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Finite Element and Theoretical Study on The Rectangular Reinforced Concrete Slab

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Abstract

Plate is a flat structural element which has smaller thickness (h) than the other dimensions. A large number of structural components in engineering structures can be classified as plates. Previously, slabs have been designed and analysed according to classical plate theories, however, the innovation of computers and appearance of many software, based on the FEM, made classic methods become less applicable. Present study aimed to compare analytical results, obtained from Classical Plate Theory, with the results of FEM (ABAQUS software). Results of the present study confirmed that values of maximum deflections according to Timoshenko's theory and FEA nearly match about 83.6% and 86.8% for rectangular slab with the thickness 90 mm and 60 mm, respectively. However, bending stresses, matched about 78.9% and 64.8% for rectangular slabs with thickness of 90 mm and 60 mm, respectively. The low percentage of differences can attributed to the fact that the assumptions in which classical plate theory depends on. Results of bending stresses were a bit different comparing classical approach and FEA, thus, present study concluded that there were no significant difference between classical plate theories and FEM.

Keywords: Bending moment, Concrete, FEM, Rectangular slabs, Stress.

I. INTRODUCTION

Probably the first imputation to a mathematical statement of plate problems, was done by Euler in 1776, which performed a free vibration analysis of plate problems, followed by Chladni which discovered the various modes of free vibrations [1] and [2]. Plate is initially a straight structural element which has smaller thickness (h) than the other dimensions [3]. Plate bounded and limited by two parallel planes which called faces and the surface called edge, the distance between the plane faces is called the thickness (h) of the plate [2]. A large number of structural components in engineering structures can be classified as plate, the most familiar examples for plate are floor, lock-gates, bridge decks, foundation slabs, table-tops, street manhole covers, side panels, turbine disks, bulkheads, and tank bottoms [4]. Additionally, many other previous studies considered parts of machineries and other mechanical devices as a plate, and reported them as slightly curved skins such as wings and a large part of the fuselage of an aircraft [5].

Generally, according to the static aspects plate has free, simply supported and fixed boundary conditions, which includes elastic supports and elastic restrains or even in some cases point supports [6]. The static and dynamic loads carried by plates, which included both holding load and lifting load, are perpendicular to the surface of the plate which called upper face, these type of loads are resisted by internal bending, torsional moments and also by transverse shear forces [7]. A study by Patil and Sigi, reported that flat plate slab construction has been in practice for a long time, however, the technology has seen large scale use only in last decade and is one of the rapidly developing technologies in Indian building industry today [8]. A studies by [9] and [3], concluded that concrete slabs behave primarily as flexural members and in design similar to that of beams.

Previously, slabs have been designed and analysed according to the theory which proposed by [10]. At the time of computer innovation and appearance of computer programs based on the FEM classic methods become less applicable [11]. Many studies preferred FFA over classical methods [12] and [3].

The present study aimed to apply classical plate theory for evaluation of bending stress and maximum deflection values in rectangular reinforced concrete slabs. Analytical results obtained from Classical Plate Theory are compared with the results of ABAQUS software.

II. METHODOLOGY

I.Classical Plate Theory

Classical Plate Theory is the thin plate theory based on Love-Kirchhoff's hypothesis which makes assumptions similar to those made by the Bernoulli-Navier hypothesis used in the theory of thin or shallow beams. It is also called as small deflection theory.

II.Governing Differential equation for rectangular plate

The mentioned below approach have been used for analysis of the simply supported rectangular RC slabs, with a plate thickness of 90 and 60mm which have subjected to the uniform distributed load.

$$\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = \frac{q}{D} \ldots\ldots (1)$$

Deflection and stresses determined for simply supported rectangular plate subjected to uniform load, using theoretical solution:

$$w = \frac{16q}{D\pi^6} \sum_m \sum_n \frac{\left(\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right)}{mn\left((m/a)^2 + (n/b)^2\right)^2} \ldots\ldots (2)$$

Where a and b are the horizontal and vertical dimension of the all plate, and x and y the coordinates of the point at which the

transverse deflection *w* is calculated. Standard values which are assumed at elastic stage of work were:
Plate dimensions =950 mm length & 950 mm width.
Poisson's ratio of reinforcement =0.3.
Young's modulus of reinforcement=200 GPa.
Steel reinforcement Ø10@100mm both directions.
Poisson's ratio of concrete = 0.2.
Modulus Young's of concrete=23.5 GPa.
Applied uniformly distributed load=10 kPa.
Final equations of maximum bending moment and deflection for simply supported plate, were as below:

$$W_{max} = \frac{0.00416\,P\,a^4}{D} \ldots\ldots(3)$$

$$M_{xmax} = M_{ymax} = 0.0492\,qa^2 \ldots\ldots\ldots(4)$$

$$D = \frac{Eh^3}{12(1-v^2)} \ldots\ldots(5)$$

Corresponding bending stresses can be found from the moments $\boldsymbol{M_x}$ and $\boldsymbol{M_y}$ by the expression

$$\sigma = \frac{6M}{h^2} \ldots\ldots\ldots(6)$$

III. ABAQUS Solution

ABAQUS software has been used, which is one of the finite element analysis based software and consider as the most powerful and effective approaches for analysing and investigating the stress state of the materials under different types of loading [13].
In the present investigation, the finite element models introduced three-dimensional 8-node first order fully integration continuum elements (C3D8 - Bricks) which modelled concrete rectangular slab and loads. Also, reinforcing bars modelled as three -dimensional truss elements, three-dimensional 2-node first order truss elements (T3D2 - Truss) used to model the steel reinforcing bars in the FE model of the rectangular concrete slab.
According to thickness of slabs the specimens are subdivided into 6500-10000 3D small elements of simple cube shapes connected at nods, thus, the stress of all small elements have been calculated, and there was a complete oblique of the stress-strain state of the entire specimen as shown in Figure (1).

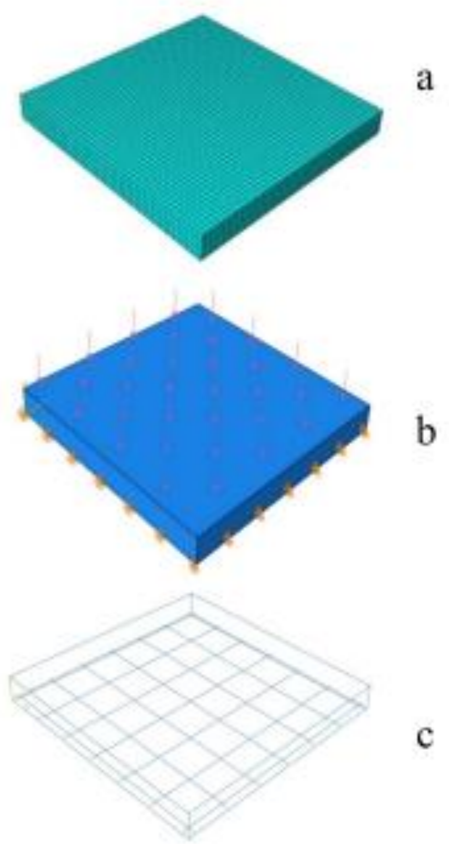

Figure 1. Reinforced concrete slab, a-slab meshing, b-Loading & Supporting of the slab c-Reinforcement

III. RESULTS AND DISCUSSIONS

Deflections of simply supported rectangular slabs with two thicknesses (90 and 60 mm) under uniform distributed load (10 kPa.) have been predicted and estimated according classical plate theory (Timoshenko) and FEM which were 0.0228 and 0.0269 respectively, for slabs with the thickness of 90mm, while 0.077 and 0.0879 respectively, for slabs with the thickness of 60mm, as shown in Table 1.

Table 1. Deflection of simply supported rectangular concrete slab under uniform distributed load.

Thickness (mm)	Timoshenko (Theory)	FEM
90	0.0228	0.0269
60	0.077	0.0879

Stresses of simply supported rectangular slabs with two thicknesses (90 and 60 mm) under uniform distributed load (10 kPa.) have been predicted and estimated according classical plate theory (Timoshenko) and FEM which were 0.329 and

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0.266 respectively, for slabs with the thickness of 90 mm, while 0.741 and 0.519 respectively, for slabs with the thickness of 60 mm, as shown in Table 2.

Table 2. Stress of simply supported rectangular concrete slab under uniform distributed load.

Thickness (mm)	Timoshenko(Theory)	FEM
90	0.329	0.266
60	0.741	0.519

Results of present investigation determined that maximum deflection of rectangular plate located at the centre of slab and decreases gradually which reaches zero at support as shown in Figure (2). The absolute value of the maximum deflection (*wz*) obtained by both analytical approach (Timoshenko) and FEM were compared as shown in Figure (3), results nearly match about 83.6 % and 86.8 % for rectangular slab with the thickness 90 mm and 60 mm, respectively.

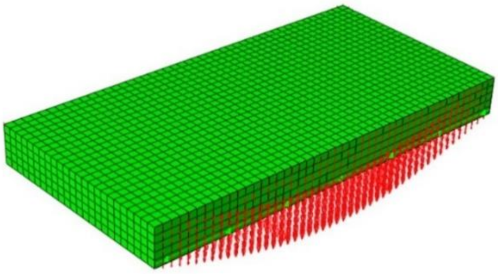

Figure 2. Vertical deflection which shows gradually decreased at the edges.

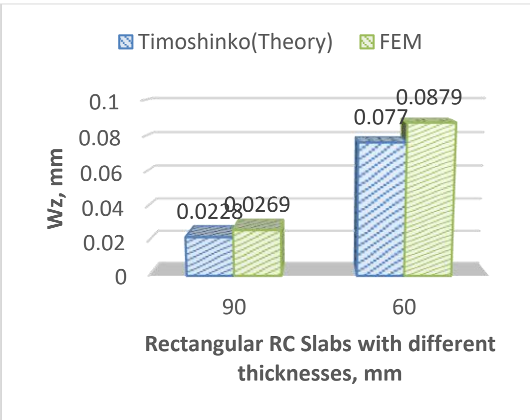

Figure 3. The absolute values of the maximum deflection (wz) which obtained by both analytical approach and FEM were shown graphically.

The results of bending stresses obtained by (Classical plate theory) and (FEM) have been compared as shown in Figure (4), result nearly matched about 78.9% and 64.8% for rectangular slabs with thickness of 90 mm and 60 mm, respectively. The differences due to the following points

Assumptions in classical plate theory: plate is flat structural elements with uniform thickness of homogeneous isotropic material, thickness is not more than about one-quarter of the least transverse dimension and the maximum deflection is not more than about one-half the thickness, all forces loads and reactions are normal to the plane of the plate, and plate is nowhere stressed beyond the elastic limit. FE analysis represents the exact model of concrete rectangular slab and reinforcements, 3D modelled slab with all properties of exact material.

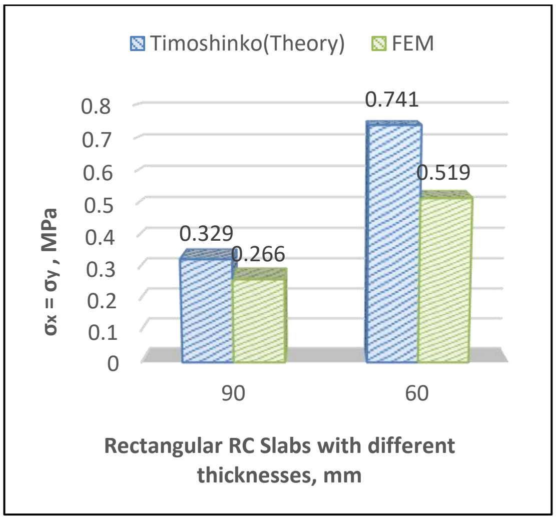

Figure 4. Bending Stress illustration, comparative evaluation of the maximum σ by both analytical approach and FEM were shown graphically.

Distribution of the stresses shown in Figure (5) which determined by FEM software and reached its maximum value at the centre of the slab.

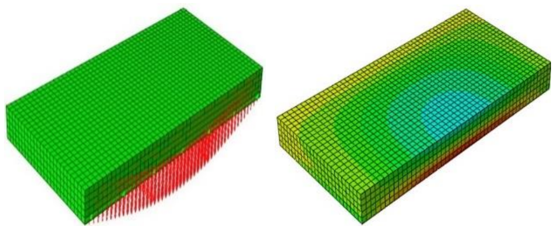

Figure 5. Location of maximum bending stresses and deflection in rectangular concrete slab

Previously, irregular type of slabs with different types of boundary conditions and loadings, have been designed according to the classical plate theory which proposed by [10], however this theory faced challenges due to the differences between yield-line load and experimental load [14]. At the time of computer innovation and appearance of computer programs based on the FEM, for example, ABAQUS, the classic methods for calculation of slab reinforces become less applicable [11].

In most cases, as structural engineers conduct finite element analysis (FEA) of plates, they are primarily interested for determining the maximum stress and displacement value, an indicator of their position, which is absolutely correct in terms of subsequent design [12]. A study by Patel et. al., adopted computers software which based on Finite Element Analysis because equivalent frame method was not satisfactory for hand calculations [3]. Previous study by Mabsout et. al., used finite element method successfully to investigate the effect of span length, slab width, and wheel load conditions on simply supported, one-span, reinforced concrete slab bridges [15].

IV. CONCLUSIONS

The results of FEA were accurate and 3D simulation in ABAQUS software were pure, as FE analysis represented the exact model of concrete rectangular slab and reinforcements. This investigation also confirmed that results of bending stresses were a bit different comparing classical approach and FEA, thus, classical plate theory was applicable. Thickness of rectangular concrete slabs significantly affected on the displacement and bending stress.

Generally, performed investigations confirm imitation of 3D finite element modelling of the rectangular RC slabs, which opens fundamentally new horizons in the analysis of the plates by using numerical methods.

REFERENCES

[1] L. Euler, "De motu vibratorio tympanorum," Novi commentarii academiae scientiarum Petropolitanae, pp. 243-260, 1766.

[2] E. Ventsel, T. Krauthammer, and E. Carrera, "Thin plates and shells: theory, analysis, and applications," Appl. Mech. Rev., vol. 55, pp. B72-B73, 2002.

[3] J. J. Patel, N. Dubey, J. J. Patel, and N. Dubey, "Comparison of Analysis Methods of Flat Slab," International Journal or Innovative Research in Science & Technology (IJIRST), pp. 459-463, 2016.

[4] a. S. A. A. NASIH HABEEB ASKANDAR, "APPLICABILITY OF THE PLATE THEORIES ON THE REINFORCED CONCRETE SLABS IN ELASTIC STAGE AND IT'S PUNCHING SHEAR STRENGTH IN FAILURE STAGE," Master Thesis, vol. College of Engineering, University of Salahaddin – Erbil, 2011.

[5] A. K. Sharma and N. Mittal, "Free vibration analysis of laminated composite plates with elastically restrained edges using FEM," Central European journal of engineering, vol. 3, pp. 306-315, 2013.

[6] J. S. Kumar, T. D. Raju, and K. V. K. Reddy, "Vibration analysis of composite laminated plates using higher-order shear deformation theory with zig-zag function," Indian journal of science and technology, vol. 4, pp. 960-966, 2011.

[7] N. Iyengar, Structural stability of columns and plates: Ellis Horwood Chichester, 1988.

[8] S. Patil and R. Sigi, "Analysis And Design Of Flat Slabs Using Various CodeS," International Journal of Research in Engineering and Technology, vol. 3, pp. 2319-1163, 2014.

[9] A. A. Adedeji, "Searching for the Optimal Design Variables of RC Flat Slab by the Reactive Taboo," Trends in Applied Sciences Research, vol. 6, pp. 375-385, 2011.

[10] K. W. Johansen, Yield-line formulae for slabs: CRC Press, 1972.

[11] K. Cichocki and M. Ruchwa, "Distribution of damages in unconventionally reinforced concrete slabs subjected to impact loads," in Advances in Mechanics: Theoretical, Computational and Interdisciplinary Issues: Proceedings of the 3rd Polish Congress of Mechanics (PCM) and 21st International Conference on Computer Methods in Mechanics (CMM), Gdansk, Poland, 8-11 September 2015, 2016, p. 129.

[12] D. Bäcker, M. Kuna, and C. Häusler, "Measurement of Kirchhoff's stress intensity factors in bending plates," in Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems 2014, 2014, p. 90611Q.

[13] S. Courtin, C. Gardin, G. Bezine, and H. B. H. Hamouda, "Advantages of the J-integral approach for calculating stress intensity factors when using the commercial finite element software ABAQUS," Engineering Fracture Mechanics, vol. 72, pp. 2174-2185, 2005.

[14] P. Desayi and A. Kulkarni, "Membrane action, deflections and cracking of two-way reinforced concrete slabs," Matériaux et Construction, vol. 10, pp. 303-312, 1977.

[15] M. Mabsout, R. Jabakhanji, K. Tarhini, and G. R. Frederick, "Finite element analysis of concrete slab bridges," in Computing in Civil and Building Engineering (2000), ed, 2000, pp. 1045-1050.