International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 6 (2019), pp. 891-898

Theoretical Investigation of Stresses and Displacement in RC Annular Slabs

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Abstract

The aim of this paper is the static bending analysis of reinforced concrete (RC) annular slabs using analytical methods, i.e. classical plate theories and finite element method (FEM). The axisymmetric bending of the annular slab is considered in the present study. Three cases of annular slabs, in which the diameter of the central opening varies, and they are simply supported at outer edge have been considered. The bending stresses in both radial and circumferential directions and deflection were investigated by using different approaches of the classical theory of plates based on Love-Kirchhoff's (L-K) hypothesis, then compared the achieved results to the numerical analysis-FEM results. For this purpose, the 3D-modelling and simulation, with the subsequent analysis of annular slab were done in ABAQUS computer program. Three-dimensional 8-node first order fully integration continuum elements (C3D8 - Bricks) are used to model the concrete annular slab and loading. In addition, three-dimensional 2-node first order truss elements (T3D2 - Truss) are used to model the steel reinforcing bars. Once deflection, bending moments and bending stresses are obtained by both methods, some approaches show close results with no significant difference, and others show results with not acceptable difference. This difference in results can be explained by the fact, that FE analysis represents the exact model of annular plate (i.e. 3D model representation) which is consist of concrete and reinforcements 3D modelled slab with all properties for each material. All of these properties and the combination of materials can't be done throw the classical plate theories equations. Also, classical plate theories depend on a number of assumption which has a significant effect on the results.

Keywords: stress, circumferential, radial, displacement, FEM.

1. INTRODUCTION

Nowadays, circular slabs are an integral part of residential and administrative buildings in modern cities. If from a practical point of view, the application of circular and annular plates is less important than rectangular, however, the theoretical aspects are quite interesting. When using classical methods of analysis and by making use of axial symmetry, the exact solution for a large number of problems comes within reach. This even holds for incremental elastic-plastic calculations and anisotropic material behaviour (Vrouwenvelder and Witteveen, 2003).

A long time ago, the effectiveness of classical methods in the analysis of annular homogeneous plates (steel plates) was proven. Another thing, when these methods are faced with such plates as reinforce concrete slabs. However, the appearance of the computer made a turn towards numerical methods, especially the development of the computer programs based on the FEM, for example, ABAQUS, which can solve all the nuances of the adequate modelling during simulation (Desayi and Muthu, 2013; Jomehzadeh et al., 2009; Reddy, 2006; Reddy et al., 1999; Vrouwenvelder and Witteveen, 2003).

Known, that in classical plate theory, because of the smallness of thickness dimension, it is often not necessary to model them using 3D elasticity equations. Simple 2D plate theories can be developed to study the strains and stresses in plate structures. However, in the presence of powerful computing tools, it is worthwhile to take a look at the stress state of plates modelled in 3D form (Iwaki et al., 1985).

The classical theory of L-K satisfactorily describes the stress state of a plate in the case when the relative thickness of the plate is a value substantially less than unity. As the plate thickness increases, the errors of this theory are significant. In this connection, the transition to the 3D formulation of the problem of plate bending is very relevant.

When structural engineers carry out finite element analysis (FEA) of plates, in most cases, they are primarily interested in determining the maximum value of the stresses and displacement, an indication of their location, that is absolutely correct in terms of subsequent design (Bäcker et al., 2014; Hui and Zehnder, 1993; Shmukler et al., 2010; Skibeli, 2017).

In modern literature, the issues of analysis of annular reinforced concrete plates are not thoroughly covered. It complicates the use of results which were obtained in theory and often leads to approximate methods application, due to the difficulty of applying exact methods (Iwaki et al., 1985; Le et al., 2010).

The flexural properties of annular plates largely depend on its thickness rather than its diameter, supporting and the ratio of the radius of opening to the radius of the entire plate. The value of deflection and stresses can be determined by solving the differential equations of an appropriate plate theory using

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 12, Number 6 (2019), pp. 891-898

different analytical approaches (Gujar and Ladhane, 2015; Timoshenko and Woinowsky-Krieger, 1959).

The main aim of this research is a comparative evaluation of the value of the maximum deflection, radial and circumferential stresses in the annular reinforced concrete slab and the most vulnerable locations in terms of these functions applying classical and numerical methods.

2. STRUCTURE ANALYSIS

Classical plate theory will be used for calculating flexural parameters, i.e. investigating the deflection and stresses of the annular slab. The analytical results obtained from classical plate theory will be studied and compared with the numerical results.

Known, that creating an adequate computational model is the first important step in the finite element analysis of plates. Before embarking on modelling, however, the geometry and boundary conditions of the structure along with the applied loads, must be clearly defined. Proper modelling starts with a good conceptual understanding of the physical behaviour of the plate, which includes the anticipated stress and deflection patterns (Szilard, 2004).

For a more realistic approach of the FE modelling, it was decided to create a three-dimensional model of the annular slab. 3D modelling helps to comprehensively investigate the value and position of the maximum stresses and deflection under uniformly distributed load (UDL) (Vainiunas et al., 2004).

2.1 Classical Theory

Classical plate theory is the thin plate theory based on L-K hypothesis which makes assumptions similar to those done by the Bernoulli-Navier hypothesis used in the theory of thin or shallow beams. It is also called as small deflection theory. Passing through the literature of the last two centuries, it is easy to notice various approaches to the analysis of plates in general, and annular plates in particular, although the source of all is the classical theory L-K (Bäcker et al., 2014).

In this work, the polar coordinates r and θ was used, and the following fundamental assumptions are considered in the classical small deflection theory of thin homogenous elastic plates (Szilard, 2004; Timoshenko and Woinowsky-Krieger, 1959; Ventsel and Krauthammer, 2001).

1. Straight line initially normal to the middle surface to the plate remains straight and normal to the deformed middle surface of the plate and unchanged in length.
2. Displacement w is assumed to be very small. This means the slope of the deflected surface is small and hence square of the slope would be negligible in comparison with unity.
3. The normal stresses σ_x and σ_y in-plane shear stress τ_{xy} are assumed to be zero at middle surface of the plate. i.e. (w<<h).
4. Stress σ_z, i.e. normal transverse stress is small as compared to other stress components and may be neglected in stress-strain relationship $\sigma_z \ll (\sigma_x, \sigma_y, \tau_{xy})$
5. The midplane remains unstrained after bending.
6. Governing Differential equation for symmetrical bending of circular plate:

$$\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr} = \frac{Q}{D} \quad \dots\dots\dots \ (1)$$

In another form, it can be written as

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\frac{dw}{dr})\right] = \frac{Q}{D} \quad \dots\dots\dots \ (2)$$

In this research, it is customary to use the following approaches for analysis of the simply supported at the outer edge annular RC slabs, with a central hole diameter of 200, 300, and 400 mm and subjected to the UDL.

2.1.1 Reddy approach

According to the Reddy approach the required unknowns at the section of the annular plates can be calculated as follows (Reddy, 2006):

$$w = \frac{qa^4}{64D}\left\{-[1-\left(\frac{r}{a}\right)^4] + \frac{2\alpha_1}{1+\nu}[1-\left(\frac{r}{a}\right)^2] - \frac{4\alpha_2\beta^2}{1-\nu}\,log\left(\frac{r}{a}\right)\right\} \quad \dots\dots\dots \ (3)$$

$$M_\theta = \frac{qa^2}{16}\left\{(3+\nu)[1-\left(\frac{r}{a}\right)^2] + \beta^2[(5\nu-1) + (3+\nu)\left(\frac{r}{a}\right)^2] + 4(1+\nu)\beta^2\kappa[1+\left(\frac{r}{a}\right)^2]\right\} \quad \dots\dots\dots \ (4)$$

$$M_r = \frac{qa^2}{16}\left\{(3+\nu)[1-\left(\frac{r}{a}\right)^2] - \beta^2(3+\nu)[1-\left(\frac{r}{a}\right)^2] + 4(1+\nu)\beta^2\kappa[1-\left(\frac{r}{a}\right)^2] + 4(1+\nu)\beta^2\,log\left(\frac{r}{a}\right)\right\} \quad \dots\dots\dots \ (5)$$

where

$$\alpha_1 = (3+\nu)(1-\beta^2) - 4(1+\nu)\beta^2\kappa, \quad \alpha_2 = (3+\nu) + 4(1+\nu)\kappa$$

$$\kappa = \frac{\beta^2}{1-\beta^2}log\,\beta\,, \quad \beta = \frac{b}{a}\,, \quad D = \frac{Eh^3}{12(1-\nu^2)}$$

2.1.2 Young's approach.

Considering Young's approach, we will carry out the following calculation formulas (Budynas et al., 2011):

$$w = w_b + \theta_b r F_1 - q\frac{r^4}{D}G_{11} \quad \dots (6)$$

$$M_\theta = \frac{\theta D(1-v^2)}{r} + v\, M_r \quad \dots (7)$$

$$M_r = \theta_b \frac{D}{r} F_7 - qr^2 G_{17} \quad \dots (8)$$

where

$$w_b = -\frac{qa^4}{D}\left(\frac{C_1 L_{17}}{C_7} - L_{11}\right), \qquad \theta_b = \frac{qa^3}{DC_7}L_{17},$$

$$\theta = \theta_b F_4 - q\frac{r^3}{D}G_{14}$$

Values of C_1, C_7, F_4, F_7, L_{11}, L_{17}, G_{14} and G_{17} can be found from Budynas et al. (2011).

2.1.3 Rahul approach

Rahul determined the hoop and radial stresses at the sections of the annular plate by using the following equations (Rahul, 2013).

$$M_\theta = \frac{-1}{16}qr^2 + (0.25qb^2)[log_e\left(\frac{r}{a}\right) + \frac{3}{4}\left(\frac{-1}{3} + \frac{a^2}{b^2} + \frac{a^2}{r^2}\right) + \frac{a^2+r^2}{a^2-b^2}\left(\frac{b}{r}\right)^2 log_e(\frac{a}{b})] \quad \dots\dots (9)$$

$$M_r = \frac{-3}{16}qr^2 + (0.25qb^2)[log_e\left(\frac{r}{a}\right) + \frac{3}{4}\left(1 + \frac{a^2}{b^2} - \frac{a^2}{r^2}\right) + \frac{a^2-r^2}{a^2-b^2}\left(\frac{b}{r}\right)^2 log_e(\frac{a}{b})] \quad \dots\dots (10)$$

Using expression $\sigma=6M/h^2$ the corresponding bending stresses can be found from the moments M_r and M_θ for the above mentioned approaches.

In the current study, the bending stresses in both radial and circumferential directions and deflection according to the equations mentioned above are determined, then compared the achieved results to the FEA results.

Fig.1 presents the schematic illustration of annular RC slabs under UDL and investigated in this research stresses in circumferential and radial directions, as well as the vertical deflection in the z-direction.

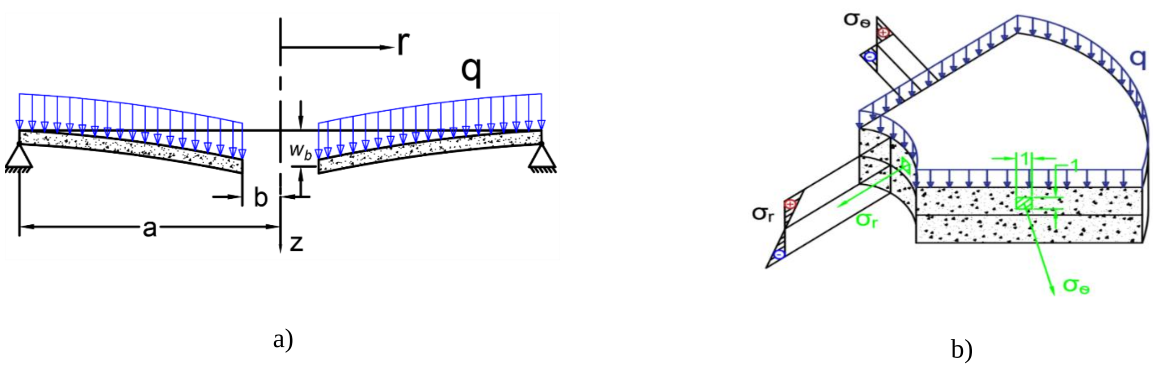


Figure 1. Schematic illustration of the simply supported annular RC slab under UDL, a-dimensions and deflection w_z, b- circumferential and radial stresses distribution across slab section, and σ_θ and σ_r at given point of cross-section

2.2 Numerical Analysis

It is of fundamental importance in the numerical analysis of the structures to know how the applied load is transferred to the stress and what is the stress and displacement distribution pattern within the investigated models. FEM is one of the most powerful and effective approaches for analyzing and investigating the stress state of the materials under different types of loading. FEM is an analytical technique which in its turn is the basis for computational analysis using software programs (Jofriet and McNeice, 1971; Kwak and Filippou, 1990; Milligan, 2018; Setiawan et al., 2018).

This goal could not be achieved without the help of a powerful computer program like ABAQUS. For this purpose, studied annular plates were modelled and simulated under lateral UDL applied to the top face of the slabs.

The solid (or continuum) elements in ABAQUS can be used for linear analysis and complex nonlinear analyses involving contact, plasticity, and large deformations. Regarding the finite element models introduced in this work, three-dimensional 8-node first order fully integration continuum elements (C3D8 - Bricks) are used to model the concrete annular slab and loading. In addition, reinforcing bars are modeled as three -dimensional truss elements. Three-dimensional 2-node first order truss elements (T3D2 - Truss) are used to model the steel reinforcing bars in the FE model of the concrete annular slab ("Abaqus 2016 Documentation," 2016).

3. ILLUSTRATIVE EXAMPLES

Three cases of the simply supported at outer edge annular slabs are considered for the present study:

1) Simply supported annular plate with a 200 mm diameter of the opening.
2) Simply supported annular plate with a 300 mm diameter of the opening.
3) Simply supported annular plate with a 400 mm diameter of the opening.

The geometrical and physical properties of all three investigated annular plates are presented below:

Plate Diameter =1000 mm, & plate thickness (h)=80mm.

Hole Diameter for Case 1=200 mm, Case 2=300 mm, Case 3=400 mm.

Poisson's ratio of Reinforcement (ν_s)=0.3.

Young's modulus of Reinforcement (E_s)=200 GPa.

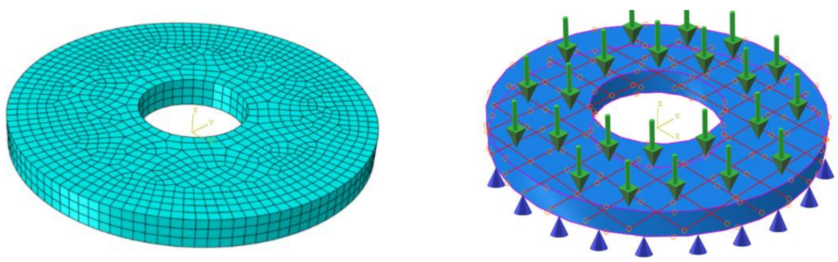
Steel Reinforcement Ø10 @100mm both directions.

Poisson's ratio of Concrete (ν_c) = 0.2.

modulus Young's of Concrete (E_c)=23.5 GPa.

Applied uniformly distributed Load=20 kPa.

To achieve more accurate results, after meshing the specimens are subdivided into 3000-3600 3D small elements of simple shapes connected at nods. Thus stress of all small elements was calculated, and there was a complete oblique of the stress-strain state of the entire specimen. Fig. 2 presents the meshing of the specimen, and it's loading, supporting, and reinforcement.



a) b)

Figure 2. Annular reinforced concrete slab, a-slab meshing, b- loading, supporting, and reinforcement of the slab.

4. RESULTS AND DISCUSSION

This paper proposed to create a three-dimensional model of the annular slabs under lateral load, then find all post analysis parameters, including the stress fields in both radial and circumferential directions and deflection for investigated in this study three cases, then compared the achieved results by FEM with the analytical results by above mentioned approaches on the basis of classical plate theory.

After numerous trials, the locations of occurrence of the maximum 6_{θ} (circumferential or hoop stress), 6_{r} (radial stress), and w_z (z-directed deflection) were reliably determined, which coincides, with great accuracy, with the results of some methods used in this work (Fig.3). At the same time, it is not difficult to identify significant discrepancies with other methods.

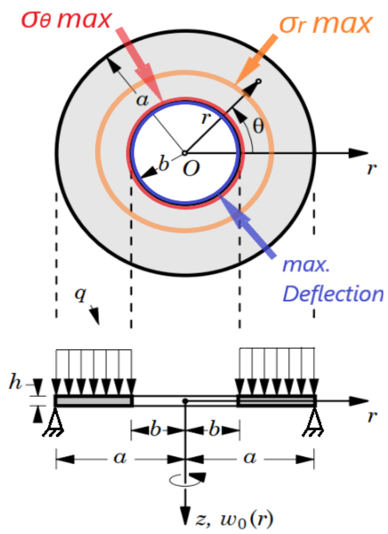


Figure 3. Location of maximum bending stresses and deflection in annular slab

The stress fields, as well as the distribution of radial and circumferential stresses, are presented in Figs. 4, 5, which shows that by the transition from green to red or blue color of the stress fields the value of the stresses was increased, and radial stress reached its maximum at the edge of the hole of the slab, however the maximum value of circumferential stress located near the edge of inner hole of the slab, i.e.(a-b)/3 calculated from the edge of the hole.

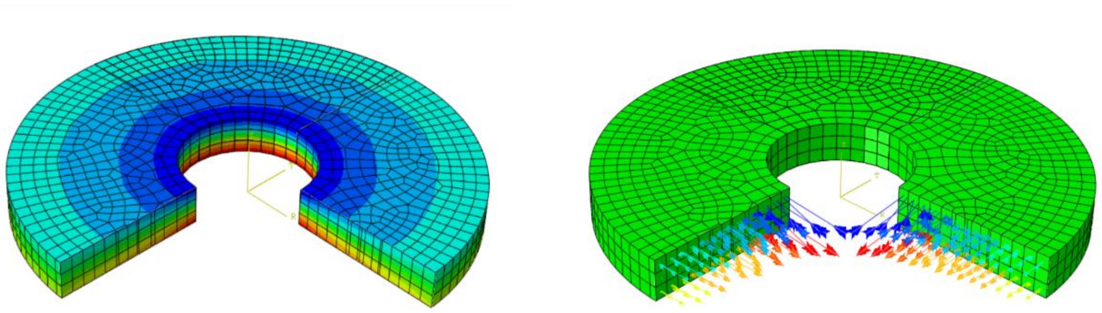
It is easy to observe that the results obtained for maximum θ-directed bending stress from analytical equations give a variety of results, each of them is more or less differ from obtained by FEM.

Sectioning of the slab in the computer model demonstrates the pattern of the distribution of stress in θ-direction in both field and vector forms (Fig.4 a, b). Fig.4c presents the good agreement between the results obtained according to Eq.(4), as reported by Reddy (2006), and FEA results. These results are close and differ only by 16-19%, there is no significant difference.

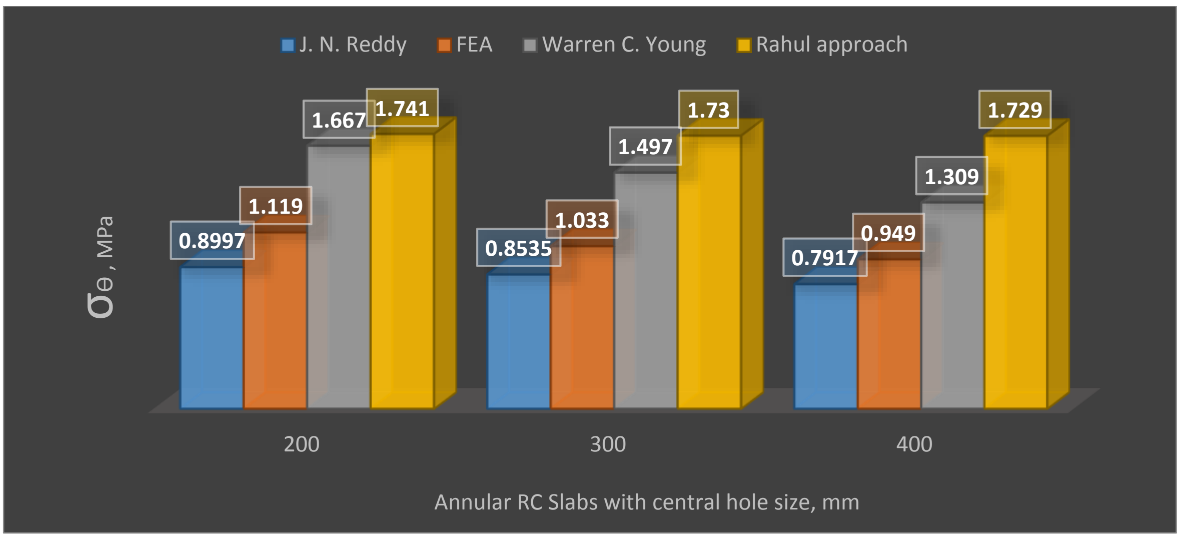
The discrepancy between the results rapidly increases and reaches 35% when applying Eq.(7 and 9), which can be explained by the considering the following assumptions in these equations (Budynas et al., 2011; Rahul, 2013):

1) The plate is flat, of uniform thickness, and of homogeneous isotropic material; 2) the thickness is not more than about one-quarter of the least transverse dimension, and the maximum deflection is not more than about one-half the thickness; 3) all forces loads and reactions are normal to the plane of the plate; 4) the plate is nowhere stressed beyond the elastic limit.

On the other hand, in this work, it was carried out the comparative evaluation of the maximum stresses as a function of the diameter of the central hole in the annular RC slab. The results presented in the diagram of the Fig.4c show that in all cases (diameter of central hole 200mm, 300mm and 400mm), and with the use of various methods of analysis (classical and numerical) with increasing diameter of the hole, the maximum circumferential stress not significantly, but still decreases.



a) b)



c)

Figure 4. Circumferential (hoop) Stress illustration, a- 6 $_{\theta}$ field within the slab, b- vector presentation of 6 $_{\theta}$ across the slab, c- graphically comparative evaluation of the maximum 6 $_{\theta}$ by different methods

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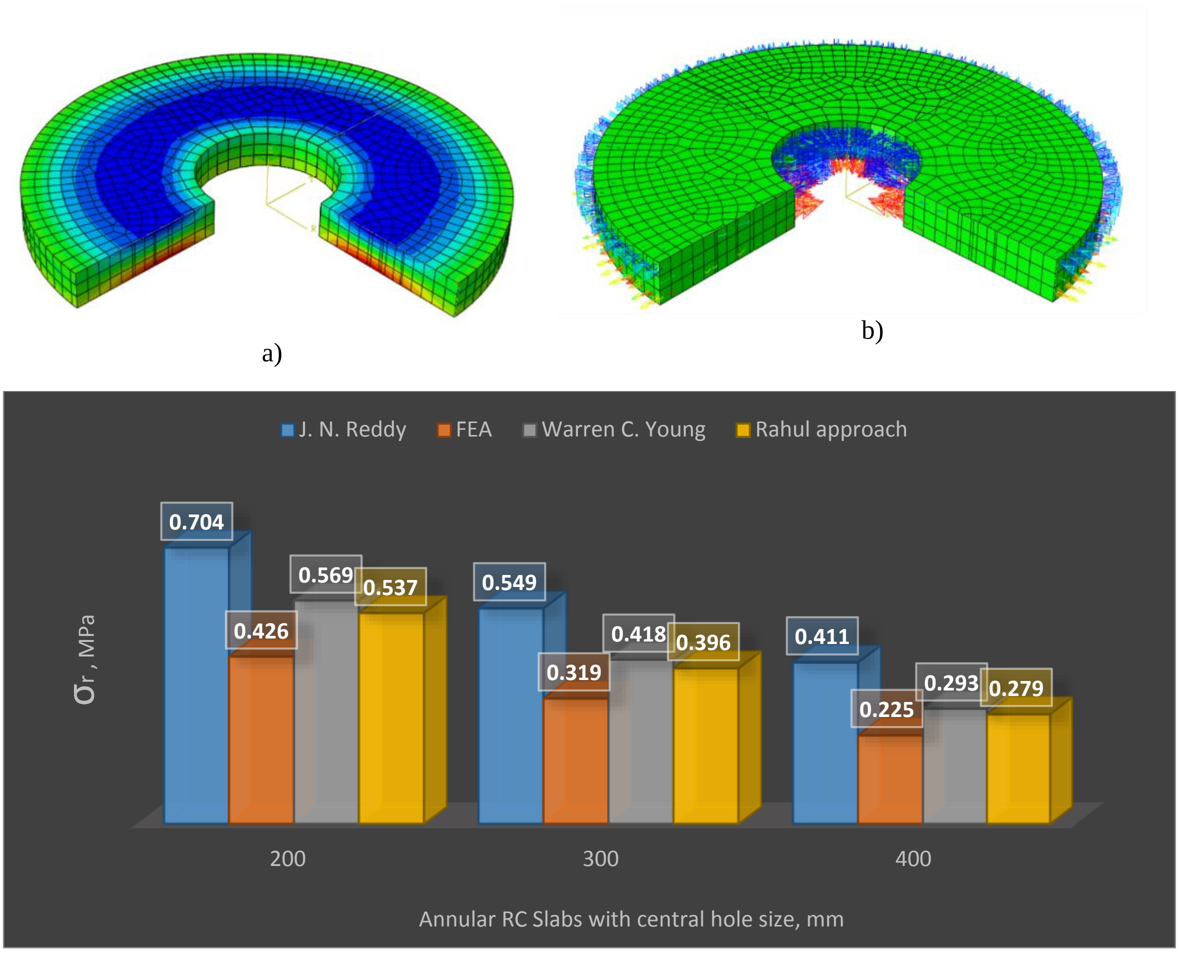
As for stress in the radial direction (σ_r), there is a slight difference to some extent between obtained results by FEM and analytical equations compared with maximum θ-directed bending stress (σ_θ).

Fig.5a, b represents a slab section, modelled by ABAQUS program, which shows the distribution of stress in radial direction in both field and vector illustrations. And Fig.5c shows graphically comparative evaluation of the maximum σ_r by different methods. According to Fig.5c the results obtained by FEA give fewer values than analytical Eq.(5, 8, 10), which are calculated by using the same approaches mentioned above.

This difference in results can be explained by the fact that FE analysis represents the exact model of annular slab which is consist of concrete and reinforcements, 3D modelled slab with all properties for each material. All of these properties and the combination of materials can't be done throw the classical plate theories equations. In addition, classical plate theories depend on a number of assumption which has a significant effect on the results.

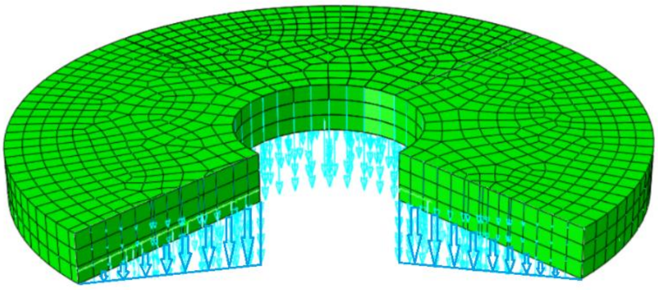


c)

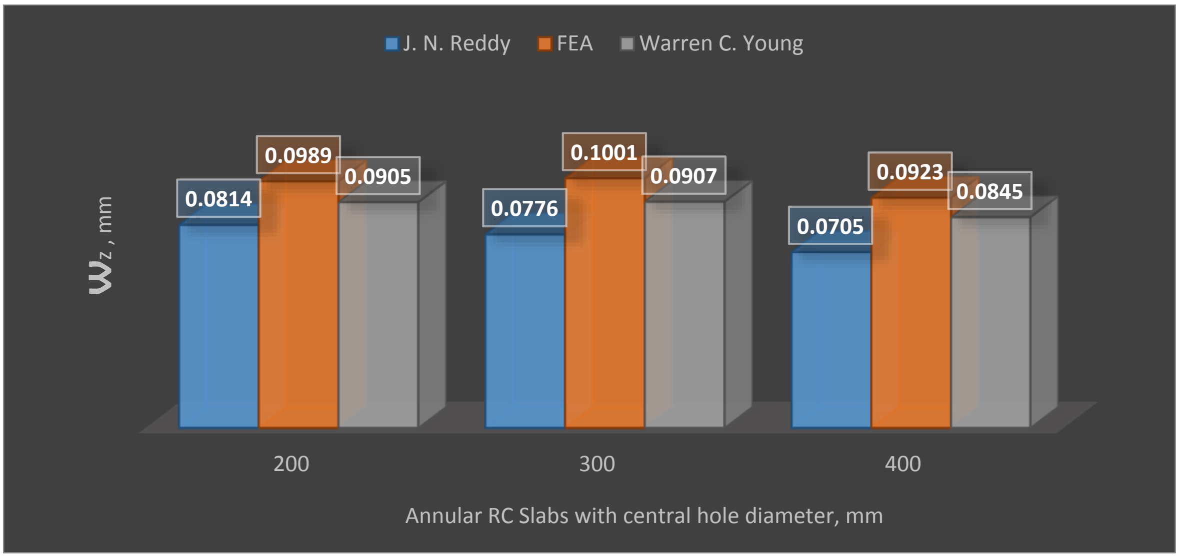
Figure 5. Radial stress presentation, a- σ_r field within the slab, b- vector illustration of σ_r across the slab, c-graphically comparative evaluation of the maximum σ_r by different methods

In addition to stress investigation, in this research work, it was carried out the comparative assessment of the maximum deflection of the slab in the z-direction, analysis performed by FEM and Eq.(3, 6) (Budynas et al., 2011; Reddy, 2006). Qualitative diagram of the vertical deflection-w_z presented in Fig.6a. And the absolute value of the maximum deflection (w_z) obtained by analytical approaches and FEM illustrated in Fig.6b.

Obviously that the location of maximum deflection of the annular slab is at the edge of the hole, this value decreases gradually and become zero at support. And according to Fig. 6b results obtained from the analytical methods by different approaches and ABAQUS nearly matches about 76-81% which with great confidence confirms the adequacy of the proposed 3D model and the accuracy of the analysis using the finite element method.



a)



b)

Figure 6 a.- vertical deflection diagram, **b.** the absolute value of the maximum z-directed deflection (w_z) obtained by analytical approaches and FEM

5. CONCLUSION

The comparison of the FEA results with the different plate theory approaches demonstrates the correctness of the FE modelling of annular RC slab, as well as the purity of the 3D simulation in ABAQUS software environment. This research showed that annular slab solid FE created according to the developed technology possess all necessary strength and rigidity attributes for laterally loading. Suggested modelling and simulation with high accuracy represent the real annular slab, and the selected type of the finite elements is adequate to the investigation of the stress-strain state of the structure.

As well as this research showed not sufficient accuracy of some approaches to the classical theory in the search for hoop stresses, and other approaches for radial. The use of these approaches in determining vertical displacement seems more acceptable and the results are very close to the output of the FEA. The investigation of behaviour RC annular slabs with different size of the central hole showed that the diameter of the opening not significantly affected on the location of the concentration of the maximum radial and hoop stresses.

In general, worth noting that the performed investigations confirm representativeness of 3D finite element modelling of the annular RC slabs, which opens fundamentally new horizons in the analysis of the plates by using numerical methods. Such a convergence and discrepancy between the results of 3D analysis and classical methods can be explained by the lack of clear factors that take into account the nature of concrete, that in our view, the proposed model is closer to the real concrete slab than previous approaches.

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