# Enhancing Entropy Based Spectrum Sensing using Eigen Value Decomposition in Cognitive Radio Networks

Baljeet Singh Negi<sup>1</sup>, OP Singh<sup>1</sup>, CN Khairnar<sup>2</sup>

<sup>1</sup> ASET, Amity University, Lucknow Campus, UP-226010, India. <sup>1</sup>ASET, Amity University, Lucknow Campus, UP-226010, India. <sup>2</sup> Military College of Telecommunication, Mhow, MP-453441, India.

<sup>1</sup>ORCID : 0000-0003-0485-3468; <sup>1</sup>ORCID : 0000-0002-9946-4141 <sup>2</sup>ORCID : 0000-0002-5099-9540

## Abstract

Spectrum sensing or detection of primary user (PU) signals is the most fundamental requirement of a Cognitive Radio Networks (CRN). Most of the traditional detectors in CRN suffer deterioration in low SNR region due to noise uncertainty factor. Entropy based detection (EBD) is found to be independent of this issue. Here we propose a new detection scheme wherein the signal received has been decomposed twice at the receiver so as to improve entropy based detection. First time, we decompose the received signal by deriving wavelet packet entropy components. Second time, we decompose the signal as we derive eigen values of the sample covariance matrix of the wavelet packet entropy components. To arrive at a decision to confirm occupation or vacation of spectrum by the licensed user, we compare the ratio of Maximum and minimum eigen values (MME) with a threshold. A comparative study of wavelet packet entropy based detection (WPEBD) and Eigen value based WPEBD (EV-WPEBD) has been carried out by implementing Monte Carlo experiments for various SNR values.

**Keywords**—CRN, spectrum sensing, WPEBD, eigen values, entropy.

## I. INTRODUCTION

There has been exponential growth in usage of wireless devices with advent of technological advancement in communication technologies. Demand for spectrum is ever growing and has no scope of slowing down in future too. The traditional practice of static allotment of spectrum has created acute shortage of available radio spectrum [1]. By 2020, over 50 billion wireless devices will be connected to one or many wireless networks which are mostly going to demand access to the high speed internet [2]. With static allocation of the radio spectrum to licensed users, some portions of the radio spectrum are heavily used while some others are rarely used. Lack of access to radio spectrum among unlicensed users can result in undesirable denial of service events. Hence exploration of techniques for resolution of the spectrum scarcity issue is at the forefront of future network research as this needs to be addressed as soon as possible.

CRN offers to solve the spectrum inadequacy by allowing the users who are unlicensed or are called as secondary users (SU) to avail every opportunity to utilize or exploit the spectrum

portion temporarily left unused by the PU with an assurance that it will be vacated as soon the PU desires to utilize it [3]. The CRN users utilize the spectrum without interfering with PU's communication networks. Hence, the most fundamental and important factor in CRN is spectrum sensing or detection of the PU signals so that it is not interfered with. Lot of research has gone into identifying and optimizing different spectrum sensing techniques for CRN, specially in last two decades [4].

The traditional spectrum sensing techniques have been classified into two categories: narrowband and wideband. Narrowband sensing analyzes one frequency channel at a time while wideband sensing analyzes a number of frequencies at a time. Examples of the former include energy detection [5-9], cyclostationary features detection [10-14], matched filter detection [15-17], covariance based-detection [18-21] and machine learning-based sensing [22-25]. In the latter, the spectrum is usually divided into multiple sub-bands and then they are sensed, either sequentially or simultaneously, using the narrowband sensing techniques [26].

However, traditional spectrum sensing or detection techniques suffer from a common problem and that is noise uncertainty. At low SNR, noise walls exists which deteriorates the detector's performance severely [27][28]. Hence at low SNR, CNR may cause interference with PU communication with spectrum sensing techniques which are affected by noise uncertainty.

Noise uncertainty issue can be overcome by entropy based detector (EBD). EBD is robust to noise uncertainty [29]. In a scenario, where Gaussian noise signals are involved, entropy sensing works on the premise that entropy of a stochastic signal is more. However, in cases where modulated signals of the primary users are contained in the received signal, reduction in entropy is observed. The time domain signal is converted to frequency domain signal to remove noise uncertainty. Once the signal is received at the receiver, its information content is evaluated using Shannon entropy[30]. Entropy is not affected by the factor of noise. Hence the results obtained, demonstrate robustness to noise uncertainty. The results show improvement in performance of detection. Various EBDs have been researched upon like DFT based EBD (DEBD) [31][32], wavelet entropy based detection (WEBD) and WPEBD [33]

DEBD provides only two dimensional information about the signal, that is, about its different frequency components and their respective amplitude. WEBD uses wavelet transform

(WT) of the signal being tested and covers all the three parameters of a signal i.e, frequency components, their respective amplitude and the time which gives the location of different frequency components on the period axis [34]. But WT excludes higher frequency components. WPEBD model utilizes wavelet packet transform which includes both lower as well as higher frequency components [35]. It gives a complete decomposition of the tested signal. Hence WPEBD supposedly yields the best results [36].

In this paper, we improve upon WPEBD detection by decomposing the received signal second time by using Maximum and Minimum Eigen values with it. Hence, the technique has been named as EV-WPEBD.

This paper is arranged in various sections. Section II briefly describes the methodology used to carry out entropy based detection using WPEBD and EV-WPEBD. Section III contains simulation results. Conclusion of the paper is given in section IV

## II. METHODOLOGY

Here the problem of spectrum sensing has been represented as a binary hypothesis.

 $H_0$  represents a situation where PU signal is absent and  $H_1$  represents a situation where PU signal is not absent.

In subsequent paragraphs, we have first explained WPEBD. Here, we calculate the wavelet packet entropy of the received signal. This entropy is matched with the threshold entropy. Based on the output, we arrive at a decision about the existence of PU signals.

Next we have elaborated upon the proposed method of EV-WPEBD where the received signal is subjected to wavelet packet transform before calculating the entropy. Sample covariance matrix is obtained for the wavelet packet entropy ensemble of the received signal. Based on random matrix theories (RMT) [37-40], MME of the covariance matrix is obtained. Ratio of  $\lambda_{max}/\lambda_{min}$  is matched with a threshold value  $\Lambda$  which is always greater than 1. If the ratio is greater than threshold, it implies that the PU signal exists.

A comparative analysis has been carried out using simulations. Results using receiver's operating characteristics (ROC) curves displays performance of both methods discussed above at various levels of SNRs. Both methods are robust to noise uncertainty, however, their efficiency differs and the same is attempted to be established in this paper. Performance of both the methods have been compared with the help of ROC curves through probabilities of detection represented by  $P_d$  and false alarm probabilities expressed as  $P_f$ .

#### **II.I** WPEBD Algorithm For Detection Of PU Signals



Figure 1: Basic WPEBD model.

Fig. 1 represents the model where the received signal is put through wavelet packet transform before calculating its entropy. For j levels, wavelet packet transform (WPT) decomposes x(n), that is, a noisy signal into  $2^{j}$  sub bands. The corresponding wavelet packet coefficients can be expressed as [41][42].

$$d_{i,m}^{j} = WP\{x(n), j\} n = 1, \dots, N$$
(1)

 $d_{i,m}^{j}$  represents the m<sup>th</sup> coefficient that belongs to the i<sup>th</sup> sub band of level j where m=1...N/2<sup>j</sup> and i=1...N/2<sup>j</sup>

The energy calculated for sub-band i and the level j [43] is given as,

$$E_i^j = \sum_m \left| d_{i,m}^j \right|^2 \tag{2}$$

Wavelet packet coefficients' total energy  $E_{total}^{j}$  is given as

$$E_{total}^{j} = \sum_{m} \left| d_{i,m}^{j} \right|^{2} = \sum_{l=1}^{2^{J}} E_{i}^{j}$$
(3)

For every level, probability distribution can be computed by

$$P_i^j = \frac{E_i^j}{E_{total}^j} \tag{4}$$

Normalized wavelet packet energy is given as  $\sum_i P_i^j = 1$ .

For j level, the wavelet packet entropy is expressed as,

$$S_{wp}^{(j)} = -\sum P_i^{j} \log_2[P_i^{j}]$$
(5)

 $H_0$  is the hypothesis where the signal received as x(n) = w(n) contains noise. Wavelet packet entropy for w(n) is calculated by (5) is given as,

$$S_{wp}^{(j)} = S_{wp}^{(j)}(w(n)) = -\sum P_i^j \log_2[P_i^j]$$
(6)

The w(n) is a random variable which is additive White Gaussian noise.

 $H_1$  is the hypothesis where the signal that is received as x(n) = s(n) + w(n) contains both noise and the primary signal. Here wavelet packet entropy for x(n) calculated using (5) is given as,

$$S_{wp}^{(j)} = S_{wp}^{(j)}(x(n)) = -\sum P_i^j \log_2[P_i^j]$$
(7)

 $T_j(X)$  is the test statistic which is concerned with wavelet decomposition level j having N as the sample size.

The test statistic obtained is expressed as,

$$T_j(X) = -\sum_i^{2^j} P_i^j \log_2[P_i^j]$$

$$= \sum_{i}^{2^{j}} \frac{E_{i}^{j}}{E_{total}^{j}} \log_{2} \left[ \frac{E_{i}^{j}}{E_{total}^{j}} \right] \begin{cases} \leq \lambda^{j} : decideH_{1} \\ \geq \lambda^{j} : decideH_{0} \end{cases}$$
(8)

 $\lambda^{j}$  is the detection threshold for level *j* obtained for a target false alarm ratio P<sub>f</sub> and is given by,

$$\lambda^{j} = S_{wp}^{(j)}(w(n)) + Q^{-1}(1 - P_{f})\sigma_{e}$$
(9)

Here,  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} exp(-\tau^2/2)d\tau$ ,  $Q^{-1}(x)$  is calculated as the inverse function of Q-function.

#### **II.II** EV-WPEBD Algorithm For Detection Of PU Signals



Figure 2: Basic EV-WPEBD model.

By (7), wavelet packet entropy for x(n) in WPEBD is given as,

$$S_{wp}^{(j)} = S_{wp}^{(j)}(x(n)) = -\sum P_i^j \log_2[P_i^j]$$

Equation (7) gives the wavelet packet entropy ensemble of the received signal. As shown in Fig. 2, we compute the covariance matrix of this entropy ensemble as,

$$\operatorname{Rx}(\mathbf{N}) \stackrel{\text{def}}{=} \frac{1}{N} \sum_{n=L-1}^{L-2+N} S_{wp}^{(j)} \check{\mathbf{x}}(n) \check{\mathbf{x}}^{\dagger}(n)$$
(10)

Rx(N) is the sample covariance matrix with N collected samples.  $\uparrow$  is the Hermitian (transpose-conjugate).  $\check{x}(n)$  is output of x(n) that is the received signal after applying smoothening factor L. We calculate eigen values,  $\lambda_1, \lambda_2, \lambda..., \lambda_n$ for the above sample covariance matrix. We obtain the maximum and minimum eigen values as  $\lambda_{max}$  and  $\lambda_{min}$ . Ratio of  $\lambda_{max}/\lambda_{min}$  is matched with a threshold value  $\Lambda$  which is always greater than 1. If the ratio is greater than threshold, it implies that the PU signal exists (H<sub>1</sub>); otherwise it does not exist (H<sub>0</sub>). We can represent the decision condition as,

$$Decision \to \begin{cases} \left(\frac{\lambda_{max}}{\lambda_{min}}\right) \le \Lambda, & H_0\\ otherwise, & H_1 \end{cases}$$

 $\Lambda$  is the threshold value. Using only N, L and P<sub>f</sub>, we can calculate threshold before hand. It is not affected by signal and noise. It is given as-

$$\Lambda = \left(\frac{\sqrt{N} + \sqrt{L}}{\sqrt{N} - \sqrt{L}}\right)^2 \left(1 + \frac{\left(\sqrt{N} + \sqrt{L}\right)^{-2/3}}{(NL)^{1/6}} F_1^{-1} (1 - P_f)\right)$$
(10)

where  $P_f$  is the probability of false alarm for MME, and cumulative distribution function (CDF) of Tracy-Widom distribution of order 1 is denoted by  $F_1$ . In 1996, Tracy and Widon derived the Tracy-Widon distributions. For random matrices, largest eigen value is defined by Tracy and Widon distribution which acts as a limiting law for it [44][45].  $F_1^{-1}$  is calculated as inverse of  $F_1$ .

#### **III SIMULATION RESULTS**

Performance of WPEBD and EV-WPEBD has been evaluated by plotting the ROC graph between  $P_d$  which is the probability of detection and  $P_f$  which is the probability of false detection. If PU is correctly detected, then  $P_d$  represents the probability of correct detection. It satisfies  $H_1$ . If PU is falsely detection,  $P_f$  represents the probability of this false detection. High  $P_d$ with low  $P_f$  even at low SNR conditions is highly desirable. The present study is estimated for various SNR levels of the received signal.

Experiments of Monte Carlo are executed for over 10,000 runs. L=15,  $P_f = 0.08$ . The primary and the binary phase shift keying BPSK signal is modulated with frequency carrier of fc=40 KHz. Here the sampling frequency fs=100 KHz and the sampling time is 5ms.



**Figure 3**. Detection performance vs. SNR of the WPEBD and EV-WPEBD scheme, Sample size (*N*=18000)

From fig. 3 we can observe that the detection probability is decreasing as SNR (dB) value is increasing. EV-WPEBD performs better than WPEBD.



**Figure 4**. Comparison for Detection performance vs. false alarm probability for WPEBD and EV-WPEBD (SNR=-25 dB) and sample Size (N=18,000).



**Figure 5**: Comparison for Detection performance vs. false alarm probability for WPEBD and EV-WPEBD (SNR=-15 dB) and sample Size (N=18,000).



Figure 6. Comparison for Detection performance vs. false alarm probability for WPEBD and EV-WPEBD (SNR=-5 dB) and sample Size (N=18,000).

The detection performance curves vs. false alarm probability of WPEBD and EV-WPEBD with SNR of -25dB, -15db and -5dB is shown in fig 4-6 respectively. We can observe that EV-WPEBD (denoted by red line) outperforms WPEBD for all SNR values. EV-WPEBD is definitely more efficient than WPEBD for all levels of decomposition.

## **IV. CONCLUSION**

In this paper, performances of two entropy based detection techniques have been evaluated. We have established the efficiency of each detection technique being used for detecting the PU signal by carrying out Monte Carlo experiments. A comparative analysis between detection techniques based on entropy of wavelet packets and detection based on eigen values have been carried out using ROC curves of probability of detection and probability of false alarm for each technique. It is quite apparent that SNR plays an important role in wireless communication. The analyses as mentioned previously have been carried out for various SNR scenarios like -25 dB, -15 dB and -5 dB. Outcome of the analyses has been encouraging. EV-WPEBD has proved to be a more efficient detection technique even at low SNR of -25dB. This outcome will surely facilitate further research in design and development of detection techniques for CRN.

## REFRENCES

- Kaabouch N., Hu W.C. Handbook of Research on Software-Defined and Cognitive Radio Technologies for Dynamic Spectrum Managemen. Volume 2. IGI Global; Hershey, PA, USA: 2014.
- [2] Al-Fuqaha A., Guizani M., Mohammadi M., Aledhari M., Ayyash M. Internet of Things: A Survey on Enabling Technologies, Protocols, and Applications. IEEE Commun. Surv. Tutor. 2015;17:2347–2376. doi: 10.1109/ COMST.2015.2444095.
- [3] Rawat P., Singh K.D., Bonnin J.M. Cognitive radio for M2M and Internet of Things: A survey. Comput. Commun. 2016;94:1–29.]
- [4] Mitola J., Maguire G.Q. Cognitive radio: Making software radios more personal. IEEE Pers. Commun. 1999;6:13–18.
- [5] Ranjan A., Singh B. Design and analysis of spectrum sensing in cognitive radio based on energy detection; Proceedings of the International Conference on Signal and Information Processing; Vishnupuri, India. 6–8 October 2016; pp. 1–5.
- [6] Alom M.Z., Godder T.K., Morshed M.N., Maali A. Enhanced spectrum sensing based on Energy detection in cognitive radio network using adaptive threshold; Proceedings of the International Conference on

Networking Systems and Security; Dhaka, Bangladesh. 5–8 January 2017; pp. 138–143.

- [7] Arjoune Y., El Mrabet Z., El Ghazi H., Tamtaoui A. Spectrum sensing: Enhanced energy detection technique based on noise measurement; Proceedings of the IEEE Computing and Communication Workshop and Conference (CCWC); Las Vegas, NV, USA. 8–10 January 2018; pp. 828–834.
- [8] Eslami A., Karamzadeh S. Performance analysis of double threshold energy detection-based spectrum sensing in low SNRs over Nakagami-m fading channels with noise uncertainty; Proceedings of the Signal Processing and Communication Application Conference; Zonguldak, Turkey. 16–19 May 2016; pp. 309–312.
- [9] Bao Z., Wu B., Ho P.-H., Ling X. Adaptive Threshold Control for Energy Detection Based Spectrum Sensing in Cognitive Radio Networks; Proceedings of the Global Telecommunications Conference; Kathmandu, Nepal. 5– 9 December 2011; pp. 1–5.
- [10] Yawada P.S., Wei A.J. Cyclostationary Detection Based on Non-cooperative spectrum sensing in cognitive radio network; Proceedings of the International Conference on Cyber Technology in Automation, Control, and Intelligent Systems; Chengdu, China. 19–22 June 2016; pp. 184–187.
- [11] Ilyas I., Paul S., Rahman A., Kundu R.K. Comparative evaluation of cyclostationary detection based cognitive spectrum sensing; Proceedings of the Ubiquitous Computing, Electronics, and Mobile Communication Conference; New York, NY, USA. 20–22 October 2016; pp. 1–7.
- [12] Damavandi M.A., Nader-Esfahani S. Compressive Wideband Spectrum Sensing in Cognitive Radio Systems Based on Cyclostationary Feature Detection; Proceedings of the International Conference on Next Generation Mobile Applications, Services, and Technologies; Cambridge, UK. 9–11 September 2015; pp. 282–287.
- [13] Cohen D., Eldar Y.C. Compressed cyclostationary detection for Cognitive Radio; Proceedings of the International Conference on Acoustics, Speech, and Signal Processing; New Orleans, LA, USA. 5–9 March 2017; pp. 3509–3513.
- [14] Sharma S.K., Bogale T.E., Chatzinotas S., Le L.B., Wang X., Ottersten B. Improving robustness of cyclostationary detectors to cyclic frequency mismatch using Slepian basis; Proceedings of the International Symposium on Personal, Indoor, and Mobile Radio Communications; Hong Kong, China. 30 August–2 September 2015; pp. 456–460.
- [15] Zhang X., Chai R., Gao F. Matched filter-based spectrum sensing and power level detection for cognitive radio network; Proceedings of the Global Conference on Signal

and Information Processing; Atlanta, GA, USA. 3–5 December 2014; pp. 1267–1270.

- [16] Jiang C., Li Y., Bai W., Yang Y., Hu J. Statistical matched filter based robust spectrum sensing in noise uncertainty environment; Proceedings of the International Conference on Communication Technology; Chengdu, China. 9–11 November 2012; pp. 1209–1213.
- [17] Lv Q., Gao F. Matched filter-based spectrum sensing and power level recognition with multiple antennas; Proceedings of the Summit and International Conference on Signal and Information Processing; Chengdu, China. 12–15 July 2015; pp. 305–309.
- [18] Kumar K.S., Saravanan R., Muthaiah R. Cognitive Radio Spectrum Sensing Algorithms based on Eigenvalue and Covariance methods. Int. J. Eng. Technol. 2013;5:595– 601.
- [19] Zeng Y., Liang Y.C. Covariance based signal detections for cognitive radio; Proceedings of the 2007 2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks; Dublin, Ireland. 17–20 April 2007; pp. 202–207.
- [20] Zeng Y., Liang Y.C. Maximum-minimum eigenvalue detection for cognitive radio; Proceedings of the 2007 IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications; Athens, Greece. 3–7 September 2007; pp. 1–5.
- [21] Zeng Y., Liang Y.C. Spectrum-sensing algorithms for cognitive radio based on statistical covariances. IEEE Trans. Veh. Technol. 2009;58:1804–1815.
- [22] Balaji V., Kabra P., Saieesh P., Hota C., Raghurama G. Cooperative Spectrum Sensing in Cognitive Radios Using Perceptron Learning for IEEE 802.22 WRAN. Elsevier Procedia Comput. Sci. 2015;54:14–23.
- [23] Lu Y., Zhu P., Wang D., Fattouche M. Machine Learning Techniques with Probability Vector for Cooperative Spectrum Sensing in Cognitive Radio Networks; Proceedings of the IEEE Wireless Conference and Networking Conference; Doha, Qatar. 3–6 April 2016; pp. 1–6.
- [24] 24. Guangming N. Byzantine Defense in Collaborative Spectrum Sensing Via Bayesian Learning. IEEE Access. 2017;5:20089–20098.
- [25] Cheng Z., Song T., Zhang J., Hu J., Shen L., Wu J. Selforganizing map-based scheme against probabilistic SSDF attack in cognitive radio networks; Proceedings of the IEEE Wireless Communications and Signal Processing Conference; Nanjing, China. 11–13 October 2017; pp. 1– 6.

- [26] 26. Lu Q., Yang S., Liu F. Wideband Spectrum Sensing Based on Riemannian Distance for Cognitive Radio Networks. Sensors. 2017;17:661.
- [27] Tandra, R., & Sahai, A. (2008). SNR walls for signal detection. IEEE Journal of selected topics in Signal Processing, 2(1), 4-17.
- [28] Tandra, R., & Sahai, A. (2008, September). Overcoming SNR walls through macroscale features. In Communication, Control, and Computing, 2008 46th Annual Allerton Conference on (pp. 583-590). IEEE.
- [29] Nagaraj, S. V. (2009). Entropy-based spectrum sensing in cognitive radio. Signal Processing, 89(2), 174-180.
- [30] Cover T.M. & Thomas J.A.: 'Elements of information theory'(John Wiley & Sons, 1991), pp. 14-15
- [31] Cover T.M. & Thomas J.A.: 'Elements of information theory'(John Wiley & Sons, 1991), pp. 228-229.
- [32] Ya Lin Zhang, Qin Yu Zhang, Tommaso Melodia, "A frequency-domain entropy based detector for robust spectrum sensing in cognitive radio networks". IEEE Communications letters, Vol-16, No. 6, June 2010.
- [33] Zi-Long Jiang, Qin-Yu Zhang, Ye Wang & Xue-Quan Shang, "Wavelet packet entropy based spectrum sensing in cognitive radio". Proceedings of IEEE, 2011.
- [34] Goswami, J. C., & Chan, A. K. (2011). Fundamentals of wavelets: theory, algorithms, and applications (Vol. 233). John Wiley & Sons.
- [35] HE Zheng-you, Chen Xiaoqing, Luo Guoming, "Wavelet entropy measure definition and its application for transmission line fault detection and identification". IEEE, International Conference on Power System Technology, 2006, 1-6.
- [36] Baljeet Singh Negi, O.P. Singh, C. N. Khairnar, "Performance analysis of wavelet entropy based detection of primay user signals in Cognitive Radio Networks", IJERT (ISSN 0974-3154), Vol 12, No.6, June 2019, pp. 796-801.
- [37] A. M. Tulino and S. Verdu', Random Matrix Theory and Wireless Communications. Hanover, USA: now Publishers Inc., 2004.
- [38] I. M. Johnstone, "On the distribution of the largest eigenvalue in principle components analysis," The Annals of Statistics, vol. 29, no. 2, pp. 295–327, 2001.
- [39] K. Johansson, "Shape fluctuations and random matrices," Comm. Math. Phys., vol. 209, pp. 437–476, 2000.
- [40] Z. D. Bai, "Methodologies in spectral analysis of large dimensional random matrices, a review," Statistica Sinica, vol. 9, pp. 611–677, 1999.

- [41] Y. Ghanbari, M. Reza, and Karami-Mollaei, "A new approach for speech enhancement based on the adaptive thresholding of the wavelet packets," Speech Communication, vol. 48, pp. 927-940, Aug. 2006.
- [42] Sheng Li, JianQi Wang, XiJing Jing, Tian Liu. "Nonacoustic sensor speech enhancement based on wavelet packet entropy," CSIE(6)'2009. ppA47-450.
- [43] S Blanco, A. Figliola, R. Q Quiroga, 0 A. Rosso, and E. Serrano, "Time-frequency analysis of electro encephalogram series. 111. Wavelet packets and information cost function," Phys. Rev, vol. 57, pp. 932 -940, 1998.
- [44] C. A. Tracy and H. Widom, "On orthogonal and symplectic matrix ensembles," Comm. Math. Phys., vol. 177, pp. 727–754, 1996.
- [45] C. A. Tracy and H. Widom, "The distribution of the largest eigenvalue in the gaussian ensembles," in Calogero-Moser-Sutherland Models (J. van Diejen and L. Vinet, eds.), (New York), pp. 461–472, Springer,2000.