$ as the following diagram.

$$A = \{a, b, c, d, e, f, g\} \quad \supseteq \quad M_I = \{a, b, c, g\}$$

$$\begin{array}{ccc}
 & U & \\
 & \nearrow \quad \nwarrow & \\
\{1,2,3,5\} & & \{1,2,3,4\} \\
\uparrow & \nwarrow \nearrow & \\
\{1,2,5\} & & \{1,2,3\} \\
\uparrow & \nwarrow \nearrow & \uparrow \\
\{2,5\} & \{1,2\} & \{1,3\} \\
\nwarrow & \uparrow & \nearrow \\
 & \emptyset &
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & U & \\
 & \nearrow \quad \nwarrow & \\
\{1,2,3,5\} & & \{1,2,3,4\} \\
\uparrow & \nwarrow \nearrow & \\
\{1,2,5\} & & \{1,2,3\} \\
\uparrow & \nwarrow \nearrow & \uparrow \\
\{2,5\} & \{1,2\} & \{1,3\} \\
\nwarrow & \uparrow & \nearrow \\
 & \emptyset &
\end{array}$$

$$mL(U, A, F) \quad \cong \quad mL(U, M_I, F_{M_I})$$

4. Conclusion

In particular, we showed that every m-dependent attribute is generated by some m-independent attributes and the family of all the m-independent attributes determines all m-concepts of a given m-context. Also, we showed that a reduction of a soft concept lattice $mL(U, A, F)$ is obtained by the family of all m-independent attributes. In the next research, we will study a variety of ways to reduce the soft concept lattices using any family of m-independent attributes and investigate how to combine soft concepts and m-concepts to efficiently reduce the soft concepts lattices.

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