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# Performance Analysis of Wavelet Entropy Based Detection of Primary User Signals in Cognitive Radio Networks

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**Abstract**

The spectrum sensing in CRN has a limitation. Most of the traditional detectors suffer from noise uncertainty issue. Noise uncertainty factor renders them ineffective at low SNR. In recent research, entropy based detection (EBD) is found to be independent of the factor of noise. We propose a scheme wherein the signal received is decomposed efficiently at the receiver so as to get better entropy results. Instead of the traditional method of deriving DFT components of the received signal, we propose to derive Wavelet components. Then wavelet entropy is matched with the threshold value to conclude if Primary User (PU) is using the subband or not. In this paper we have used wavelet transform (WT) and wavelet packet transform (WPT) to decompose the received signal before calculating the entropy which is further utilized to identify PU. A comparative study of DFT, WT and WPT based detection is carried out using Monte Carlo experiments.

**Keywords—**CRN, WT, WPT, EBD.

## I. INTRODUCTION

Cognitive Radio Networks (CRN) propose to solve the spectrum inadequacy by helping the unlicensed users to temporarily utilize the spectrum which is allocated to a licensed user or primary user (PU) when it is lying unused [1][2]. Traditional techniques like matched filter[3], energy detection[4], feature detection[5] etc. suffer from a common problem of noise uncertainty[6][7].

Entropy based detector or EBD is robust to noise uncertainty [8]. In a scenario, where Gaussian noise signals are involved, entropy sensing works on the premise that entropy of a stochastic signal is more. However, in cases where modulated signals of the primary users are contained in the received signal, reduction in entropy is observed.

In DFT based EBD (DEBD), the time domain signal is converted to frequency domain signal to remove noise uncertainty. Once the signal is received at the receiver, its information content is evaluated using Shannon entropy[9]. Entropy is not affected by the factor of noise. Hence the results obtained, demonstrate robustness to noise uncertainty. The results show improvement in performance of detection.

However, DFT has a limitation. It provides only two dimensional information about a signal i.e, about its different frequency components and their respective amplitude. To overcome this, we propose using wavelet transform.

Wavelet transform covers all the three parameters of a signal i.e, frequency components, their respective amplitude and the time which gives the location of different frequency components on the period axis [10][11]. Hence, wavelet transform entropy based detection (WEBD) should yield better results as compared to DFT entropy based detection.

However, wavelet transform also suffer from the limitation of excluding higher frequency components. Hence we propose Wavelet packet transform based detection model which includes both lower as well as higher frequency components [12]. It gives a complete decomposition of the tested signal. Hence it is supposed that wavelet packet entropy based detection (WPEBD) should yield the best results.

In this paper, we investigate DFT, WT and WPT as a measure to enhance the efficiency of entropy based detection. The received signal is first subjected to above mentioned three types of transform. Their entropy is calculated which is matched with a threshold value to arrive at the decision whether PU is present or not.

A comparative analysis is carried out using simulations. Results using receiver's operating characteristics (ROC) curves displays performance of all three methods discussed above at various levels of SNRs. All three methods are robust to noise uncertainty, however, their efficiency differs with different transforms and the same is attempted to be established in this paper.

This paper is arranged in various sections. Section II briefly describes the methodology used to carry out entropy based detection using DFT, wavelet and wavelet packet transforms. Section III contains simulation results. Conclusion of the paper is given in section IV

## II. METHODOLOGY

The problem of spectrum sensing can be represented as a binary hypothesis. $\mathbf{H_0}$ represents absence of primary signal. $\mathbf{H_1}$ represents presence of primary signal. The information that is held in the signal is quantified by entropy.

We decompose the received signal using DFT, wavelet transform and wavelet packet transforms methods. Then we calculate entropy of the each method and match it with the threshold entropy. Based on the outcome of above methodology adopted, we conclude about the existence of PU signals.

We compare the performance of each method with the help of ROC curves through probabilities of detection represented by $P_d$ and false alarm probabilities expressed as $P_f$.

### II.I DEBD Model For Detection Of PU Signals

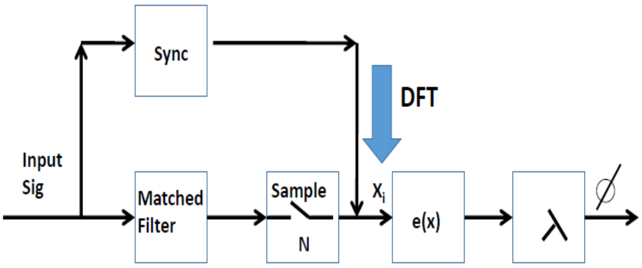


**Figure 1:** Basic DEBD model

As shown in Fig. 1 above, when DFT is applied to the input signal x(n), we get

$$\overline{X}(k) = \overline{S}(k) + \overline{W}(k), \quad k = 0,1,\dots,K-1 \tag{1}$$

Here, K denotes the size of DFT. $\overline{X}$, $\overline{S}$ and $\overline{W}$ represents the complex spectrum of received signals, signals which are primary and noise respectively. Y is a random variable which expresses the magnitude of the spectrum of the signal that is being measured.

A frequency based detection strategy is followed which includes testing information entropy[13]. This model is expressed as :

$$H_{L0}(Y) vs. H_{L1}(Y) \tag{2}$$

Here $H_{L1}(Y)$ represents entropy that consists of a number of states denoted by L [14] in hypothesis $H_i$. Probability of each state of the random variable is estimated using Histogram method. Number of states is same as L which is the bin number and represents the dimension of the probability space.

$\boldsymbol{H_0}$ is the hypothesis where x($n$) = w($n$). It represents the received signal that comprises noise which is a Gaussian random variable distributed independently.

Y follows a Rayleigh distribution with differential entropy $H_d$ [15].

$$H_d(Y) = 1 + ln\frac{\sigma_1}{\sqrt{2}} + \frac{\gamma}{2}, \tag{3}$$

Here γ represents the Euler-Mascheroni constant.

As per $\mathbf{H_1}$ hypothesis, the received signal comprises of both noise and the primary signal. In $\mathbf{H_1}$, the magnitude of the spectrum of the signal that is received adopts rice distribution that doesn't require an analytical expression of differential entropy.

Relative entropy or $(Y|H1) - (Y|H0)$ is the basis for detection.

Entropy is computed for $H_0$ as well as $H_1$. To decide presence or absence of PU, the test statistic obtained is matched with the threshold. The outcome helps in taking a decision regarding the occupation of the subband by the PU.

### II.II WEBD Model For Detection Of PU Signals

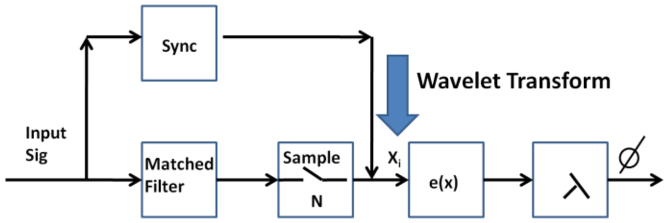


**Figure 2:** Basic WEBD model

Fig. 2 represents the model where the received signal is put through wavelet decomposition before calculating its entropy. When at a given instance $k$ and scale $j$, a discrete signal is wavelet transformed, it gets decomposed into higher frequency components $d_i(k)$ and lower frequency components $a_j(k)$. Signal components $Dj(k)$, $Aj(k)$ comprise of the frequency band of information which is obtained by reconstruction.[16][17]

When, $Dj(k) : [2^{-(j+1)} F_s, 2^{-j} F_s]$, (where $j = 1,2,3,\dots,m$)

then, $$Aj(k) : [0, 2^{-(j+1)} F_s] \tag{4}$$

Now, original signal x(n) is given as

$$x(n) = \sum_{j=1}^{J} Dj(n) + Aj(n) \tag{5}$$

Let $A_j(n) = D_{J+1}(n)$ for the purpose of unification.

Then we get,

$$x(n) = \sum_{j=1}^{J+1} Dj(n) \tag{6}$$

For scale j, $D_j(n)$ is the multiple resolution depiction of the signal x(n). This acts as the feature subset of its classification. The states and probabilities of an event define its uncertainty. We can have a sample space $\{x_1, x_2, x_3 \dots x_n\}$ such that it is aggregation of all possible states of an event, then probability of each piece of information is given by

$$P(x_i) = P_i, \quad 0 \le P_i \le 1 \quad \& \quad \sum P_i = 1 .$$

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The self information value of $x_i$ will be given by,

$$I(x_i) = -\log_a P(x_i) = -\log_a P \tag{7}$$

$x_i$ in the above equation denotes events. Here, any changes in information will change the value of the random variable, $I(x_i)$. Therefore, it is not appropriate to use it for measuring the complete information source. To overcome this, we can use mathematical expectation of self- information, that is, the mean self–information for the information source. This is entropy and is represented as $H(X)$.

Where,

$$H(X) = E[I(x_i)] = E[-\log_a P_i] = -\sum P_i \log_a P_i \tag{8}$$

The spectrum entropy is thus defined in frequency domain using information entropy [18].

The power spectrum is,

$$S(\omega) = \frac{1}{2\pi n} |X(\omega)|^2 \Delta\omega.$$

Where,

$S = \{S_1, S_2, \ldots\ldots S_n\}$ and represents partition of the original signal.

The proportion for i-th power spectrum is given as,

$$P_i = \frac{S_i}{\sum_{i=0}^{n} S_i} \tag{9}$$

Then, corresponding information entropy is given as,

$$H = -\sum_{i=0}^{n} P_i \log P_i \tag{10}$$

Here the value for $P_i$ can be calculated by (9).

The motive of our experiment is to decide on the signals received and to analyze if the primary signal is present or not. The algorithm we proposed would help in making this decision that depends upon the wavelet transform estimation of entropy from samples of the signals.

**H₀** is the hypothesis that shows the signal received contains noise that is Gaussian random variable and distributed independently. Wavelet transform entropy for the value of w(n) is given by various levels { i=1, 2 . . .n}

**H₁** is the hypothesis that shows the signal that is received contains noise and also the primary signal. Here wavelet transform entropy for x(n) is calculated by (10).

Entropy is computed for both hypotheses. The test statistic obtained is matched with the threshold to detect primary user signals.

### II.III WPEBD Model For Detection Of PU Signals

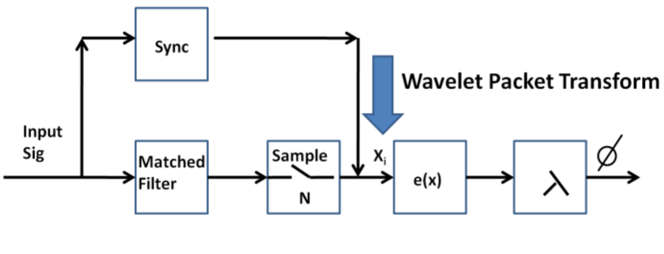


**Figure 3**: Basic WPEBD model.

Fig. 3 represents the model where the received signal is put through wavelet packet transform before calculating its entropy. Approximation space as well as the detail space is decomposed by wavelet packet transform which gives a better frequency resolution of the decomposed signal as compared to wavelet transform.

Suppose we take j levels, wavelet packet transform decomposes x(n), that is, a noisy signal into $2^j$ sub bands. The corresponding wavelet packet coefficients can be expressed as [19][20].

$$d_{i,m}^{j} = WP\{x(n), j\}\; n = 1, \ldots\ldots\ldots\; .., N \tag{11}$$

$d_{i,m}^{j}$ represents the m[th] coefficient that belongs to the i[th] sub band of level j where m=1. . . $N/2^j$ , i=1. . . $N/2^j$

Wavelet coefficients' energy defines the wavelet packet entropy of the subband. [21]. The energy calculated for sub-band i and the level j is given as,

$$E_i^j = \sum_m \left|d_{i,m}^{j}\right|^2 \tag{12}$$

The formula to measure $d_{i,m}^{j}$is shown in (11). $E_{total}^{j}$ is the wavelet packet coefficients' total energy, and is given as

$$E_{total}^{j} = \sum_m \left|d_{i,m}^{j}\right|^2 = \sum_{I=1}^{2^J} E_i^j \tag{13}$$

For every level, probability distribution can be computed by

$$P_i^j = \frac{E_i^j}{E_{total}^j} \tag{14}$$

Here $P_i^j$ denotes the wavelet packet energy that is normalized. $\sum_i P_i^j = 1.$

These normalized wavelet packets energy bands carry the important information regarding the location of frequencies in the sub bands.

For j level, the wavelet packet entropy is expressed as,

$$S_{wp}^{(j)} = -\sum P_i^j \log_2[P_i^j] \tag{15}$$

The motive of our experiment is to decide on the signals received and to analyze if the primary signal is present or not. The algorithm that has been proposed here would help in making this decision that depends upon the estimation of entropy of wavelet packets from signal samples.

**H₀** is the hypothesis where the signal received as x(n) = w(n) contains noise. Wavelet packet entropy for different levels(j=1,2,3…) for w(n) by (15) is given by

$$S_{wp}^{(j)} = S_{wp}^{(j)}(w(n)) = -\sum P_i^j \log_2[P_i^j] \tag{16}$$

Here the value for $P_i^j$could be calculated by using the formula (14). The w(n) is a random variable which is additive White Gaussian noise. Consequently, results show $P_1^j = P_2^j = \ldots\ldots. P_2^j$. Say for level j=1, $P_1^1 = P_2^1$.

**$H_1$** is the hypothesis where the signal that is received as x(n) =s(n) + w(n) contains both noise and the primary signal. Here wavelet packet entropy for x(n) is calculated using (15)

$$S_{wp}^{(j)} = S_{wp}^{(j)}(x(n)) = -\sum P_i^j \log_2[P_i^j] \qquad (17)$$

The value for $P_i^j$ is calculated by using the formula as shown in (14). For a given level $j$ of wavelet decomposition, entropy is calculated using formula given in (15) for both the hypotheses. The test statistic obtained is expressed as,

$$T_j(X) = -\sum_i^{2^j} P_i^j \log_2[P_i^j]$$

$$= \sum_i^{2^j} \frac{E_i^j}{E_{total}^j} \log_2\left[\frac{E_i^j}{E_{total}^j}\right] \begin{cases} \leq \lambda^j : & decide H_1 \\ \geq \lambda^j : & decide H_0 \end{cases} \qquad (18)$$

$\lambda^j$ is the detection threshold for level $j$ obtained for a target false alarm ratio $P_f$ and is given by

$$\lambda^j = S_{wp}^{(j)}(w(n)) + Q^{-1}(1 - P_f)\sigma_e \qquad (19)$$

Here the assumption is that Gaussian distribution is followed by noise entropy that has a theoretical mean value obtained by (16) and $\sigma_0^2$ as the variance.

Here, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty exp(-\tau^2/2)d\tau$ , $Q^{-1}(x)$ is calculated as the inverse function of Q-function.

$T_j(X)$ is the test statistic which is concerned with wavelet decomposition level j having N as the sample size. When the decomposition level of the wavelet is fixed, the entropy of wavelet packets in hypothesis $H_0$ remains constant and is not affected by noise. Also, the $P_f$ does not vary.

## III SIMULATION RESULTS

Performance of DEBD, WEBD and WPEBD are evaluated by considering the $P_d$ and $P_f$.

Experiments of Monte Carlo are executed for over 10,000 runs. $L$= 15, $P_f$ = 0.08. The primary and the binary phase shift keying BPSK signal is modulated with frequency carrier of $fc$= 40 KHz. Here the sampling frequency $fs$= 100 KHz and the sampling time is 5ms.

The detection performance curves vs. false alarm probability of DEBD, WEBD and WPEBD with SNR of -25dB, -15db and -5dB is shown in fig 4-6 respectively. It is seen that the detection probability is decreasing as SNR (dB) value is increasing. We can observe that WPEBD (denoted by blue line) outperforms both DEBD and WEBD for all SNR values.

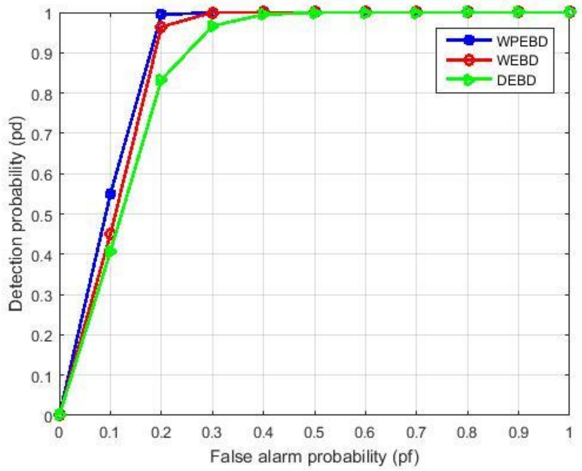


**Figure 4.** Detection performance vs. false alarm probability for DEBD, WEBD and WPEBD (SNR=-25 dB) and sample Size (N=10,000).

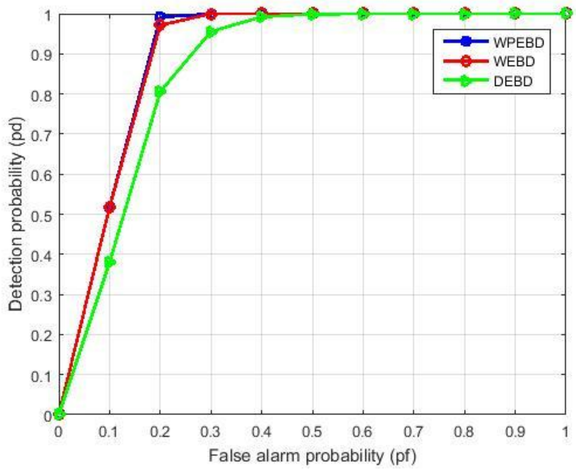


**Figure 5.** Detection performance vs. false alarm probability for DEBD, WEBD and WPEBD (SNR=-15 dB) and sample Size (N=10,000).

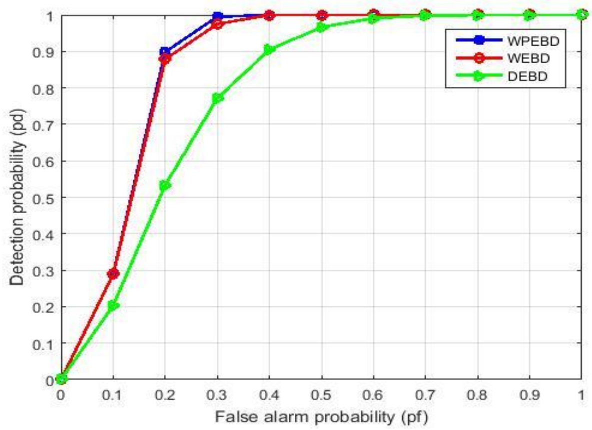


**Figure 6.** Detection performance vs. false alarm probability for DEBD, WEBD and WPEBD (SNR=-5 dB) and sample Size (N=10,000).

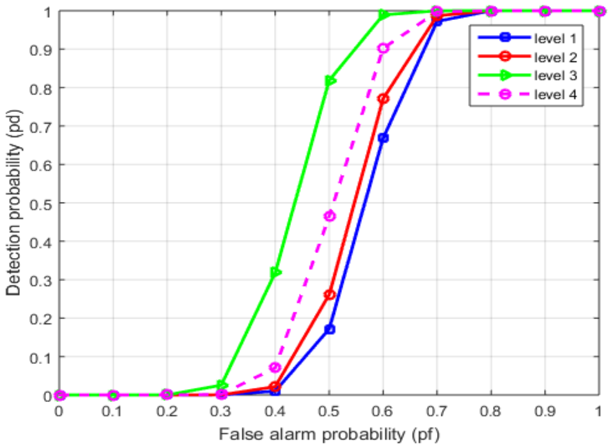


**Figure 7.** Detection performance vs. false alarm probability of WEBD at various levels j, SNR=-25 dB and sample Size N=10,000

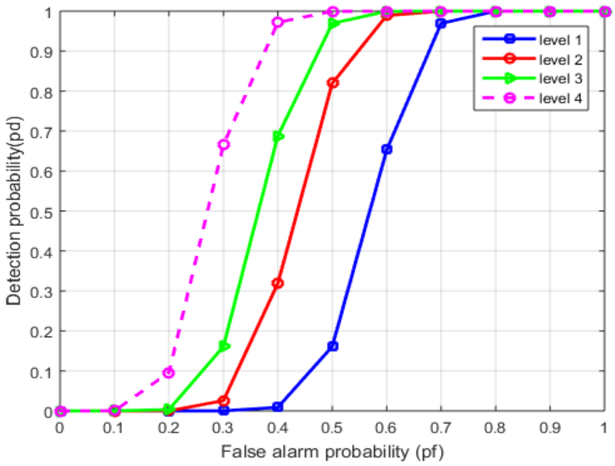


**Figure 8.** Detection performance vs. false alarm probability of WPEBD at various levels j, SNR=-25 dB and sample Size (N=10,000).

Figure 7 and 8 shows the performance curves of detection for different levels of decomposition j for probability of false alarm for WEBD and WPEBD that is constant. We can see from the Fig 7 and 8 that WPEBD is definitely more efficient than WEBD for all levels of decomposition.

## IV. CONCLUSION

Effectiveness of a CRN depends on how accurately and efficiently the receiver can detect occupancy of a frequency subband by the PU who is its authorized or licensed user. In this paper, performances of three entropy based detection techniques have been analyzed. Monte Carlo experiments have been carried out to establish the efficiency of each detection techniques being used for detecting the PU signal. A comparative analysis between detection techniques based on DFT entropy, wavelet entropy and entropy of wavelet packets have been carried out using ROC curves of probability of detection and probability of false alarm for each technique. SNR plays a important role in wireless communication. The analyses have been carried out for various SNR scenarios like -25 dB, -15 dB and -5 dB. Wavelet packet entropy based detection or WPEBD has proved to be the most efficient entropy based detection technique even at low SNR of -25dB. This finding can go a long way in further research and design for detection techniques for CRN.

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