

An Efficient Method for Hand-Torn Document Reconstruction

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Abstract:

In this paper, an efficient method for hand-torn document reconstruction is proposed. The proposed method comprises three major stages, which are preprocessing, feature extraction, and matching respectively. The preprocessing stage starts with extracting the green-channel of the input fragment, and morphological opening and binarization is performed, followed by contour extraction and simplification. A number of features, such as vertex angle, line distance and orientation are extracted in the feature extraction stage. Afterwards, the method searches for the best matching between two fragments, correct the rotation angle, and join those fragments, and this procedure is performed iteratively until the whole document is reconstructed. A simple but effective procedure is proposed to measure the performance of the proposed reconstruction method. Two criteria, namely True Positive Rate (*TPR*) and False Negative Rate (*FNR*) have been used to evaluate the performance of the proposed method. The experimental results show that the proposed algorithm achieves 0.9777 and 0.0223 average values in terms of *TPR* and *FNR* respectively.

Keywords: Image Reconstruction; Torn Document; Feature Matching; Polygonal Approximation.

1. INTRODUCTION

The problem of hand-torn document reconstructions is often essential for forensic investigators and intelligence gathering operators to prepare a preliminary analysis on discarded documents of confidential or sensitive information associated with organized crimes, art crimes, embezzlement, forgery, etc. It is also essential for historians, to reconstruct valuable historical documents those might be severely damaged. Reconstruction of torn documents is done by trained professionals. However, such manual reconstruction of the torn documents is tedious and tough due to the large number of fragments and permutations associated with fragment arrangements process. Therefore, automated reconstruction of shredded documents is feasible in terms of time and efforts as compared to the manual reconstruction process.

Several methods have been proposed in the literature for the reconstruction of torn documents. Zhu et al. [1] presented a novel global method for the reconstruction of hand-torn documents using partial curve matching by determining

candidate matches of the fragments. Then, the candidate matches are disambiguated iteratively by maximizing a global consistency criterion to reconstruct the document. The overall performance of that method indicates the possibility of reconstructing up to 50 fragments of torn documents automatically. However, the global reconstruction method requires more theoretical analysis. For example, in the relaxation process, the candidate matches are assumed to be well connected, and without considering the color or text it may fail to distinguish the resulting identical matches from the identical fragments. Pimenta et al. [2] proposed a method for document reconstruction using a dynamic programming. The polygonal approximation is firstly performed to simplify the boundaries complexity and extract the most important features from that polygon to be fed later to the LCS dynamic programming algorithm. Finally, the best matching fragments are determined using modified Prim's algorithm by calculating the LCS score obtained from the extracted features. The results showed that Pimenta's method makes an improvement of 18% in the number of reconstructed fragments as compared to global search algorithms. Kleber et al. [3] proposed an automated method for torn documents reconstruction based on shape and content, which performs a pre-calculation of each fragment and cluster the input data. Based on the handwritten text or printed information, the main orientation is calculated. Other calculations, such as rotational analysis and color of the ink/paper are performed on each fragment to reduce the search space for matching process. Although the method seems efficient to reconstruct the torn documents as the reconstruction relies on the shape and content, the method may fail if one of the document fragments is missing. A semi-automatic method for torn document reconstruction has been proposed by Patrick [4]. Partial reconstruction is performed using an iterative and interactive process of acceptance, selection or fine tuning. The outer frames are reconstructed then the frames are filled in with non-border fragments if necessary. The method makes use of novel set of interactive tools for global reconstruction. However, the method did not consider the large scale experiments. Cho et al. [5] considers the overall case of rectangular image fragments. A confidence propagation algorithm and probabilistic model are employed to identify the optimal fragment configuration. In case there is no available information on shape, the similarity measurement is used based on the respective fragment borders, and various compatibility metrics are evaluated. Minimum spanning tree

has been employed by Cao et al. [6] to reconstruct torn documents, where possible matches among the document fragments are identified by using inter-fragment constraints. The method makes use of shape features and color information. A graph-based method is proposed in that work in order to assemble the torn document by identifying sub-graphs corresponding to separate documents and employ a spanning tree to search for of each sub-graph. The preprocessing stage and spanning tree of Cao et al. [6] have been adapted by Richter et al. [7] but the information obtained from the polyline and content-based constraints on partial solutions are considered. Another algorithmic method has been proposed by Richter et al. [8] for the reassembly of torn documents. Support Vector Machine (SVM) is used for identifying suitable pairs of support points for aligning their respective fragments. The purpose of using SVM is to distinguish matching points of attachment from false matches based on feature dissimilarities which utilized shape- and content-based information. Then, all fragments are aligned into groups after identifying the points of attachment. A set of geometric and content-based constraints are employed to find the optimal alignment between all groups of fragments. In each iteration, the groups of fragments are combined until the document is entirely reconstructed. The overall performance of the method indicates the possibility of reconstructing up to 32 fragments of torn documents. A generic model for reconstruction of torn documents has been proposed by Roy & Garain [9]. The model is probabilistic in nature and it addresses several issues of a probabilistic model applied to a generic reconstruction. The model indicates the simplicity of searching through the space of all possible alternatives regardless of the number of samples. The model makes use of shape and statistics based cues to compare the compatibility of two pieces probabilistically. Shang et al. [10] proposed a semi-automatic method to reconstruct torn documents based on pairwise matching of fragments. Image fragment contour is divided into curves using corner detection and presents a process to estimate the match of two curves. The curve matching process is robust to rotation and translation and it is capable of dealing with shape deformations caused by tearing by tolerating overlaps of fragments during matching. To improve matching performance, text lines alignment and color information is also utilized. Furthermore, the method is able to solve the first and second puzzles of “DARPA shredder challenge”. Wattanacheep & Chitsobhuk [11] proposed an algorithm for torn document reconstruction based on the histogram of accumulated radius of fitted ellipse to identify the image fragments with uncertain properties, such as different orientations. The reconstruction process relies on the estimation of geometric shape representation using Hough transform descriptor.

In this paper, an efficient method for hand-torn document reconstruction is proposed. The proposed method comprises three major stages, which are preprocessing, feature extraction, and matching respectively. The preprocessing stage starts with extracting the green-channel of the input fragment, and morphological opening and binarization is performed, followed by contour extraction and simplification. A number of features, such as vertex angle, line distance and orientation are extracted in the feature extraction stage.

Afterwards, the method searches for the best matching between two fragments, correct the rotation angle, and join those fragments, and this procedure is performed iteratively until the whole document is reconstructed.

The remainder of this paper is organized as follows: Section 2 describes the stages of the proposed method in details. Results and discussion are presented in Section 3, and finally, Section 4 provides conclusions.

2. PROPOSED METHOD

Document or image reconstruction is the process of assembling ripped-up or torn document fragments to obtain a clear image corresponding to the original one. Smooth edges and well defined corners are usually taken into account in traditional puzzle solving algorithms. However, it is quite complicated to deal with hand torn documents since the act of tearing a piece of paper by hand causes irregularity in the boundaries, which makes it impossible to get a perfect curve matching. All proposed reconstruction methods take image fragments as an input, and return a complete reconstructed image after processing it.

A dataset consists of 100 pages collected from magazines and books are used to measure the performance of the proposed method. Our dataset is divided into three subsets of manually shredded images with different shapes and sizes. 50 pages are shredded into 16 pieces, 30 pages are shredded into 24 pieces, and 20 pages are shredded into 32 pieces. The fragments of each page are scanned against a uniformly black background to facilitate the subsequent stages, as shown in Fig. 1. The proposed method consists of three major stages, namely preprocessing, feature extraction, and matching respectively, as demonstrated in Fig. 2.



Figure 1: Sample of hand-torn page used as an input to our proposed method

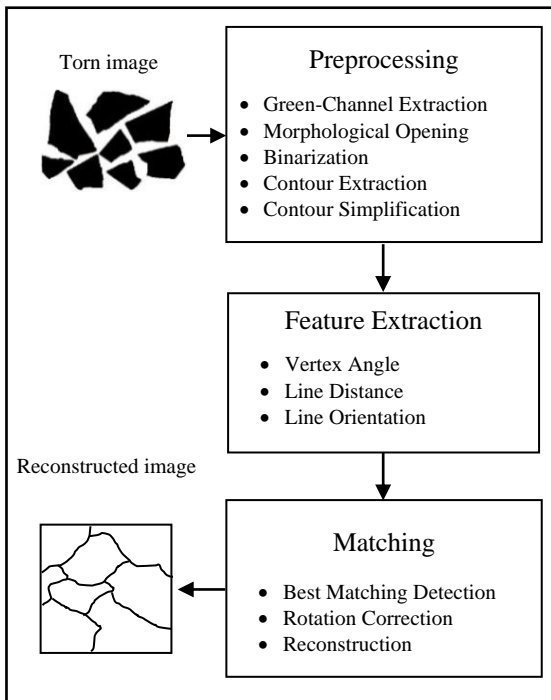


Figure 2: Block diagram of the proposed method.

2.1 Preprocessing

Hand-torn fragments $I_f (f = 1, 2, \dots, n)$ are stored in an array of size n where n represents the number of image fragments. Among the color image components (i.e. red, green, and blue), green-channel provides maximum local contrast among the image pixel values [12]. Therefore, the green-channel of the input fragments I_g is first extracted, as shown in Fig. 3(a). This is followed by morphological opening γ using a predefined structuring element (SE) to get rid of the possible holes or text within the fragment, as shown in Fig. 3(b). The concept of an opening is similar to the erosion in that it tends to remove some of the foreground pixels from the edges of regions of foreground pixels. Unlike erosion, opening operator tends to smooth the contour of an object, eliminates thin protrusions, and breaks narrow isthmuses. An opening is simply defined as erosion (ϵ) followed by dilation (δ) using the same structuring element (SE) for both operations [13]. The opening process is expressed by,

$$\gamma_{SE}(I_g) = \delta_{SE}[\epsilon_{SE}(I_f)] \quad (1)$$

Subsequently, the resulting image of the opening operation is converted to black and white so that the boundaries of each fragment image can be well identified. As mentioned previously, the fragments of each page are scanned against a uniformly black background while the fragments are relatively bright, which make it easier to select a fixed threshold value T to binarize the image fragments, as illustrated in Figs. 3(c)-(d).

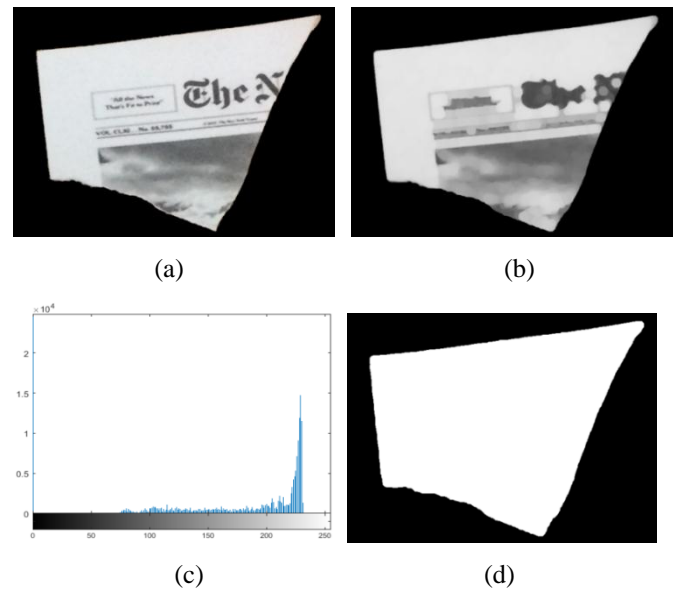


Figure 3: Binarization Steps, (a) Green-channel extracted from a sample fragment, (b) resulting image of morphological opening, (c) histogram of (b), and (d) resulting binary image of (b).

Shape representation plays an essential role in image reconstruction, and it is generally classified into two methods, which are as region-based and contour-based methods. Contour-based shape representation is further classified into global approaches (i.e. perimeter, signature, eccentricity, and compactness) and structural approaches (i.e. polygon, chain, invariants, and B spline). Modified Moore-neighbor tracing algorithm [13] is employed in the proposed algorithm in order to trace the contour of each fragment. The exact contour of the fragment tends to be large in practice as it is represented by a set of pixels. Therefore, Douglas-Peucker algorithm [14] is employed for polygon simplification by reducing the number of vertices in a hand torn fragment's contour. Each fragment is defined by $F_i = (S_i, C_i)$, where C_i denotes the image content and S_i represents a subset of support points (vertices), $S_i = \{s_1^i, s_2^i, \dots, s_n^i\} \subset P_i$, as demonstrated in Fig. 4.

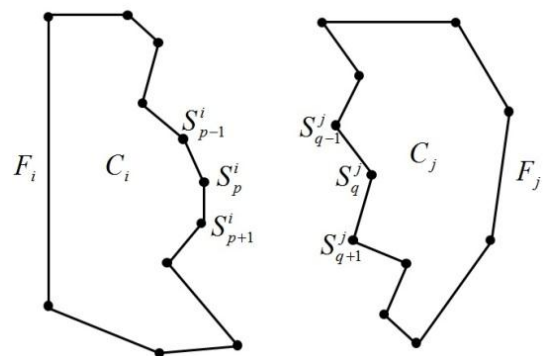


Figure 4: The approximate contour of two fragments F_i and F_j

2.2 Feature Extraction

After applying Douglas-Peucker algorithm and perform the polygon simplification task, significant features are extracted and stored in feature vector whereby the local matching is performed. Feature extraction is considered as process utilized to reduce the complexity of the polygon, as polygon is converted to a set of features. Three significant features are used in the proposed method, which are Angle Feature, Line Distance Feature, and Line Orientation Feature.

Angle feature represents angle of each vertex with respect to its two adjacent vertices. All the angle values are stored in an array A. As shown in Fig. 5, consider the vertex S_p of the polygon. In the proposed method, the angle α is also verified whether it is concave or convex. The vertex S_p , as can be seen in Fig. 5, has a convex angle while vertex S_{p+1} has a concave one. The vertex angle α is expressed by,

$$\alpha = \cos^{-1} \frac{\overline{S_p S_{p+1}} \cdot \overline{S_p S_{p-1}}}{\left| \overline{S_p S_{p+1}} \right| \left| \overline{S_p S_{p-1}} \right|} \quad (2)$$

Image fragments are classified in our method into frame-part and inner-part to reduce the number of iterations in the matching stage. This classification is performed based on the angle values of the polygon vertices. Every angle value stored in A is compared with its successor. If the angle values are different, the successor of current angle is stored in another array B. Similarly, we compare every angle value stored in B, and the successor value which differs from its preceding value is counted as d_θ . Through the ratio r between number of vertices and d_θ , we decide whether that fragment is a frame- or inner-fragment. Consequently, the current fragment is said to be a frame-fragment if it is greater than a predefined threshold [15].

Line distance feature is computed using Euclidean distance method, which represents the distances between every vertex and its adjacent one (vertices can be passed through in either clockwise- or counterclockwise direction) and stored in our feature vector.

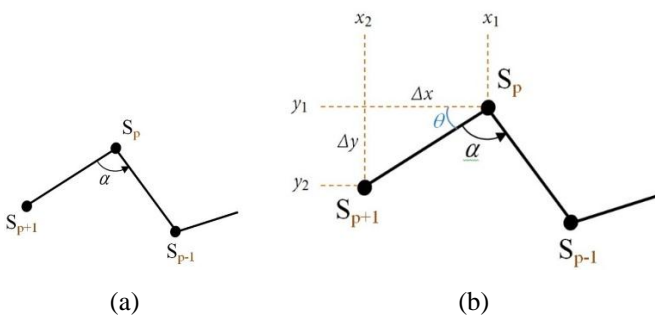


Figure 5:(a) Angle feature extracted from the polygon, (b) line orientation between the vertex S_p and its next vertex S_{p+1} .

Eventually, line orientation feature θ_p is calculated by determining the coordinates of the vertex S_p and its next vertex S_{p+1} , which are represented (x_1, y_1) and (x_2, y_2)

respectively. Based on Δx , Δy calculated using Eqns. (3) and (4), the line orientation angle calculated using Eq. (5), as follows:

$$\Delta x = |x_1 - x_2| \quad (3)$$

$$\Delta y = |y_1 - y_2| \quad (4)$$

$$\theta = \tan^{-1} \frac{\Delta y}{\Delta x} \quad (5)$$

where \tan^{-1} represents the inverse tangent of θ . Table 1 briefly describes the feature vector of sample vertices S_p , where $p=1,2,\dots, N$ number of vertices that form the fragment polygon. Four values are associated with each vertex.

Table 1: Example of the proposed feature vector of sample vertices S_p .

Vertex	Angle ^o	Line Distance		Line Orientation
	α	Next	Previous	θ
S_1	110	55	40	35
.
.
.
S_N	80	40	58	60

2.3 Matching

The proposed method selects a piece as a seed to start the matching. As mentioned in the previous section, image fragments are classified into frame-part and inner-part to reduce the number of iterations in the matching stage. The perimeter of each frame- and inner-part is calculated, and based on the perimeter values; the largest frame-part is taken as the seed fragment and compared against the largest inner-parts. The reason behind selecting the largest fragments is due to the greatest shape statistics as compared to the small fragments.

A degree of similarity is computed based on the feature vector values presented in the previous section, which indicates the quality of matching two fragments. Assume we have two fragments F_i and F_j , as demonstrated in Fig. 6(a). The feature vector of a vertex S_p of the fragment F_i is compared against the feature vectors of F_j till the best match is met. The vertex angle is one of the most significant features, and the best matching between two vertices angles S_p^α and S_q^α must be complementarity and sum up 360° . Practically, the vertices' angles might not sum up 360° since the polygonal approximation made would slightly affect the angles, and therefore a degree of tolerance should be considered. If the both vertex angles $\approx 360^\circ$, a matching angle (M^α) is set to 1.

Let D_1^p denote the Euclidean distance between the vertices S_p and S_{p+1} , and D_2^p denote the Euclidean distance between the vertices S_p and S_{p-1} .

Let D_1^q denote the Euclidean distance between the vertices S_q and S_{q+1} , and D_2^q denote the Euclidean distance between the vertices S_q and S_{q-1} .

Let θ_p denote the orientation line associated with the vertex S_p , and θ_q denote the orientation line associated with the vertex S_q .

The similarity criterion SC between those two vertices is computed by using Eq. (6). If a sequence of feature vector values associated with 5 vertices is similar, then $SC=SC+2$. Otherwise, the SC value will not be changed. As the image fragments are scanned randomly without considering their orientation, most (if not all) fragments would be rotated in order to reconstruct them correctly. To overcome the rotation problem, after calculating the SC between two fragments (e.g. F_i and F_j) and measuring the best matching between them, the fragment F_j is rotated in either clockwise or anticlockwise direction depending on the orientation angle of any two matched vertices, as described in Fig. 6(b). Assume that (x,y) refer to the source coordinates of an askew image fragment. The destination coordinates (x',y') are calculated by using the general equation of rotation, as expressed by Eq. (7).

$$SC = \begin{cases} 1 & \text{if } [(D_1^p \approx D_1^q \mid D_2^p \approx D_2^q) \& M^\alpha = 1] \\ 5 & \text{if } [(D_1^p \approx D_1^q \& D_2^p \approx D_2^q) \& M^\alpha = 1] \\ 8 & \text{if } [(D_1^p \approx D_1^q \& D_2^p \approx D_2^q) \& (\theta_p \approx \theta_q) \& M^\alpha = 1] \\ null & \text{otherwise} \end{cases} \quad (6)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (7)$$

Afterwards, a well-known algorithm proposed by Leitao and Stolfi [16] is applied to join the image fragments, as described in Algorithm 1. Assume we have a hand-torn image $IM = \{F_1, F_2, \dots, F_n\}$ composed of n fragments and F denotes an image fragment. To search for the best matching, F_1 is compared against other fragments, and the SC value defined previously is maximized accordingly until the highest value of SC is obtained, then those two fragments are joined and produce a new fragment. Once the new fragment is reconstructed, its feature vector is then updated by eliminating the matched vertices, as shown in Fig. 6(c). The algorithm is performed repeatedly until the entire document is reconstructed [17]. Fig. 7 shows a sample reconstructed image using the proposed method.

Algorithm 1

1. $IM = \{F_1, F_2, \dots, F_n\}$
2. **repeat**
3. best = Null
4. **for** $i = 2$ to n **do**
5. Compute all possible SC for F_1 and F_i
6. **if** $SC > 0$ **then**
7. best = i
8. **end if**
9. **end for**
10. **if** best \neq Null **then**
11. $F_{new} = F_1 \cup F_{best}$
12. Remove F_1 and F_{best} from IM
13. Insert F_{new} into IM
14. $n = n - 1$
15. **end if**
16. **until** $n=1$ or best \neq Null
17. **Return** F_{new}

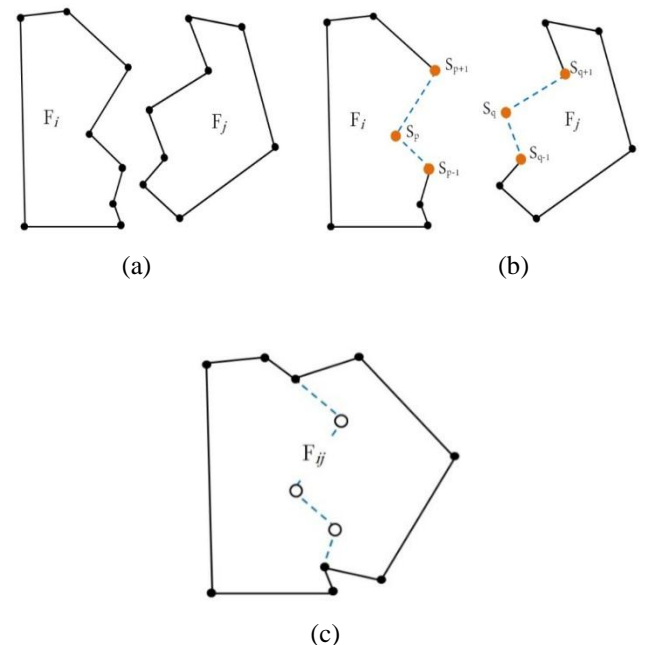


Figure 6: Best matching joining process, (a) fragments F_i and F_j , (b) best matching in terms of angles and line distances between 2 fragments, and (c) reconstructed fragment F_{ij} , where inner vertices denoted by \circ are eliminated.



Figure 7: Reconstructed image using the proposed method.

3. RESULTS AND DISCUSSION

A dataset consists of 100 pages collected from magazines and books were used to measure the performance of the proposed method. The dataset was divided into three subsets of manually shredded images with different shapes and sizes. 50 pages were shredded into 16 pieces, 30 pages were shredded into 24 pieces, and 20 pages were shredded into 32 pieces, and the fragments of each page were scanned against a uniformly black background in JPEG format at 300 dpi. To measure the performance of the proposed reconstruction method, a simple but effective procedure is proposed. Firstly, binarized fragments of each page were reconstructed to form an image BW_r of size $N \times M$, as clarified in the preprocessing stage. Afterwards, a new binary Image BW_w of size $N \times M$ was created to resemble the reconstructed image in terms of size, where $BW_w(x,y)=1$. In practice, the edges of the reconstructed fragments tend to be black due to several reasons, such as the low resolution of the scanned images, deformed edges, inaccurate rotation, etc. Therefore, the dissimilarity between the reconstructed image BW_r , which represents the actual result of reconstruction and BW_w which represents an ideal case of reconstruction would be meaningful as it indicates the error rate, as illustrated in Fig. 8.

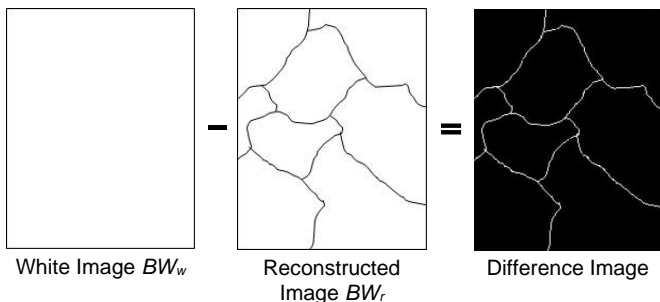


Figure 8: Example of dissimilarity between two images

The performance parameters were calculated on pixel by pixel basis using the confusion matrix shown in Fig. 9. Four values

are considered in the pixel-based evaluation, namely true positive (TP), false positive (FP), false negative (FN), and true negative (TN). TP refers to positive pixels correctly labeled as positive. FP refers to negative pixels incorrectly labeled as positive. FN refers to positive pixels incorrectly labeled as negative. Finally, TN refers to negative pixels correctly labeled as negative [18]. Thus, white lines within the difference image shown in Fig. 8 indicate the FN value, and the black pixels indicate the TP values. Two criteria, namely True Positive Rate (TPR) and False Negative Rate (FNR) were used to evaluate the performance of the proposed method, as expressed by Eqns. (8) and (9) respectively.

		Target	
		Positive	Negative
Output	Positive	TP	FP
	Negative	FN	TN

Figure 9: Confusion Matrix

$$TPR = \frac{TP}{TP + FN} \quad (8)$$

$$FNR = \frac{FN}{FN + TP} \quad (9)$$

The experiments conducted on the proposed method were implemented using MATLAB. Based on the experimental results, the proposed algorithm achieved 0.9777 and 0.0223 average values in terms of TPR and FNR respectively. Although the experimental results were satisfactory and promising, the performance of the proposed method can be improved by using high resolution images (e.g. 600 dpi).

4. CONCLUSION

In this research paper, an efficient method for hand-torn document reconstruction has been proposed. Three major stages are comprised in the proposed method, which are preprocessing, feature extraction, and matching respectively. The preprocessing stage starts with extracting the green-channel of the input fragment, and morphological opening and binarization is performed, followed by contour extraction and simplification. A number of features, such as vertex angle, line distance and orientation are extracted in the feature extraction stage. Finally, the method searches for the best matching between two fragments, correct the rotation angle, and join those fragments. The procedure is performed iteratively until the whole document is reconstructed. A

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REFERENCES

- [1] Zhu, L., Zhou, Z., and Hu, D., "Globally consistent reconstruction of ripped-up documents," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 30, no. 1, pp. 1–13, Jan. 2008.
- [2] Pimenta, A., Justino, E., Oliveira, L., and Sabourin, R., "Document Reconstruction using Dynamic Programming," in *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, Taipei, pp. 1393-1396, 2009.
- [3] Kleber, Florian, Markus Diem And Robert Sablatnig, "Torn Document Analysis As A Prerequisite For Reconstruction", *15th International Conference on Virtual Systems and Multimedia*, pp. 143-148, 2009.
- [4] Patrick De Smet, "Semi-Automatic Forensic Reconstruction Of Ripped-Up Documents", *10th International Conference on Document Analysis and Recognition*, pp. 703-707, 2009
- [5] Cho, T. S., Avidan, S., and Freeman, W. T., "A probabilistic image jigsaw puzzle solver," in *Proc. IEEE Conf. Comput. Vis. Pattern Recogn.*, pp. 183–190, 2010.
- [6] Cao, S., Liu, H., and Yan, S., "Automated assembly of shredded pieces from multiple photos," in *Proc. IEEE Int. Conf. Multimedia Expo*, pp. 358–363, 2010.
- [7] Richter, F., Ries, C. X., and Lienhart, R., "A graph algorithmic framework for the assembly of shredded documents," in *Proc. IEEE Int. Conf. Multimedia Expo*, pp. 1–6, 2011.
- [8] Richter, F., Ries, C. X., Cebron, N., and Lienhart, R., "Learning to Reassemble Shredded Documents," *IEEE Transactions on Multimedia*, Vol. 15, No. 3, 2013
- [9] Roy, A., Garain, U., "A Probabilistic Model for Reconstruction of Torn Forensic Documents," in *Proc. 12th International Conference on Document Analysis and Recognition*, pp. 494-498, 2013.
- [10] Shang, S., Sencar, H. T., Memon, N., Kong, X., "A Semi-Automatic Deshredding Method Based on Curve Matching," in *Proc. IEEE International Conference on Image Processing (ICIP)*, pp. 5537-5541, 2014.
- [11] Wattanacheep, B., and Chitsobhuk, O., "Plane alignment algorithm for torn document reconstruction," in *Proc. 12th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON)*, 2015.
- [12] Salih, N. D., Saleh, M. D., Eswaran C., and Abdullah, J., "Fast Optic Disc Segmentation Using FFT-Based Template-Matching and Region-Growing Techniques", *Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization*, 6(1): 101-112, 2018.
- [13] Gonzalez, R. C., R. E. Woods, and S. L. Eddins, *Digital Image Processing Using MATLAB*, New Jersey, Pearson Prentice Hall, 2004.
- [14] Douglas, D., and Peucker, T., "Algorithms for the reduction of the number of points required for represent a digitized line or its caricature," *Canadian Cartographer*, 10(2):112-122, 1973.
- [15] Lotus, R., Varghese, J., and Saudia, S., "An Approach to Automatic Reconstruction of Apictorial Hand Torn Paper Document," *The International Arab Journal of Information Technology*, Vol. 13, No. 4, 2016.
- [16] Leitao, H. C. G., and Stolfi, J., "A Multiscale Method for the Reassembly of Two-Dimensional Fragmented Objects," *IEEE Trans. Pattern Anal. Mach. Intel.* 24:1239–1251, 2002.
- [17] Justino, E., Oliveira, L. S., Freitas, C., "Reconstructing Shredded Documents Through Feature Matching," *Forensic Science International*, 160: 140–147, 2006.
- [18] Saleh, M. D., Eswaran, C., Mueen, A., "An Automated Blood Vessel Segmentation Algorithm Using Histogram Equalization and Automatic Threshold Selection", *Journal of Digital Imaging*, 24(4): 564-572, 2011.