

Mathematical Modeling and Optimization of the Control System for Multi-Motor Electric Drive of Conveyor Belt

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ABSTRACT:

This article discusses the problems of mathematical modeling and optimal control of a multi-motor asynchronous electric drive of synchronous rotation with the frequency converter of belt conveyor. To determine the optimal control curve for the electric drive of the conveyor, the mathematical model of the conveyor was compiled, and the Pontryagin L.S. principle method was chosen, the algorithm and program for calculating the optimal control of a multi-motorized electric drive of belt conveyor were compiled. Simulation of the electric drive of conveyor belt, the calculations were made in the Matlab software package.

Keywords - Belt conveyor, frequency regulation, mathematical modeling, multi-motor electric drive, optimal control.

I. INTRODUCTION

Multi-drive belt conveyors serve as the main means of continuous transport in mines and pits, and have been widely used as a means of ore delivery through intermediate and modular drifts, slopes, and also along inclined trunks. Used in them multi-motor AC electric drives are the most massive and energy-intensive consumers of electrical energy [1].

The problems of constructing and optimizing the operating modes of conveyor electric drives attract the wide attention of specialists. In particular, the appearance on the market of reliable, high-quality and relatively inexpensive semiconductor energy converters in combination with automation tools creates the prerequisites for the wide use of technical advances to solve problems of energy and resource saving.

The development of the theory of optimal control of electric drives is associated with an increasing demand for speed and

accuracy of electric drive control systems of industrial mechanisms. The increase in speed is possible only with the correct distribution of limited control resources, and therefore accounting for control constraints is one of the central ones in the theory of optimal control, in particular, a multi-motor electric conveyor belt [2-4]. The above indicates the relevance of the research in this work.

II. MATERIALS AND METHODS

The requirements of high reliability, preventing slippage of the belt, coordinated rotation of several electric motors and ensuring a smooth start are imposed on the electric drive of conveyors [5]. In the process of acceleration and deceleration of the belt conveyor with the multi-motor electric drive, especially high power, optimal control is one of the necessary tasks of synchronizing the operation of the drives, stabilizing the tension of the conveyor belt and reducing the power consumption during the transition of the working mechanism from one position to another. This problem can be successfully solved by the Pontryagin L.S. maximum principle method [6] with the appropriate mathematical description of the belt conveyor transients process.

Following the requirements imposed on the conveyor electric drive, the task of optimal control of the conveyor electric drive can be attributed to one of the tasks of converting the state of a multi-motor electric drive to another state, i.e. ensure a smooth start. In this case, this task is to find the control law as a function of time [6].

III. RESULTS AND DISCUSSION

The mathematical model of a three-motor asynchronous electric drive with frequency control, based on a single-motor electric drive with frequency control [7], is represented by the following system of equations [8]:

$$\left\{ \begin{aligned}
 \frac{d\Delta\omega_1}{dt} &= \frac{1}{\beta_1 T_{m1}} M_1 - \frac{1}{\beta_1 T_{m1}} M_{c1}; \\
 \frac{dM_1}{dt} &= \frac{\beta_1}{T_{E1}} \Delta\omega_{01} - \frac{\beta_1}{T_{E1}} \Delta\omega_1 - \frac{1}{T_{E1}} M_1; \\
 \frac{d\Delta\omega_{01}}{dt} &= \frac{k_{PCh1}}{T_{PCh1}} \Delta u_{RS1} - \frac{1}{T_{PCh1}} \Delta\omega_{01}; \\
 \frac{d\Delta u_{RS1}}{dt} &= \frac{k_{RS1} k_{s.m.f1}}{\beta_2 T_{m2}} (M_2 - M_{c2}) - \frac{k_{RS1} (k_{s.m.f1} + k_{s.m.f3} + k_{s.f1})}{\beta_1 T_{m1}} (M_1 - M_{c1}) + \\
 &+ \frac{k_{RS1} k_{s.m.f3}}{\beta_3 T_{m3}} (M_3 - M_{c3}) + \frac{\Delta u_3}{T_{RS1}} + \frac{k_{s.m.f1}}{T_{RS1}} \Delta\omega_2 - \frac{k_{s.m.f1} + k_{s.m.f3} + k_{s.f1}}{T_{RS1}} \Delta\omega_1 + \frac{k_{s.m.f3}}{T_{RS1}} \Delta\omega_3; \\
 \frac{d\Delta\omega_2}{dt} &= \frac{1}{\beta_2 T_{m2}} M_2 - \frac{1}{\beta_2 T_{m2}} M_{c2}; \\
 \frac{dM_2}{dt} &= \frac{\beta_2}{T_{E2}} \Delta\omega_{02} - \frac{\beta_2}{T_{E2}} \Delta\omega_2 - \frac{1}{T_{E2}} M_2; \\
 \frac{d\Delta\omega_{02}}{dt} &= \frac{k_{PCh2}}{T_{PCh2}} \Delta u_{RS2} - \frac{1}{T_{PCh2}} \Delta\omega_{02}; \\
 \frac{d\Delta u_{RS2}}{dt} &= \frac{k_{RS2} k_{s.m.f2}}{\beta_3 T_{m3}} (M_3 - M_{c3}) - \frac{k_{RS2} (k_{s.m.f2} + k_{s.m.f1} + k_{s.f2})}{\beta_2 T_{m2}} (M_2 - M_{c2}) + \\
 &+ \frac{k_{RS2} k_{s.m.f1}}{\beta_1 T_{m1}} (M_1 - M_{c1}) + \frac{\Delta u_3}{T_{RS2}} + \frac{k_{s.m.f2}}{T_{RS2}} \Delta\omega_3 - \frac{k_{s.m.f2} + k_{s.m.f1} + k_{s.f2}}{T_{RS2}} \Delta\omega_2 + \frac{k_{s.m.f1}}{T_{RS2}} \Delta\omega_1; \\
 \frac{d\Delta\omega_3}{dt} &= \frac{1}{\beta_3 T_{m3}} M_3 - \frac{1}{\beta_3 T_{m3}} M_{c3}; \\
 \frac{dM_3}{dt} &= \frac{\beta_3}{T_{E3}} \Delta\omega_{03} - \frac{\beta_3}{T_{E3}} \Delta\omega_3 - \frac{1}{T_{E3}} M_3; \\
 \frac{d\Delta\omega_{03}}{dt} &= \frac{k_{PCh3}}{T_{PCh3}} \Delta u_{RS3} - \frac{1}{T_{PCh3}} \Delta\omega_{03}; \\
 \frac{d\Delta u_{RS3}}{dt} &= \frac{k_{RS3} k_{s.m.f3}}{\beta_1 T_{m1}} (M_1 - M_{c1}) - \frac{k_{RS3} (k_{s.m.f3} + k_{s.m.f2} + k_{s.f3})}{\beta_3 T_{m3}} (M_3 - M_{c3}) + \\
 &+ \frac{k_{RS3} k_{s.m.f2}}{\beta_2 T_{m2}} (M_2 - M_{c2}) + \frac{\Delta u_2}{T_{RS3}} + \frac{k_{s.m.f3}}{T_{RS3}} \Delta\omega_1 - \frac{k_{s.m.f3} + k_{s.m.f2} + k_{s.f3}}{T_{RS3}} \Delta\omega_3 + \frac{k_{s.m.f2}}{T_{RS3}} \Delta\omega_2.
 \end{aligned} \right. \quad (1)$$

The designations of the variables of the system of differential equations (1) are presented in Table 1. The mathematical model represented by the system of equations (1) takes into account feedbacks on the mismatch of the speeds of asynchronous motors $k_{s.m.f1}$, $k_{s.m.f2}$, $k_{s.m.f3}$ three-motor asynchronous electric drives with frequency control introduced to maintain synchronous rotation of electric motors.

The equations of motion of the electric conveyor belt, to determine the optimal control, will consider on the basis of the system of equations (1), taking into account that in addition to the interconnections between conveyor motors for speed mismatch, there is also an interconnection of conveyor drive drums through a conveyor belt with a certain tension value T .

The basic relations characterizing the interconnection of electric motors through a conveyor belt, consider on the basis of the scheme presented in Fig. 1.

According to the scheme in Fig. 1, the radii of the drive drums are the same:

$$R_{B1} = R_{B2} = R_{B3}. \quad (2)$$

Elastic deformation in the kinematic chain asynchronous motor-drive drum of each of the drives is missing.

Enter a number of assumptions:

- 1) the belt is uniform and has the same length over the entire length;
- 2) the weight of the belt does not affect its deformation;
- 3) wave processes associated with the propagation of deformation along the entire length of the belt are absent;
- 4) there is no slip of the driving belt relative to the drums.

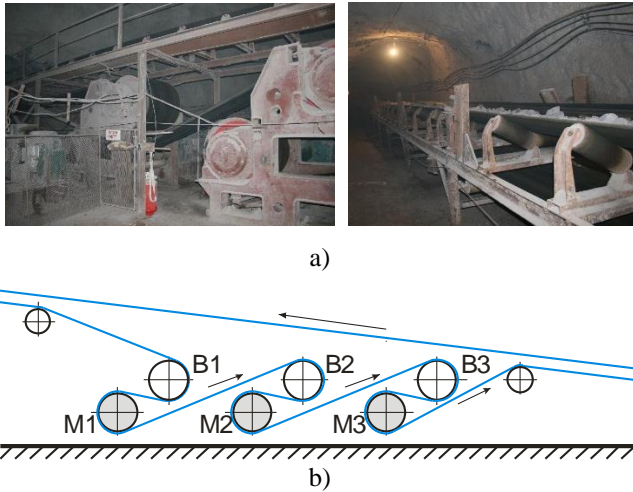


Fig. 1. General view (a) and kinematic scheme (b) of the three-motor belt conveyor

The behavior of the belt at the stretching section length l_2 is described by the following differential equation [9]:

$$\frac{d\varepsilon_2}{dt} = [v_2 - v_1(1 + \varepsilon_2)] \frac{1}{l_2}, \quad (3)$$

where $\varepsilon_2 = \frac{\Delta l_2}{l_2}$ - relative elongation;

v_2, v_1 - linear speeds of the conveyor belt at the beginning and at the end of the stretching section;

Δl_2 - absolute value of stretching.

The tension that occurs in the belt is related to the value Δl_2 of the expression

$$T_2 = c_2 \Delta l_2 = c_2 l_2 \varepsilon_2, \quad (4)$$

where c_2 - stiffness coefficient of a rubber-fabric belt under tension.

Substituting equation (3) into equation (4) and writing down the equilibrium moment equation in the first electric drive, obtain the system of equations.

$$\begin{cases} \beta_1 T_{m1} \frac{d\omega_1}{dt} = M_1 - M_{c1} + (T_2 - T_1) R_{B1}; \\ F_2 = \frac{c_2}{p} [v_2 - v_1(1 + \varepsilon_2)] \end{cases} \quad (5)$$

The system of equations (5), taking into account (3) and (4), in increments of coordinates can be written in the following form

$$\begin{cases} \beta_2 T_{m2} \frac{d\Delta\omega_2}{dt} = \Delta M_2 - \Delta M_{c2} + (\Delta T_3 - \Delta T_2) R_{B2}; \\ \frac{d\Delta T_3}{dt} = \frac{c_3 R_{B3}}{j_2} \Delta\omega_3 - \left(1 + \frac{1}{c_3 l_3} T_3^*\right) \frac{c_3 R_{B2}}{j_2} \Delta\omega_2 - \frac{R_{B2} \omega_2^*}{j_2 l_3} \Delta T_3. \end{cases} \quad (6)$$

For the three-motor asynchronous electric drive of belt

conveyor with frequency converters, the system of equilibrium moment equations can be written as

$$\begin{cases} \frac{d\Delta\omega_1}{dt} = \frac{1}{\beta_1 T_{m1}} \Delta M_1 - \frac{1}{\beta_1 T_{m1}} \Delta M_{c1} + \frac{1}{\beta_1 T_{m1}} (\Delta T_2 - \Delta T_1) R_{B1}; \\ \frac{d\Delta T_2}{dt} = \frac{c_2 R_{B2}}{j_2} \Delta\omega_2 - \left(\frac{c_2 R_{B1}}{j_1} + \frac{R_{B1} T_2^*}{j_1 l_2}\right) \Delta\omega_1 - \frac{R_{B1} \omega_1^*}{j_1 l_2} \Delta T_2; \\ \frac{d\Delta\omega_2}{dt} = \frac{1}{\beta_2 T_{m2}} \Delta M_2 - \frac{1}{\beta_2 T_{m2}} \Delta M_{c2} + \frac{1}{\beta_2 T_{m2}} (\Delta T_3 - \Delta T_2) R_{B2}; \\ \frac{d\Delta T_3}{dt} = \frac{c_3 R_{B3}}{j_2} \Delta\omega_3 - \left(\frac{c_3 R_{B2}}{j_2} + \frac{R_{B2} T_3^*}{j_2 l_3}\right) \Delta\omega_2 - \frac{R_{B2} \omega_2^*}{j_2 l_3} \Delta T_3; \\ \frac{d\Delta\omega_3}{dt} = \frac{1}{\beta_3 T_{m3}} \Delta M_3 - \frac{1}{\beta_3 T_{m3}} \Delta M_{c3} + \frac{1}{\beta_3 T_{m3}} (\Delta T_1 - \Delta T_3) R_{B3}; \\ \frac{d\Delta T_1}{dt} = \frac{c_1 R_{B1}}{j_3} \Delta\omega_1 - \left(\frac{c_1 R_{B3}}{j_3} + \frac{R_{B3} T_1^*}{j_3 l_1}\right) \Delta\omega_3 - \frac{R_{B3} \omega_3^*}{j_3 l_1} \Delta T_1, \end{cases} \quad (7)$$

where $\omega_1^*, \omega_2^*, \omega_3^*$ - initial values of the angular velocity;

T_1^*, T_2^*, T_3^* - initial values of relative tensions.

In the system of equations (7) assuming that $\frac{1}{c_3 l_3} T_3^* \ll 0$

and $\frac{1}{c_1 l_1} T_1^* \ll 0$ can be considered fair approximate equality

$$\left(1 + \frac{1}{c_3 l_3} T_3^*\right) \approx 1 \quad \left(1 + \frac{1}{c_1 l_1} T_1^*\right) \approx 1. \quad (8)$$

The system of equations (7), taking into account the accepted assumptions in (8), takes the form

$$\begin{cases} \frac{d\Delta\omega_1}{dt} = \frac{1}{\beta_1 T_{m1}} \Delta M_1 - \frac{1}{\beta_1 T_{m1}} \Delta M_{c1} + \frac{1}{\beta_1 T_{m1}} (\Delta T_2 - \Delta T_1) R_{B1}; \\ \frac{d\Delta T_2}{dt} = \frac{c_2 R_{B2}}{j_2} \Delta\omega_2 - \frac{c_2 R_{B1}}{j_1} \Delta\omega_1 - \frac{R_{B1} \omega_1^*}{j_1 l_2} \Delta T_2; \\ \frac{d\Delta\omega_2}{dt} = \frac{1}{\beta_2 T_{m2}} \Delta M_2 - \frac{1}{\beta_2 T_{m2}} \Delta M_{c2} + \frac{1}{\beta_2 T_{m2}} (\Delta T_3 - \Delta T_2) R_{B2}; \\ \frac{d\Delta T_3}{dt} = \frac{c_3 R_{B3}}{j_2} \Delta\omega_3 - \frac{c_3 R_{B2}}{j_2} \Delta\omega_2 - \frac{R_{B2} \omega_2^*}{j_2 l_3} \Delta T_3; \\ \frac{d\Delta\omega_3}{dt} = \frac{1}{\beta_3 T_{m3}} \Delta M_3 - \frac{1}{\beta_3 T_{m3}} \Delta M_{c3} + \frac{1}{\beta_3 T_{m3}} (\Delta T_1 - \Delta T_3) R_{B3}; \\ \frac{d\Delta T_1}{dt} = \frac{c_1 R_{B1}}{j_3} \Delta\omega_1 - \frac{c_1 R_{B3}}{j_3} \Delta\omega_3 - \frac{R_{B3} \omega_3^*}{j_3 l_1} \Delta T_1. \end{cases} \quad (9)$$

The mathematical description of the three-motor asynchronous electric drive of belt conveyor with frequency converters, based on (1), taking into account the resulting system of equations (9), is represented by the following system of equations.

$$\begin{cases}
 \frac{d\Delta\omega_1}{dt} = k_1\Delta M_1 - k_{38}\Delta M_{c1} + k_2(\Delta T_2 - \Delta T_1); \\
 \frac{d\Delta M_1}{dt} = k_3\Delta\omega_1 - k_{59}\Delta\omega_1 - k_4\Delta M_1; \\
 \frac{d\Delta\omega_{01}}{dt} = k_5\Delta u_{RS1} - k_6\Delta\omega_{01}; \\
 \frac{d\Delta u_{RS1}}{dt} = k_7(\Delta M_2 - \Delta M_{c2}) - k_8(\Delta M_1 - \Delta M_{c1}) + k_9(\Delta M_3 - \Delta M_{c3}) + k_{10}\Delta T_3 - k_{11}\Delta T_2 + k_{12}\Delta T_1 - k_{13}\Delta\omega_1 + k_{14}\Delta\omega_2 + k_{15}\Delta\omega_3 + k_{16}\Delta u_3; \\
 \frac{d\Delta T_2}{dt} = k_{17}\Delta\omega_2 - k_{18}\Delta\omega_1 - k_{19}\Delta T_2; \\
 \frac{d\Delta\omega_2}{dt} = k_{20}\Delta M_2 - k_{60}\Delta M_{c2} + k_{21}(\Delta T_3 - \Delta T_2); \\
 \frac{d\Delta M_2}{dt} = k_{22}\Delta\omega_2 - k_{61}\Delta\omega_2 - k_{23}\Delta M_2; \\
 \frac{d\Delta\omega_{02}}{dt} = k_{24}\Delta u_{RS2} - k_{25}\Delta\omega_{02}; \\
 \frac{d\Delta u_{RS2}}{dt} = k_{26}(\Delta M_1 - \Delta M_{c1}) - k_{27}(\Delta M_2 - \Delta M_{c2}) + k_{28}(\Delta M_3 - \Delta M_{c3}) + k_{29}\Delta T_1 + k_{30}\Delta T_2 - k_{31}\Delta T_3 + k_{32}\Delta\omega_1 - k_{33}\Delta\omega_2 + k_{34}\Delta\omega_3 + k_{35}\Delta u_3; \\
 \frac{d\Delta T_3}{dt} = k_{36}\Delta\omega_3 - k_{37}\Delta\omega_2 - k_{38}\Delta T_3; \\
 \frac{d\Delta\omega_3}{dt} = k_{39}\Delta M_3 - k_{62}\Delta M_{c3} + k_{40}(\Delta T_1 - \Delta T_3); \\
 \frac{d\Delta M_3}{dt} = k_{41}\Delta\omega_3 - k_{63}\Delta\omega_3 - k_{42}\Delta M_3; \\
 \frac{d\Delta\omega_{03}}{dt} = k_{43}\Delta u_{RS3} - k_{44}\Delta\omega_{03}; \\
 \frac{d\Delta u_{RS3}}{dt} = k_{45}(\Delta M_1 - \Delta M_{c1}) - k_{46}(\Delta M_2 - \Delta M_{c2}) + k_{47}(\Delta M_3 - \Delta M_{c3}) - k_{48}\Delta T_1 + k_{49}\Delta T_2 + k_{50}\Delta T_3 + k_{51}\Delta\omega_1 + k_{52}\Delta\omega_2 - k_{53}\Delta\omega_3 + k_{54}\Delta u_3; \\
 \frac{d\Delta T_1}{dt} = k_{55}\Delta\omega_1 - k_{56}\Delta\omega_3 - k_{57}\Delta T_1.
 \end{cases} \tag{10}$$

where k_1, \dots, k_{57} – coefficients entered to simplify the form of the mathematical model, are presented in Table 1 below.

Table 1. Parameters of the electric drive of the three-motor asynchronous electric drive of belt conveyor

Parameter	Value
Mechanical characteristics stiffness	$\beta_1 = \beta_2 = \beta_3 = 1098.039$
Electromechanical time constant, s	$T_{M1} = T_{M2} = T_{M3} = 0.344$
Radius of drive drums, m	$R_{B1} = R_{B2} = R_{B3} = 0.645$
Electromagnetic time constant, s	$T_{E1} = T_{E2} = T_{E3} = 0,086$
Gear ratios PCh1-3	$k_{PCh2} = k_{PCh3} = k_{PCh3} = 2;$
Time constants PCh1-3, s	$T_{PCh1} = T_{PCh2} = T_{PCh3} = 0.001$
Gear ratios RS1-3	$k_{RS1} = k_{RS2} = k_{RS3} = 20$
Feedback coefficients of velocity mismatch M1-3	$k_{s.m.f1} = k_{s.m.f2} = k_{s.m.f3} = 0.6$
Feedback coefficients of velocity	$k_{s.f1} = k_{s.f2} = k_{s.f3} = 0.4$
Time constants RS1-3, s	$T_{RS1} = T_{RS2} = T_{RS3} = 0.1$
The stiffness of the rubber conveyor belt	$c_1 = c_2 = c_3 = 500000$
The distance between the first and third motor through a loaded branch, m	$l_3 = 1980$
The distance between the first and second motor, m	$l_1 = l_2 = 5$
The transmission gear ratio	$j_1 = j_2 = j_3 = 20$
Ideal angular velocity M1-3, rad/s	$\omega_1^* = \omega_2^* = \omega_3^* = 157$

The above Table 1 takes into account the identity of the electric motors of the three-motor system.

Table 2 presents the expressions and numerical values of the coefficients k_1, \dots, k_{57}

Table 2. Expressions and numerical values of the coefficients k_1, \dots, k_{57}

$k_1 = k_{58} = \frac{1}{\beta_1 T_{m1}} = 0.003;$	$k_{39} = k_{62} = \frac{1}{\beta_3 T_{m3}} = 0.003;$	$k_{20} = k_{60} = \frac{1}{\beta_2 T_{m2}} = 0.003;$
$k_2 = \frac{R_{B1}}{\beta_1 T_{m1}} = 0.002;$	$k_{40} = \frac{R_{B3}}{\beta_3 T_{m3}} = 0.002;$	$k_{21} = \frac{R_{B2}}{\beta_2 T_{m2}} = 0.002;$
$k_3 = k_{59} = \frac{\beta_1}{T_{E1}} = 12767.895;$	$k_{22} = k_{61} = \frac{\beta_2}{T_{E2}} = 12767.895;$	$k_{41} = k_{63} = \frac{\beta_3}{T_{E3}} = 12767.895;$
$k_4 = \frac{1}{T_{E1}} = 11.628;$	$k_{23} = \frac{1}{T_{E2}} = 11.628;$	$k_{42} = \frac{1}{T_{E3}} = 11.628;$
$k_5 = \frac{k_{PCh1}}{T_{PCh1}} = 2000;$	$k_{24} = \frac{k_{PCh2}}{T_{PCh2}} = 2000;$	$k_{43} = \frac{k_{PCh3}}{T_{PCh3}} = 2000;$
$k_6 = \frac{1}{T_{PCh1}} = 1000;$	$k_{25} = \frac{1}{T_{PCh2}} = 1000;$	$k_{44} = \frac{1}{T_{PCh3}} = 1000;$
$k_7 = \frac{k_{RS1} k_{s.m.f1}}{\beta_2 T_{m2}} = 0.032;$	$k_{26} = \frac{k_{RS2} k_{s.m.f1}}{\beta_1 T_{m1}} = 0.032;$	$k_{47} = \frac{k_{RS3} k_{s.m.f2}}{\beta_2 T_{m2}} = 0.032;$
$k_9 = \frac{k_{RS1} k_{s.m.f3}}{\beta_3 T_{m3}} = 0.032;$	$k_{45} = \frac{k_{RS3} k_{s.m.f3}}{\beta_1 T_{m1}} = 0.032;$	$k_{28} = \frac{k_{RS2} k_{s.m.f2}}{\beta_3 T_{m3}} = 0.032;$
$k_{14} = \frac{k_{s.m.f1}}{T_{RS1}} = 6;$	$k_{32} = \frac{k_{s.m.f1}}{T_{RS2}} = 6;$	$k_{51} = \frac{k_{s.m.f3}}{T_{RS3}} = 6;$
$k_{15} = \frac{k_{s.m.f3}}{T_{RS1}} = 6.$	$k_{34} = \frac{k_{s.m.f2}}{T_{RS2}} = 6;$	$k_{52} = \frac{k_{s.m.f2}}{T_{RS3}} = 6.$
$k_{13} = \frac{k_{s.m.f1} + k_{s.m.f3} + k_{s.f1}}{T_{RS1}} = 16;$	$k_{33} = \frac{k_{s.m.f2} + k_{s.m.f1} + k_{s.f2}}{T_{RS2}} = 16;$	$k_{53} = \frac{k_{s.m.f3} + k_{s.m.f2} + k_{s.f3}}{T_{RS3}} = 16;$
$k_{16} = \frac{1}{T_{RS1}} = 10;$	$k_{35} = \frac{1}{T_{RS2}} = 10;$	$k_{54} = \frac{1}{T_{RS3}} = 10;$
$k_{17} = \frac{c_2 R_{B2}}{j_2} = 16125;$	$k_{36} = \frac{c_3 R_{B3}}{j_2} = 16125;$	$k_{55} = \frac{c_1 R_{B1}}{j_3} = 16125;$
$k_{18} = \frac{c_2 R_{B1}}{j_1} = 16125;$	$k_{37} = \frac{c_3 R_{B2}}{j_2} = 16125;$	$k_{56} = \frac{c_1 R_{B3}}{j_3} = 16125;$
$k_{19} = \frac{R_{B1} \omega_1^*}{j_1 l_2} = 1.0126;$	$k_{38} = \frac{R_{B2} \omega_2^*}{j_2 l_3} = 1.0126;$	$k_{57} = \frac{R_{B3} \omega_3^*}{j_3 l_1} = 0.005.$
$k_8 = \frac{k_{RS1} (k_{s.m.f1} + k_{s.m.f3} + k_{s.f1})}{\beta_1 T_{m1}} = 0.085;$	$k_{11} = \frac{k_{RS1} k_{s.m.f1} R_{B2}}{\beta_2 T_{m2}} + \frac{k_{RS1} (k_{s.m.f1} + k_{s.m.f3} + k_{s.f1}) R_{B1}}{\beta_1 T_{m1}} = 0.075;$	
$k_{27} = \frac{k_{RS2} (k_{s.m.f2} + k_{s.m.f1} + k_{s.f2})}{\beta_2 T_{m2}} = 0.085;$	$k_{12} = \frac{k_{RS1} k_{s.m.f3} R_{B3}}{\beta_3 T_{m3}} + \frac{k_{RS1} (k_{s.m.f1} + k_{s.m.f3} + k_{s.f1}) R_{B1}}{\beta_1 T_{m1}} = 0.075;$	
$k_{46} = \frac{k_{RS3} (k_{s.m.f3} + k_{s.m.f2} + k_{s.f3})}{\beta_3 T_{m3}} = 0.085;$	$k_{30} = \frac{k_{RS2} k_{s.m.f1} R_{B1}}{\beta_1 T_{m1}} + \frac{k_{RS2} (k_{s.m.f2} + k_{s.m.f1} + k_{s.f2}) R_{B2}}{\beta_2 T_{m2}} = 0.075;$	
$k_{10} = \frac{k_{RS1} k_{s.m.f1} R_{B2}}{\beta_2 T_{m2}} - \frac{k_{RS1} k_{s.m.f3} R_{B3}}{\beta_3 T_{m3}} = 0;$	$k_{31} = \frac{k_{RS2} k_{s.m.f2} R_{B3}}{\beta_3 T_{m3}} + \frac{k_{RS2} (k_{s.m.f2} + k_{s.m.f1} + k_{s.f2}) R_{B2}}{\beta_2 T_{m2}} = 0.075;$	
$k_{29} = \frac{k_{RS2} k_{s.m.f2} R_{B3}}{\beta_3 T_{m3}} - \frac{k_{RS2} k_{s.m.f1} R_{B1}}{\beta_1 T_{m1}} = 0;$	$k_{48} = \frac{k_{RS3} k_{s.m.f3} R_{B1}}{\beta_1 T_{m1}} + \frac{k_{RS3} (k_{s.m.f3} + k_{s.m.f2} + k_{s.f3}) R_{B3}}{\beta_3 T_{m3}} = 0.075;$	
$k_{49} = \frac{k_{RS3} k_{s.m.f3} R_{B1}}{\beta_1 T_{m1}} - \frac{k_{RS3} k_{s.m.f2} R_{B2}}{\beta_2 T_{m2}} = 0;$	$k_{50} = \frac{k_{RS3} k_{s.m.f2} R_{B2}}{\beta_2 T_{m2}} + \frac{k_{RS3} (k_{s.m.f3} + k_{s.m.f2} + k_{s.f3}) R_{B3}}{\beta_3 T_{m3}} = 0.075;$	

The equations of motion of a multi-motor asynchronous electric drive of belt conveyor will have the form of a system of equations (11), where, for convenience of calculation, the transformations of variables x_i of the system of equations (10) are adopted, in accordance with Table 3.

Table 3. Transformations of variables x_i of the system of equations (10)

$\Delta\omega_1 = x(1);$	$\Delta\omega_2 = x(6);$	$\Delta\omega_3 = x(11);$
$M_{E1} = x(2);$	$M_{E2} = x(7);$	$M_{E3} = x(12);$
$\Delta\omega_{01} = x(3);$	$\Delta\omega_{02} = x(8);$	$\Delta\omega_{03} = x(13);$
$\Delta u_{RS1} = x(4);$	$\Delta u_{RS2} = x(9);$	$\Delta u_{RS3} = x(14);$
$\Delta T_2 = x(5);$	$\Delta T_3 = x(10);$	$\Delta T_1 = x(15).$

$$\left\{ \begin{array}{l} px(1)=k_1x(2)+k_2x(5)-k_2x(15); \\ px(2)=-k_59x(1)-k_4x(2)+k_3x(3); \\ px(3)=-k_6x(3)+k_5x(4); \\ px(4)=-k_{13}x(1)-k_8x(2)-k_{11}x(5)+k_{14}x(6)+k_7x(7)+k_{10}x(10)+k_{15}x(11)+k_9x(12)+k_{12}x(15)+k_{16}\Delta u_3; \\ px(5)=-k_{18}x(1)-k_{19}x(5)+k_{17}x(6); \\ px(6)=-k_{21}x(5)+k_{20}x(7)+k_{21}x(10); \\ px(7)=-k_{61}x(6)-k_{23}x(7)+k_{22}x(8); \\ px(8)=-k_{25}x(8)+k_{24}x(9); \\ px(9)=k_{32}x(1)+k_{26}x(2)+k_{30}x(5)-k_{33}x(6)-k_{27}x(7)-k_{31}x(10)+k_{34}x(11)+k_{28}x(12)+k_{29}x(15)+k_{35}\Delta u_3; \\ px(10)=-k_{37}x(6)-k_{38}x(10)+k_{36}x(11); \\ px(11)=-k_{40}x(10)+k_{39}x(12)+k_{40}x(15); \\ px(12)=-k_{63}x(11)-k_{42}x(12)+k_{41}x(13); \\ px(13)=-k_{44}x(13)+k_{43}x(14); \\ px(14)=k_{51}x(1)+k_{45}x(2)+k_{49}x(5)+k_{52}x(6)+k_{47}x(7)+k_{50}x(10)-k_{53}x(11)-k_{46}x(12)-k_{48}x(15)+k_{54}\Delta u_3; \\ px(15)=k_{55}x(1)-k_{56}x(11)-k_{57}x(15). \end{array} \right. \quad (11)$$

One of the main tasks of optimal control is the selection of a process quality criterion. In our case, the most acceptable criterion for the quality of transient processes is the integral criterion for the quality of the form:

$$Q = \frac{1}{2} \int_0^T \left(\sum_{i=1}^6 x_i^2 + cu^2 \right) dt, \quad (12)$$

where c – weight coefficient of control.

The selected optimality criterion is an integral optimality criterion, the parts of which prohibit the long existence of the deviation x_i [3], which corresponds to the requirement of the technological process of the conveyor.

According to the maximum principle method, the function H for the system is written in the form:

$$\begin{aligned} H = & \frac{1}{2} P_0 \left(\sum_{i=1}^{12} x_i^2 + cu^2 \right) + P_1 [k_1x(2) + k_2x(5) - k_2x(15)] + P_2 [-k_{59}x(1) - k_4x(2) + k_3x(3)] + \\ & P_3 [-k_6x(3) + k_5x(4)] + P_4 [-k_{13}x(1) - k_8x(2) - k_{11}x(5) + k_{14}x(6) + k_7x(7) + \\ & k_{10}x(10) + k_{15}x(11) + k_9x(12) + k_{12}x(15) + k_{16}\Delta u_3] + P_5 [-k_{18}x(1) - k_{19}x(5) + k_{17}x(6)] + \\ & P_6 [-k_{21}x(5) + k_{20}x(7) + k_{21}x(10)] + P_7 [-k_{61}x(6) - k_{23}x(7) + k_{22}x(8)] + \\ & P_8 [-k_{25}x(8) + k_{24}x(9)] + P_9 [k_{32}x(1) + k_{26}x(2) + k_{30}x(5) - k_{33}x(6) - k_{27}x(7) - \\ & k_{31}x(10) + k_{34}x(11) + k_{28}x(12) + k_{29}x(15) + k_{35}\Delta u_3] + P_{10} [-k_{37}x(6) - k_{38}x(10) + k_{36}x(11)] + \\ & P_{11} [-k_{40}x(10) + k_{39}x(12) + k_{40}x(15)] + P_{12} [-k_{63}x(11) - k_{42}x(12) + k_{41}x(13)] + \\ & P_{13} [-k_{44}x(13) + k_{43}x(14)] + P_{14} [k_{51}x(1) + k_{45}x(2) + k_{49}x(5) + k_{52}x(6) + k_{47}x(7) + \\ & k_{50}x(10) - k_{53}x(11) - k_{46}x(12) - k_{48}x(15) + k_{54}\Delta u_3] + P_{15} [k_{55}x(1) - k_{56}x(11) - k_{57}x(15)], \end{aligned} \quad (13)$$

where P_i – coordinates of the conjugate system, $P_0 = -1$.

Conjugated system will look like, assuming here that

$$\frac{dP_i}{dt} = -\frac{\partial H}{\partial x_i}, \quad (14)$$

$$\begin{aligned} \frac{dP_1}{dt} &= -\frac{\partial H}{\partial x_1} = x_1 + k_{59}P_2 + k_{13}P_4 + k_{18}P_5 - k_{32}P_9 - k_{51}P_{14} - k_{55}P_{15}; \\ \frac{dP_2}{dt} &= -\frac{\partial H}{\partial x_2} = x_2 - k_1P_1 + k_4P_2 + k_8P_4 - k_{26}P_9 - k_{45}P_{14}; \\ \frac{dP_3}{dt} &= -\frac{\partial H}{\partial x_3} = x_3 - k_5P_2 + k_6P_3; \\ \frac{dP_4}{dt} &= -\frac{\partial H}{\partial x_4} = x_4 - k_5P_3; \\ \frac{dP_5}{dt} &= -\frac{\partial H}{\partial x_5} = x_5 - k_2P_1 + k_{11}P_4 + k_{19}P_5 + k_{21}P_6 - k_{30}P_9 - k_{49}P_{14}; \\ \frac{dP_6}{dt} &= -\frac{\partial H}{\partial x_6} = x_6 - k_{14}P_4 - k_{17}P_5 + k_{61}P_7 + k_{33}P_9 + k_{37}P_{10} - k_{52}P_{14}; \\ \frac{dP_7}{dt} &= -\frac{\partial H}{\partial x_7} = x_7 - k_7P_4 - k_{20}P_6 + k_{23}P_7 + k_{27}P_9 - k_{47}P_{14}; \\ \frac{dP_8}{dt} &= -\frac{\partial H}{\partial x_8} = x_8 - k_{22}P_7 + k_{25}P_8; \\ \frac{dP_9}{dt} &= -\frac{\partial H}{\partial x_9} = x_9 - k_{24}P_8; \\ \frac{dP_{10}}{dt} &= -\frac{\partial H}{\partial x_{10}} = x_{10} - k_{10}P_4 - k_{21}P_6 + k_{31}P_9 + k_{38}P_{10} + k_{40}P_{11} - k_{50}P_{14}; \\ \frac{dP_{11}}{dt} &= -\frac{\partial H}{\partial x_{11}} = x_{11} - k_{15}P_4 - k_{34}P_9 - k_{36}P_{10} + k_{63}P_{12} + k_{53}P_{14} + k_{56}P_{15}; \\ \frac{dP_{12}}{dt} &= -\frac{\partial H}{\partial x_{12}} = x_{12} - k_9P_4 - k_{28}P_9 - k_{39}P_{11} + k_{42}P_{12} + k_{46}P_{14}; \\ \frac{dP_{13}}{dt} &= -\frac{\partial H}{\partial x_{13}} = x_{13} - k_{41}P_{12} + k_{44}P_{13}; \\ \frac{dP_{14}}{dt} &= -\frac{\partial H}{\partial x_{14}} = x_{14} - k_{43}P_{13}; \\ \frac{dP_{15}}{dt} &= -\frac{\partial H}{\partial x_{15}} = x_{15} + k_2P_1 - k_{12}P_4 - k_{29}P_9 - k_{40}P_{11} + k_{48}P_{14} + k_{57}P_{15}. \end{aligned} \quad (15)$$

The final values of P_i will be written:

$$P_i(T) = 0, \quad (i = 1, 15). \quad (16)$$

The specified initial conditions of system (11) for x_i will be:

$$x_i(0) = x_i^0. \quad (17)$$

The first value ($z=$) is the value of the initial condition of the variables of system (11).

Optimal control u is determined by the algorithm of the method of successive approximations, the block diagram of which is presented in Fig. 3.

III. I. Description of the algorithm flowchart

The first block of the optimal control calculation of algorithm flowchart describes the initial data:

- a) the coefficients of variable differential equations of the transition process of a multi-motor electric belt conveyor – k_i ;
- b) the initial conditions of the differential equations of the transition process (x_i^*), and the final conditions of the conjugated system of differential equations ($P_i(T)$);
- c) the accuracy of the solution (e) and the number of iterations (N).

In the second block of the algorithm, the initial control approximation ($qy(i)$) is specified.

In the third block of the algorithm, the differential equations ($x_i(i) \div x_n(i)$) are calculated with optimal control $qy(i)$, first given by constant values $qy(i) = 0.001$. The calculation is made according to the finite difference scheme.

In the fourth block of the algorithm, the variables ($P_i(i) \div P_n(i)$) of the conjugated system of differential equations are also calculated according to a finite difference scheme.

In the fifth block of the algorithm, optimal control $qh(i)$ is calculated taking into account the control weighting factor C .

In the sixth block of the algorithm, the value of the absolute difference of the given initial approximation of the control and the obtained control value is determined during its calculation.

In the seventh block of the algorithm, a check on the end of the count is performed according to the condition:

a) if the absolute difference of the initial and final controls is greater than the predetermined accuracy of this difference, then the process of reassigning the initial and final controls $qy(i) = qh(i)$ with the transition to the third block of the account is carried out:

b) if the absolute difference of controls is less in front of the specified accuracy value (e), then the process of displaying the result of the account ($u(i)$) and stopping it is performed.

In the eighth block of the algorithm, the control is reassigned, i.e. the current control is equated with the value of the initial control and the counting starts again from the third block (Fig. 3).

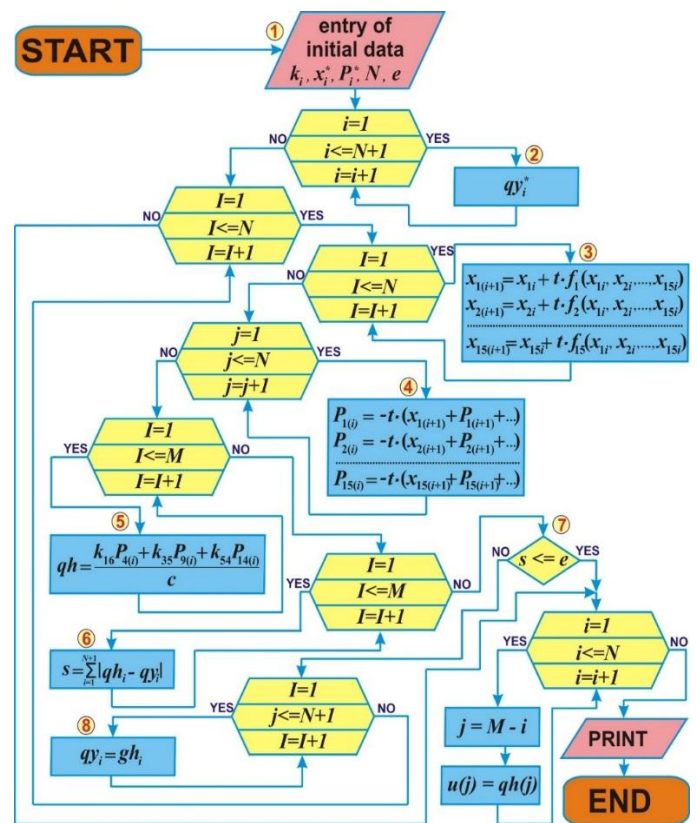


Fig. 3. Algorithm flowchart for calculating the optimal control $u(t)$

The algorithm program is shown below (Fig. 4), which is written in the algorithmic language of the MATLAB system.

```

N=input('N=');
t=0.001;
e=0.001;
k1=0.003;    k2=0.002;    k3=12767.895;    k4=11.628;    k5=2000;    k6=1000;
k7=0.032;    k8=0.085;    k9=0.032;    k10=0;    k11=0.075;    k12=0.075;
k13=16;    k14=6;    k15=6;    k16=10;    k17=16125;    k18=16125;
k19=0.9998;    k20=0.003;    k21=0.002;    k22=12767.895;    k23=11.628;    k24=2000;
k25=1000;    k26=0.032;    k27=0.085;    k28=0.032;    k29=0;    k30=0.075;
k31=0.075;    k32=6;    k33=16;    k34=6;    k35=10;    k36=16125;
k37=16125;    k38=0.9998;    k39=0.003;    k40=0.002;    k41=12767.895;    k42=11.628;
k43=2000;    k44=1000;    k45=0.032;    k46=0.085;    k47=0.032;    k48=0.075;
k49=0;    k50=0.075;    k51=6;    k52=6;    k53=16;    k54=10;
k55=16125;    k56=16125;    k57=0.0025;    k59=12767.895;    k61=12767.895;    k63=12767.895;
M=N+1;
for i=1:M
    qy(i)=0.001;
end;
x1(1)=25;    x2(1)=0;    x3(1)=25;    x4(1)=12.5;    x5(1)=0;
x6(1)=25;    x7(1)=0;    x8(1)=25;    x9(1)=12.5;    x10(1)=0;
x11(1)=25;    x12(1)=0;    x13(1)=25;    x14(1)=12.5;    x15(1)=0;
p1(M)=0;    p2(M)=0;    p3(M)=0;    p4(M)=0;    p5(M)=0;
p6(M)=0;    p7(M)=0;    p8(M)=0;    p9(M)=0;    p10(M)=0;
p11(M)=0;    p12(M)=0;    p13(M)=0;    p14(M)=0;    p15(M)=0;
i=1;
while (i<=N)
    for i=1:N
        x1(i+1)=t*(k1*x2(i)+k2*x5(i)-k2*x15(i))+x1(i);
        x2(i+1)=t*(-k59*x1(i)-k4*x2(i)+k3*x3(i))+x2(i);
        x3(i+1)=t*(-k6*x3(i)+k5*x4(i))+x3(i);
        x4(i+1)=t*(-k13*x1(i)-k8*x2(i)-k11*x5(i)+k14*x6(i)+k7*x7(i)+k10*x10(i)+k15*x11(i)+k9*x12(i)+k12*x15(i)+k16*qy(i))+x4(i);
        x5(i+1)=t*(-k18*x1(i)-k19*x5(i)+k17*x6(i))+x5(i);
        x6(i+1)=t*(-k21*x5(i)+k20*x7(i)+k21*x10(i))+x6(i);
        x7(i+1)=t*(-k61*x6(i)-k23*x7(i)+k22*x8(i))+x7(i);
        x8(i+1)=t*(-k25*x8(i)+k24*x9(i))+x8(i);
        x9(i+1)=t*(k32*x1(i)+k26*x2(i)+k30*x5(i)-k33*x6(i)-k27*x7(i)-k31*x10(i)+k34*x11(i)+k28*x12(i)+k29*x15(i)+k35*qy(i))+x9(i);
        x10(i+1)=t*(-k37*x6(i)-k38*x10(i)+k36*x11(i))+x10(i);
        x11(i+1)=t*(-k40*x10(i)+k39*x12(i)+k40*x15(i))+x11(i);
        x12(i+1)=t*(-k63*x11(i)-k42*x12(i)+k41*x13(i))+x12(i);
        x13(i+1)=t*(-k44*x13(i)+k43*x14(i))+x13(i);
        x14(i+1)=t*(k51*x1(i)+k45*x2(i)+k49*x5(i)+k52*x6(i)+k47*x7(i)+k50*x10(i)-k53*x11(i)-k46*x12(i)-k48*x15(i)+k54*qy(i))+x14(i);
        x15(i+1)=t*(k55*x1(i)-k56*x11(i)-k57*x15(i))+x15(i);
    end;
    for j=1:N
        i=M-j;
        L=i+1;
        p1(i)=-t*(x1(L)+k59*p2(L)+k13*p4(L)+k18*p5(L)-k32*p9(L)-k51*p14(L)-k55*p15(L))+p1(L);
        p2(i)=-t*(x2(L)-k1*p1(L)+k4*p2(L)+k8*p4(L)-k26*p9(L)-k45*p14(L))+x2(L);
        p3(i)=-t*(x3(L)-k3*p2(L)+k6*p3(L))+p3(L);
        p4(i)=-t*(x4(L)-k5*p3(L))+p4(L);
        p5(i)=-t*(x5(L)-k2*p1(L)+k11*p4(L)+k19*p5(L)+k21*p6(L)-k30*p9(L)-k49*p14(L))+p5(L);
        p6(i)=-t*(x6(L)-k14*p4(L)-k17*p5(L)+k61*p7(L)+k33*p9(L)+k37*p10(L)-k52*p14(L))+p6(L);
        p7(i)=-t*(x7(L)-k7*p4(L)-k20*p6(L)+k23*p7(L)+k27*p9(L)-k47*p14(L))+p7(L);
        p8(i)=-t*(x8(L)-k22*p7(L)+k25*p8(L))+p8(L);
        p9(i)=-t*(x9(L)-k24*p8(L))+p9(L);
        p10(i)=-t*(x10(L)-k10*p4(L)-k21*p6(L)+k31*p9(L)+k38*p10(L)+k40*p11(L)-k50*p14(L))+p10(L);
        p11(i)=-t*(x11(L)-k15*p4(L)-k34*p9(L)-k36*p10(L)+k63*p12(L)+k53*p14(L)+k56*p15(L))+p11(L);
        p12(i)=-t*(x12(L)-k9*p4(L)-k28*p9(L)-k39*p11(L)+k42*p12(L)+k46*p14(L))+p12(L);
        p13(i)=-t*(x13(L)-k41*p12(L)+k44*p13(L))+p13(L);
        p14(i)=-t*(x14(L)-k43*p13(L))+p14(L);
        p15(i)=-t*(x15(L)+k2*p1(L)-k12*p4(L)-k29*p9(L)-k40*p11(L)+k48*p14(L)+k57*p15(L))+p15(L);
    end;
    for i=1:M
        qh(i)=(k16*p4(i)+k35*p9(i)+k54*p14(i))/10000000;
    end;
    s=0;
    for i=1:M
        s=s+abs(qh(i)-qy(i));
    end;
    if s<=e
        break;
    end;
    for i=1:M
        qy(i)=qh(i);
    end;
    i=i+1;
end;
disp(['    t ',' Control values']);
disp(['    0 ',' 0 ']);
for i=1:N
    j=M-i;
    u(j)=qh(j);
    fprintf('%9.0i %15.3f\n',i, u(j));
end;
    
```

Fig. 4. The program for calculating the optimal control of the electric belt conveyor

According to the obtained values, the control curve is plotted, shown in Fig. 5.

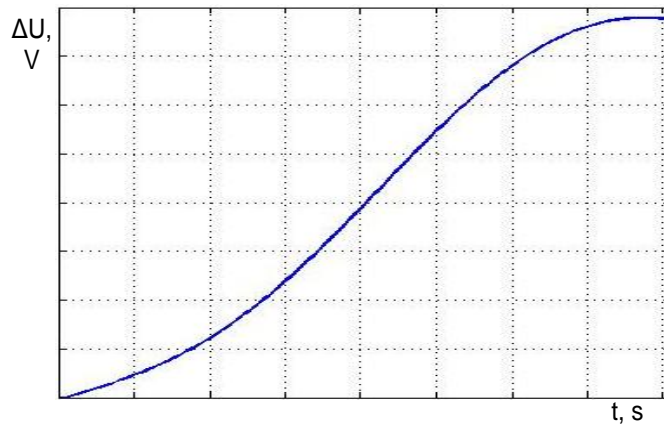


Fig. 5. The optimal control signal curve

The optimal control signal curve, obtained in Fig. 5, is described by a second-order equation, which has the form [10]:

$$\tau_1\tau_2 \frac{d^2y}{dt^2} + (\tau_1 + \tau_2) \frac{dy}{dt} + y = kx, \quad (19)$$

where the time constants τ_1, τ_2 and the constant k can be determined by the graph-analytical method or the method of A.N. Krylov.

The transfer function of the optimal curve of the control signal has the form [11]:

$$\frac{u_{1,2,3}(p)}{u_i(p)} = \frac{k}{(\tau_1 p + 1)(\tau_2 p + 1)}. \quad (20)$$

The resulting transfer function on the optimal curve of the control signal determines the optimal law of control of a multi-motor asynchronous electric drive of synchronous rotation with the frequency converters of the belt conveyor.

For the study of transients with a control signal u , which is described by equation (20), in accordance with Fig. 6, the model has been compiled.

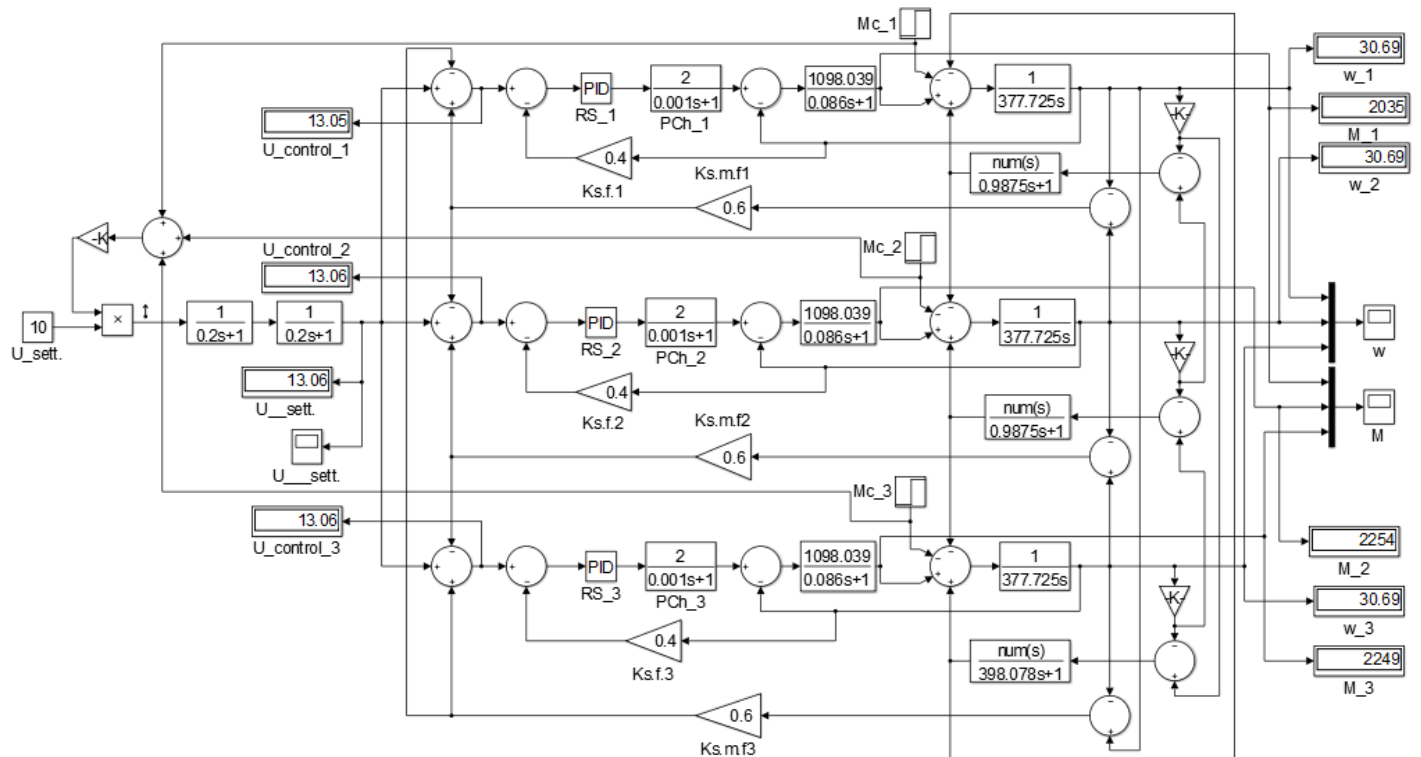


Fig. 6. Model in Matlab of three-motor asynchronous electric belt conveyor with optimal control

The oscillograms of the master signal u , the angular velocities $\Delta\omega_1, \Delta\omega_2, \Delta\omega_3$, moments M_1, M_2, M_3 are shown in Fig. 7 and 8.

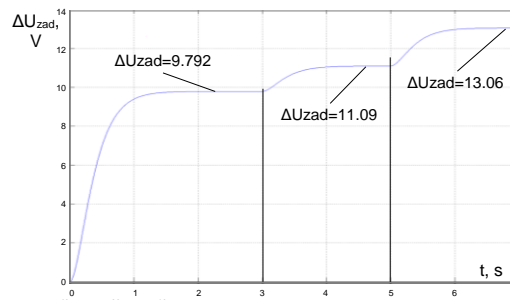


Fig. 7. Oscillograms of the general master control of the model of the multi-motor asynchronous electric drive of belt conveyor with optimal control

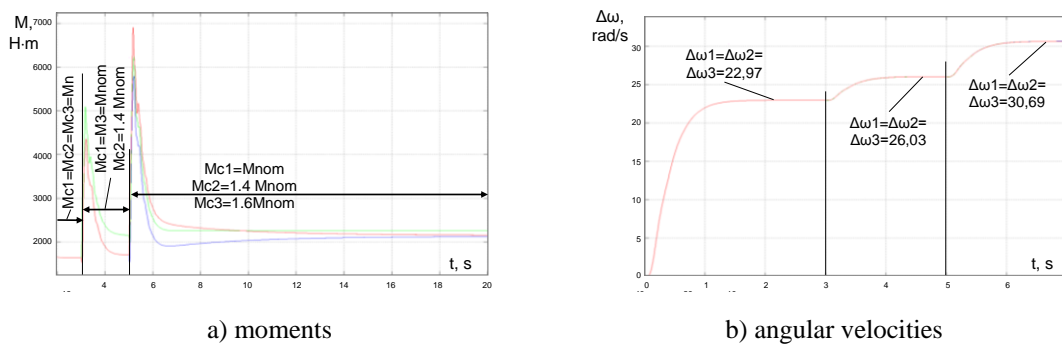


Fig. 8. Oscillograms of the moments and angular velocities of the model of the multi-motor asynchronous electric drive of belt conveyor with optimal control

In the process of modeling, the values of the static moments of asynchronous motors are selected in the same way as shown in Fig. 8, a.

The simulation results show that the control signal, depending on the static loads of the motors, varies according to Fig. 5. The resulting control equation improves the quality of regulation. The resulting model allows a smooth start, as well as a smooth control of the speeds of the multi-motor asynchronous electric drive of synchronous rotation with the frequency converter of belt conveyor.

IV. CONCLUSIONS

As a result of the research, the mathematical model of the three-motor asynchronous electric drive of synchronous rotation with the frequency converter of belt conveyor was developed, which takes into account feedbacks on the mismatch of motor speeds, as well as the relationship of conveyor drive drums through a conveyor belt with a certain tension value. The resulting model ensures the synchronous rotation of the electric motors of the system, thereby ensuring the service life of the belt conveyor. The optimal control law as a function of time is determined, ensuring smooth start and braking, reducing the cost of electricity during the transition of the conveyor from one mode to another. To implement the tasks of optimal control of a multi-motor electric belt conveyor, the Pontryagin L.S. principle method is applied.

The algorithm and program for calculating the optimal control of the multi-motor asynchronous electric drive of synchronous rotation with the frequency converter of belt conveyor were compiled and the “S” shaped optimal control curve was determined. The functionality of modern industrial frequency converters makes it possible to successfully implement the obtained optimum acceleration curve of a three-motor asynchronous electric drive of belt conveyor.

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