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Balancing of Multi-Stage Pump Using the Coupling Hub

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Abstract

In this paper, we present an investigation to perform a temporary on-site balancing of machines which rotor is not accessible, such as multi-stage oil centrifugal pumps, in order to bring back the vibration magnitude below the alarm threshold, waiting for the scheduled general review of the machine.

We have shown that if the vibration measured on the Driver End Support (DE) is very higher than that measured on the Non Driver End Support (NDE), we can consider the Coupling Hub fitted to the Pump (CHP) a correction plane in order to apply on site balancing by means of the Influence Coefficients Method. This permits to overcome the problem of the inaccessibility of the rotor and to solve, in the shortest possible time, the unbalance occurred during the working. We have experimentally verified this hypothesis in a multistage pump of a petroleum industrial plant by two different approaches. The first performs Single Plane Balancing (SPB) and vibration measurements on two supports, acquired with eddy probe sensors. The second approach, which may be more simple and easy to apply, aims to apply the SPB using vibration measurements on only one support acquired with an accelerometer. The results of the different approaches have been compared.

Keywords: Coupling Hub, Influence Coefficients Method, On-site balancing, Vibration alarm threshold.

I. INTRODUCTION

The rotating machinery vibration problems remain a concern, despite progress in recent years in their design. Wear, deformations, shifts of the assembled elements, deposition of material (dirk) on the rotor cause a change in the state of their equilibrium, which induces a deterioration of vibration levels [1-6]. The imbalances are an important cause of vibration. Particularly, the unbalance caused by deposition of material on the rotor happens very often in the multi-stage centrifugal pumps used in heavy residual oils refining process and, generally, it represents the more important part of their total unbalance. Normal balancing practices need the total shutdown of the machine, removal the rotor and balancing it on a balancing machine. This operation is very expensive and induces additional costs of missed production caused by the long downtime, mostly if it is unscheduled. The literature review highlights the interest of designing balancing approaches that reduce the number of test runs required and the machines downtime so that to minimize the production loss generated in industrial plants [7-16].

The goal of the present study is to propose a method for temporary on-site balancing of this kind of pump, in order to bring back the vibration magnitude below the alarm threshold in a little bit time, waiting for the scheduled general review of the machine.

In the first part of this paper, we describe the problem and the theoretical approach for its resolution. The key assumption of the proposed method is that, in presence of particular conditions, the coupling hub fitted to the pump (CHP) is used as a single balancing plane of a multi stage centrifugal pump (MSCP). In the second part, we perform the experimental verifications on a multistage centrifugal pump of an industrial plant by use of the Influence Coefficients Method (ICM), which is recognized for its efficiency and for its ease of implementation in industrial context. The pump considered in this study is used in the process of heavy petrochemical products. Indeed, the product in question is a residue (vacuum residue) of a first stage of refining crude oil, which is reused for feeding a second phase. The product is very viscous and is fed at a temperature of 306 °C. For local unfavourable conditions, a dirt adheres to rotor impellers causing a phenomenon of unbalance of the rotor with consequent unacceptable levels of vibration. The pump and the measurement instruments are descripted in details and the balancing process based on the coupling hub is shown. We have used the ICM with two approaches. The first performs Single Plane Balancing (SPB) by means of vibration measurements on two supports; they are acquired with fixed eddy probe sensors. The second approach aims to apply the SPB using vibration measurements on only one support, acquired with a removable accelerometer. The latter has the advantage to be relatively simpler and more easily applicable whereas many machines, for cost reason, are not equipped with fixed sensors. The results of the different approaches have been compared.

II. THEORETICAL APPROACH

The shaft of a rotor can be considered, at same time, as rigid, if it is operating much below its first critical speed, or flexible, when it is operating near or above the first critical speed. An eccentricity of the center of gravity of a rigid rotor causes a static unbalance. On the contrary, a uniformly distributed unbalance along the length of a rigid rotor causes a couple unbalance. The latter cannot be detected without to rotate the shaft. The effect of the static and of the couple unbalance together is named dynamic unbalance, which is what occurs generally in industrial machines. However, once the rotor approaches a critical speed, its centerline bends and whirls around and new centrifugal forces set-up. In this case,

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eccentricity and its angular orientation may change in three-dimension continuously from one end of the shaft to another. Basic principles of rigid and flexible rotor balancing are quite different. Various types of practical balancing techniques have been proposed [17-32]. One of the most representative balancing method is the influence coefficient method (ICM), which is largely employed in industrial sector. This method is the more appropriate to perform on site-balancing because it uses only experimental information; indeed, it has the advantage of not requiring a mathematical modelling of the system, with the consequent uncertainties related to the discretization of the system itself. Consequently, it can apply to rotors assumed to be rigid or flexible. We summarize it in the in the next section.

II.I ICM- Influence Coefficients Method

The machine is described by p measurement plans and n corrective plans (or balancing plans). This method involves the system linearity assumption and it is based on the construction of a matrix modelling the system, made of sensitivity to unbalance experimentally measured. The vibration measurements can be carried out either by arranging two sets of p transducers, in correspondence with the p measurement planes, on two planes orthogonal to each other and containing the rotation axis, or by arranging a single series of p coplanar transducers and an indicator of phase.

For a given rotation speed, the coefficient $\boldsymbol{c}_{ij}$ means how much the vibration on the sensor i is influenced by the imbalance on the plane j, i.e.

$$\boldsymbol{c}_{ij} = \frac{\boldsymbol{w}_{ij} - \boldsymbol{w}_i}{m_{tj}} \qquad (1)$$

where: $\boldsymbol{w}_i$ is the vector of initial vibration measured by the sensor i in amplitude and phase, $\boldsymbol{w}_{ij}$ is the vector of vibration measured by the same sensor i but after adding a trial mass m_{tj} on the plane j. Each test finds one column of the matrix $[\mathbf{C}]$. Then it is needed as many experimental tests, as they are the imbalance plans. If a machine operates at different speeds, it is necessary to construct a matrix for each speed, thus grouping them together into a larger matrix $[\mathbf{C}]$. However, most of the machines has a single operating speed.

In order to cancel the vibrations of the rotor, a suitable system of correction masses is needed such that it is

$$\boldsymbol{w}_i + \sum_{j=1}^{n} \boldsymbol{c}_{ij}\, m_{cj} = 0 \qquad (2)$$

and in matrix form

$$\{\boldsymbol{W}\} + [\mathbf{C}]\{\boldsymbol{M}_c\} = \{\mathbf{0}\} \qquad (3)$$

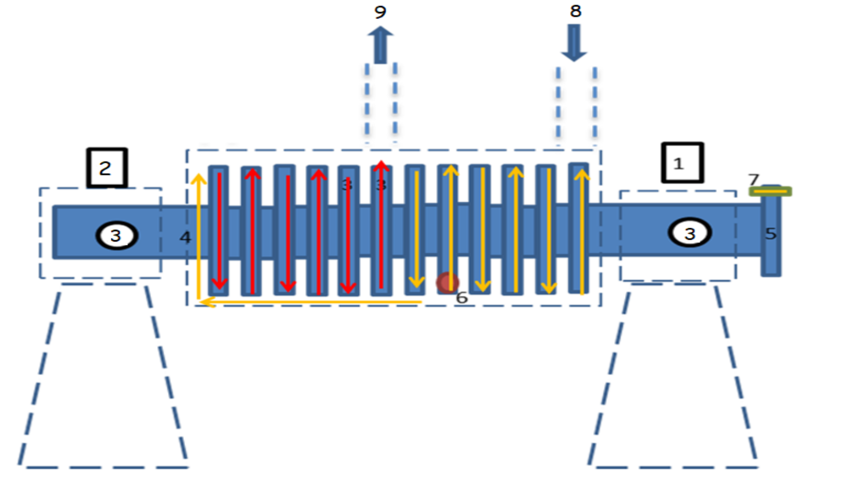
However, if the measurement plans p are more than the corrective ones n, the matrix $[\mathbf{C}]$ is rectangular, thus not invertible and eq. (3) cannot be solved. The least square method (or another equivalent) is needed to minimize the residual vibrations [33-37]. We used the method proposed in [34] which minimizes the quadratic norm of the residual vibration. In this case, the vector of the correction masses $\{\boldsymbol{M}_c\}$ is expressed by the following equation (Eq.2):

$$\{\boldsymbol{M}_c\} = -\{[\boldsymbol{C}^*]^t[\boldsymbol{C}]\}^{-1}[\boldsymbol{C}^*]^t\{\boldsymbol{W}\} \qquad (4)$$

where the symbol * is the conjugate.

II.II ICM applied to on-site balancing of a multi stage centrifugal pump

Fig.1 schematics the pump: it has 12 impellers divided into two groups with opposing flows. This design aims primarily to reduce the axial thrust. Two cylindrical hydrodynamic bearings bring the shaft. The red point in Fig. 1 represents the dirt of mass m, which causes a change in the state of equilibrium of the rotating machine.



1. Support Driver End
2. Support Non Driver End
3. Shaft
4. Impellers (number 12)
5. Pump Hub
6. Mass m (dirt, cause of unbalance)
7. Correction mass
8. Aspiration
9. Delivery

Figure1: Schematic representation of Multi-stage Centrifugal Pump unbalanced

Fig. 2 presents the schematic representation of the forces involved in the pump balancing. $\boldsymbol{F}$ represents the unbalancing force created by the mass m, while $\boldsymbol{R}_{NDE}$ and $\boldsymbol{R}_{DE}$ are the reaction forces on the supports to this unbalance. The rotor is supposed to be balanced in absence of the mass m.

The reaction forces change in $\boldsymbol{R'}_{NDE}$ and $\boldsymbol{R'}_{DE}$ if we add a correction force $\boldsymbol{Z} \propto m_{cz}$ in opposite versus of $\boldsymbol{F}$ at the end of the rotor (i.e. on the CHP). Indeed, applying $\boldsymbol{Z}$ creates a couple unbalance, but it can reduce the static unbalance; our aim is to define in which condition a proper balancing force $\boldsymbol{Z}$ can bring the measured vibration magnitude on the supports below the threshold alarm.

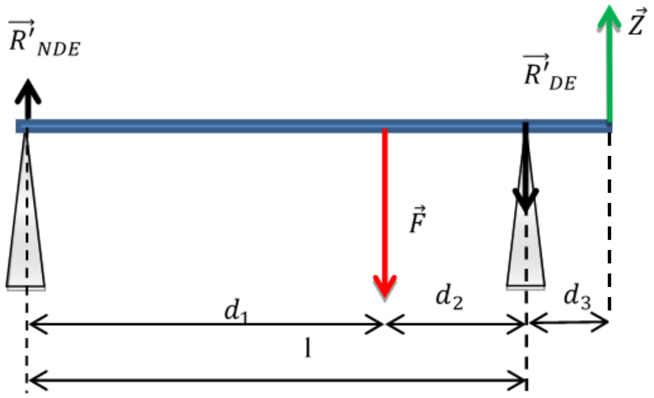


Figure 2: Forces involved in the pump balancing

The following equations give the reaction forces of the supports to the initial unbalance $\boldsymbol{F}$ and to the correction force Z:

$$R'_{NDE} = \frac{Fd_2}{l} + \frac{d_3}{l} Z \tag{5}$$

$$R'_{DE} = \frac{Fd_1}{l} - \frac{(l+d_3)}{l} Z \tag{6}$$

F, $d_1$and d_2 are unknown, but they are fixed data and linked to the state of unbalance; on the contrary, d_3 and l are known from the machine design. In general, the ratio $\frac{d_3}{l} \approx 0.1$, this means that R'_{NDE} grows slowly in function of Z, while R'_{DE} decreases faster (indeed, $\frac{(l+d_3)}{l} \approx 1.1$).

If we now assume the vibration magnitude on a support is proportional to the reaction force on the same support, then it is $\frac{w_{DE}}{w_{NDE}} = \frac{R_{DE}}{R_{NDE}} = \frac{d_1}{d_2}$, where w_{DE} and w_{NDE} are the vibration amplitudes due to the initial unbalance F. Therefore, if it is $w_{DE} \gg w_{NDE}$ at first, then R'_{DE}will decrease with the increase of Z, and bring back the vibration magnitude in the normal on the DE support, before that R'_{NDE} increase enough to cause vibration higher than the alarm threshold (see Fig.3). On the contrary, if it is $w_{DE} < w_{NDE}$, a corrective action on CHP could not have the desired effect but even worsen the situation.

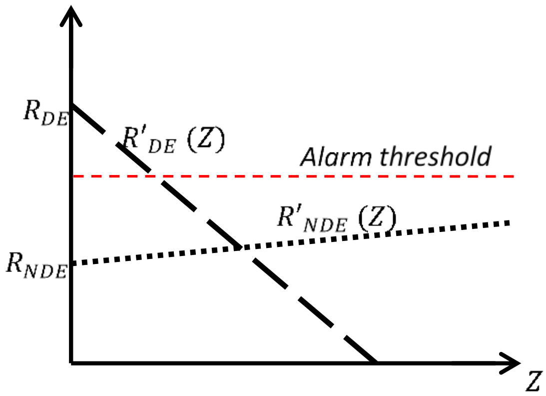


Figure 3: Trend of the **r**eaction forces as a function of the correction force if $w_{DE} > w_{NDE}$

However, the occurrence $\frac{w_{DE}}{w_{NDE}} > 2$ is common in multi-stage pumps unbalances, so we have verified experimental the possibility to use the CHP as unique correction plane. We followed two different approaches by means of the ICM.

The first performs balancing by vibration measurements on both the supports. In order to simplify the notation, in the following, we indicate with 1 the mid plane of the support NDE, with 2 the one of the support DE and with z the one of the CHP. The matrix of influence coefficients $[\mathrm{C}] = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix}$ is rectangular and his size is (2×1). Applying eq (4), the correction mass m_{cz} is:

$$m_{cz} = -\frac{c_{11}^* w_1 + c_{21}^* w_2}{c_{11}^* c_{11} + c_{21}^* c_{21}} \tag{7}$$

The second approach aims to use vibration measurements only on the support DE, basing on the observation that $R_{DE}$is very higher than R_{NDE}. In this case, it is c_{21}^*=0 and the correction mass will result simply

$$m_{cz} = -\frac{w_1}{c_{11}} \tag{8}$$

III. EXPERIMENTAL TESTS

An electric three-phase motor, which rotation speed is 1492 rev/min, drives the pump via a speed multiplier (see Fig. 4), which has a transmission ratio of approximately 3.22. Therefore, the pump rotation speed is equal to 4798 rev/min. The shaft of the pump is connected to the gearbox by a Meta-stream coupling (CHP) (see Fig.5). This hub is connected to the coupling block by 10 bolts at regular distance and angle (36 degrees).

The length of the steel shaft of the multistage pump is l=3297 mm. The rotor weighs 218 kg. The pumped liquid is vacuum residue, its capacity 120 m^3.

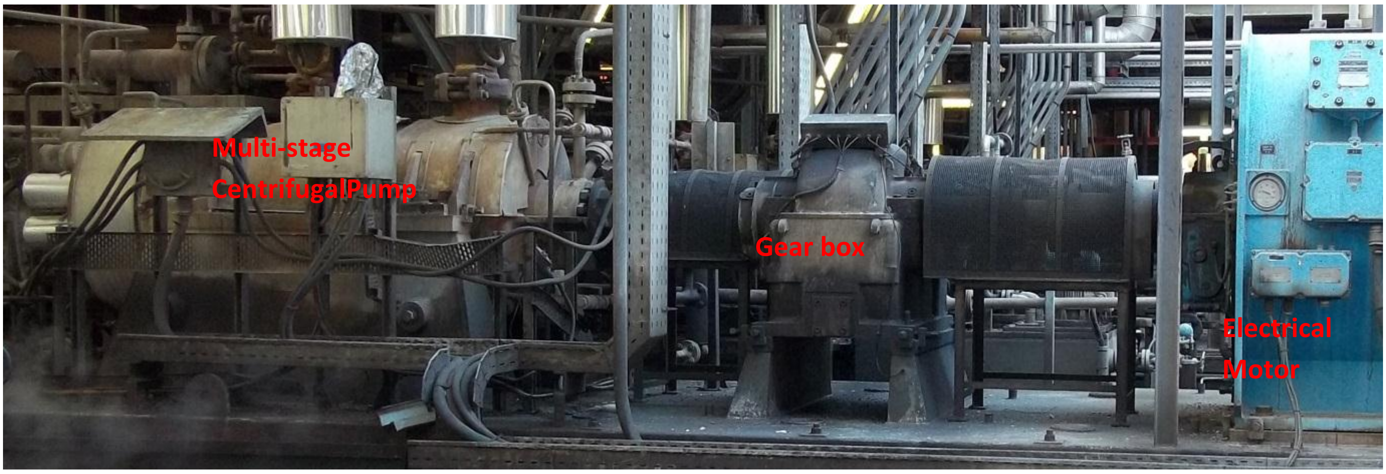


Figure 4: Global view of motor, gearbox and pump

This pump is monitoring online with eddy current probes which measure shaft motion in axial direction *z* on the support *NDE* and in direction *X'* and *Y'* for each of the two supports as shown in Fig.5 and Fig. 6. Other four eddy current sensors are installed on the gearbox supports in vertical direction. An addiction accelerometer with magnetic base is fixed only on the pump support DE in the horizontal direction. A key phasor (laser device) is installed under the rotor to connect sensors signals and phase. All the sensors are connected to a data collect SKF Microlog Analyzer GX. Displacements of the shaft are measured in micro with detection peak-to-peak, while the signal from the accelerometer is acquired in velocity by time integration. The acquisition of the measures has been done simultaneously from all the sensors [38].

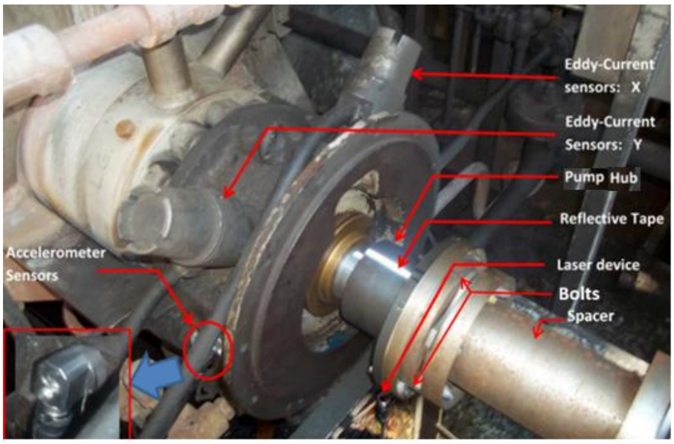


Figure 5 Pump Support DE and coupling hub CHP, devices for vibrations and phase measurement.

***Table 1** Pump allowable vibration.*

	Allowable vibration
Normal	40 µm peak to peak
Alarm	54 µm peak to peak
Shutdown	70 µm peak to peak

Table 1 reports the values of the pump allowable vibration, defined basing on historical data [39-40].

Fig.7 shows the vibration spectrum on DE support in the initial conditions, measured by the eddy current probes. The peaks detected at the frequency corresponding to the rotation speed are very high (61.69 µm in direction *X'* and 72.78 µm in direction *Y'*) than any other peak. This proves that the state of the machine vibration is inacceptable and that the main cause of high vibration is really due to the unbalance; we have confirmed it also by the vibration measured in velocity with the accelerometer.

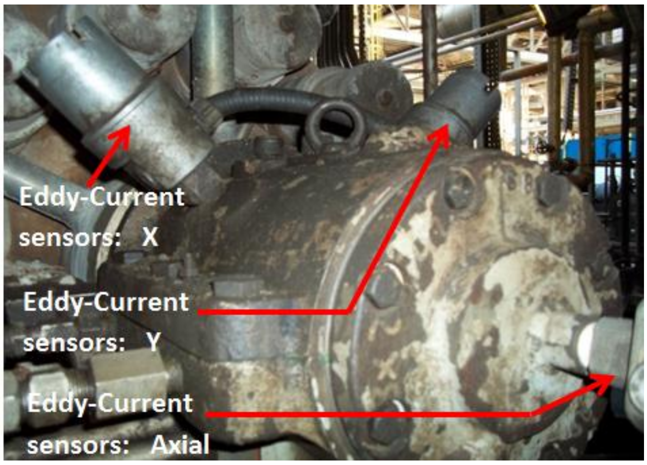


Figure 6 Pump Support NDE, devices for vibrations measurement.

In order to apply the balance with one-plane of balancing and two planes of measurement, it is sufficient one point of acquisition for each support. Since we have a pump that equipped with four sensors (two for support), we profit to compare the results of the two-way acquisition *X'* and *Y'*. Tables 2 reports the results, independently for the directions *X'* and *Y'*: initial measurement, measurement with a trial mass and with the related correction mass calculated by eq. (7). It is worth to note that, in the initial conditions, it is $w_{DE} = w_1 \gg$

$w_2 = w_{NDE}$. On the contrary, Table 3 reports the results using the vibration measurements of the support DE only, with the addition of the accelerometer in the direction *X*.

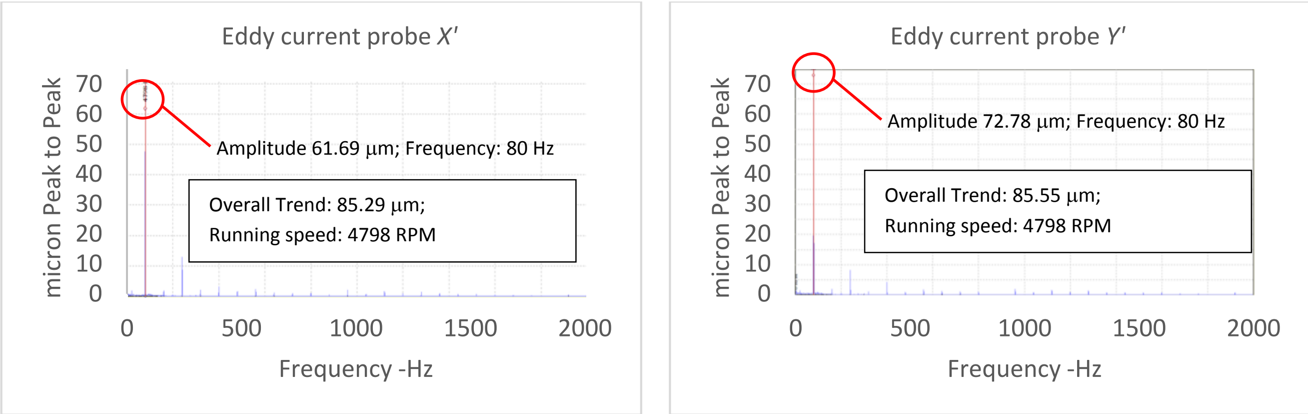


Figure 6: Vibration Spectrum on support-DE showing the initial unbalance.

Table 2: Vibrations measured on both supports by eddy currents sensors.

	Initial vibration measurement				Vibration measurement with trial mass m_{tz} (10 g, θ=144°)				Calculated correction mass	
	w_1 (DE vibration)		w_2 (NDE vibration)		w_{1z}		w_{2z}		m_{cz}	
	Modulo (µm)	Phase (deg)	Modulo (µm)	Phase (deg)	Modulo (µm)	Phase (deg)	Modulo (µm)	Phase (deg)	Modulo (g)	Position (deg)
direction X'	61.69	128	13.72	308	31.45	129	10.5	308	20.64	145
direction Y'	72.11	218	12.01	38	36.97	218	15.55	38	19.98	145

Table3: Vibrations measured on support DE only by eddy currents sensors and accelerometer.

	Vibration initial measurement		Vibration measurement with trial mass m_{tz}(10 g, θ=144°)		Calculated correction mass	
	w_1		w_{1z}		m_{cz}	
	Modulo	Phase (deg)	Modulo	Phase (deg)	Modulo (g)	Position (deg)
Eddy current probes X'	61.69 µm	128	31.45µm	129	20.39	145
Eddy current probes Y'	72.11 µm	218	36.97µm	218	20.51	145
Accelerometer X	2.89 mm/s	83	1.51 mm/s	83	20.93	145

From Table 2 and Table 3, we can observe that the values of correction mass m_{cz} calculated for the different approaches and different sensors are quite similar. Therefore, in order to balance the rotor, we considered the correction mass calculated with the second approach by the accelerometer vibration measurement (Table 3, m_{cz}=20.9g, position 145°).
The coupling hub is fitted to the pump shaft by 10 bolts at regular distance and angle (see Fig.5). Therefore, we thought to add the correction mass on the hub by increasing the weight of two bolts closer, together with their screws. The bolts closer to the calculated correction mass are at 144° (8/10π) and 180° (π), thus, it is $20.9e^{\frac{806}{1000}\pi j} = 20.43e^{\frac{8}{10}\pi j} + 0.62e^{\pi j}$. Because *0.62* is negligible in respect of 20.43, as well as the difference between 145° and 144°, we chose to fix the correction mass to m_{cz} =20.9g at position 144°, so to use only one bolt as shown in Fig.7.
Table 4 shows the residual vibrations, after the addition of the correction mass, measured on the support DE both by the accelerometer and by the two eddy current sensors in directions *X'* and *Y'*.

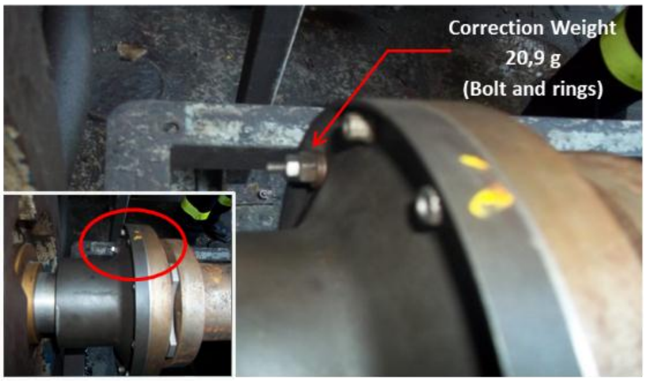


Figure 7: Addition of the correction mass (20.90g) to the position 144 °

Table 4: Residual vibration values measured after correction weight addition

Sensor	Modulo w_{c1}
Eddy Probe X'	13.97 (µm)
Eddy Probe Y'	19.26 (µm)
Accelerometer	0.51 (mm/s)

As it can be seen, the pump rotor balancing obtained by adding a correction weight to the CHP has permitted to reduce considerably the vibration level, which is turn to be normal. Obviously, we took care to verify that the added weight not increase the pump axial vibration and not cause any problem to the gearbox. Indeed, we have observed with the eddy probe sensors that the level of these vibrations have remained normal even after the balancing.

IV. CONCLUSION

The main result of this study is having proved that it is conceivable to perform the on-site balancing of a multi-stage centrifugal pump, which ordinarily cannot be balanced on-site, because the rotor is accessible only with the disassembly of the pump. The set out approach allows to solve the unbalance in the shortest possible time, reducing the time required to an hour on average. It allows to bring the machine in acceptable reliability conditions, but it not replaces definitely the normal balancing, which is necessary in other unbalance cases, as the ones due to wear, deformations, shifts to the assembled elements, or in the context of a general planned review.
The proposed balancing technique is performed using the coupling hub fitted to the pump as a single balancing plane, so overcoming the constrain of the inaccessibility of the pump impellers. However, this is possible only when the initial vibration on the driver end support is very higher than the non-drive end support one. We have experimental verified this approach on an industrial pump used in the process of heavy petrochemical products and we have obtained a good success. We used the Influence Coefficients Method, both with two measurement planes and with only one. The two approaches have provided nearly the same result. The second approach, carried out by means of a removable accelerometer, has the advantage to be relatively more simple and more easily applicable whereas many machines, for cost reason, are not equipped with fixed sensors.

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