

# Isolation Enhancement Between Two Slot Antenna Elements Using Complementary Split-Ring Resonators

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## Abstract

In this paper, a decoupling structure based on complementary split-ring resonators is proposed to mitigate the mutual coupling between two slot antenna elements. The slot antenna elements are placed side-by-side, that are operating at 5.2 GHz frequency band for WLAN applications. Unlike previous attempts of utilizing the mushroom-like electromagnetic bandgap structures, complementary split-ring resonant inclusions are placed between two slot antenna elements in order to mitigate mutual coupling effects due to space waves coupling. The deployed decoupling structure results in ease of fabrication, since only etching of resonant decoupling patterns is required, thus maintaining the antenna system low-profile without the need for extra embedded layer, as presented in earlier studies. In this work, numerical full-wave simulations are carried out in order to show the capabilities of the developed decoupling technique in mutual coupling reduction. Based on the numerical results, reduction in mutual coupling between the two slot antenna elements by more than 25 dB was achieved in comparison to the reference case without any decoupling structure between the two slot antennas.

**Keywords :** Complementary split-ring resonator, isolation enhancement, mutual coupling, slot antennas, surface waves, space waves.

## I. INTRODUCTION

Slot antennas are very attractive and among the widely used radiators in many applications, including but not limited to mobile terminals, wireless connectivity, imaging and recently in multiple-input multiple-output (MIMO) wireless communication systems [1]- [2]. In principle, slot antennas are low profile elements and are easy to fabricate and integrate with other radio frequency/microwave circuits and sub-systems. However, an inherent mutual interaction between slot antenna elements, also known as mutual coupling, becomes more dominant as elements are placed in close proximity to each others, and in many applications like mobile terminal devices, this coupling remains a challenging design task and needs to be maintained as low as possible [3].

As slot antennas fit well in a parallel-plate waveguide structures, space waves are more dominant than surface waves in such environment when thin dielectric substrates are used, i.e. for thicknesses less than  $\lambda/10$  [4]- [5], and results in affecting impedance characteristics of the slot antenna elements when in close proximity, including radiation performance.

Several attempts and techniques to reduce the mutual coupling between slot antenna elements were previously investigated [6]- [12]. Zhang et al. proposed in [6] a decoupling structure based on conductive parasitic elements in order to reduce the mutual coupling between cavity-backed slot antenna elements. The use of decoupling parasitics to decouple two slot antennas for mobile terminal applications was also presented in [7]. A different technique based on meanderlined parasitic elements was proposed in [8] in order to reduce mutual coupling between two cavity-backed slot antenna elements. A reduction of 15-dB was achieved between the two slot antennas. In [11], mutual coupling effect between slot antenna elements in a waveguide environment was reduced using incorporating additional parasitic slots. The parasitic slot elements disturb the current distribution between the nearby slot elements and hence result in mutual coupling reduction.

Another decoupling structure that had widely been used to mitigate surface waves coupling, for instance microstrip patch antennas [13], is the electromagnetic-bandgap structures (EBGs). EBG structures were firstly proposed in 1999 by Sivenpiper et al. [14]. In the context of mutual coupling reduction between slot antennas, mushroom-like electromagnetic bandgap structures (EBGs) were utilized in [9]- [10] in order to reduce the coupling effects between two slot elements. The EBG structure was embedded within a parallel plate waveguide structure hosting the two antenna elements. As a result, the two antenna elements with embedded EBG structure is a multi-layer structure and may pose some challenges in its use in many applications. Although the EBG structure had resulted in significant mutual coupling reduction between the slot antenna elements in [10], the profile of the two slot antennas with embedded EBG structure is multi-layered and incurs additional design complexity and manufacturing cost.

Recently, smart engineered materials, or well-known as metamaterials, have been a research subject of interest in many engineering applications, to name a few, miniaturized microwave circuits and subsystems, imaging, and performance enhancement of antenna systems [15]- [18]. One of the well-known building blocks of metamaterials is the artificial magnetic conductor (AMC). It comprises the use of two oppositely metallic resonant rings with slits in opposite sides, known as Split-Ring Resonators (SRRs), and patterned periodically or aperiodically in a host dielectric medium, with a periodicity being much smaller than the operating wavelength of the incoming electromagnetic wave [19]. This AMC structure can give the response of an effective magnetic permeability being negative over certain frequency band of interest, once excited by an axial time-varying magnetic field. Considering the dual-counterpart of the SRR, the complementary split-ring resonator (CSRR), which was first proposed by Falcone et. al [20], gives the response of an effective electric permittivity over the desired frequency band, upon an excitation of a normal time-varying electric field component [20]- [21]. Furthermore, slotted complementary split-ring resonators (slotted-CSRRs) structure was introduced in [22] as a building block to provide wideband filtering mitigation capability of normal electric field components. Further, the developed slotted-CSRR inclusions are also well suited to mitigate noise and interference between radiating elements within a parallel-plate waveguide environment, in cases that significant mutual coupling effects resulted due to dominant space waves radiation. It is worth noting here that the deployment of CSRRs as a decoupling structure to reduce mutual coupling between two slot antenna elements in a parallel-plate like waveguide, due to space waves coupling, has not yet been investigated.

In this work, the slotted CSRRs are adopted to reduce the mutual coupling between two slot antenna elements. This decoupling technique will be compared against the reference case without the slotted-CSRRs in between the antenna elements. Moreover, the decoupling structure is also used to provide an alternative technique for space waves coupling effects between slot antenna elements that are etched on thin dielectric substrate, instead of the use of mushroom-like EBG structures, which results in multi-layer radiating system. Numerical full-wave simulations were carried out using the Finite-Element Method based simulator HFSS of Ansys Electromagnetic Suite [23].

The rest of the paper is organized as follows. Section II provides a overview of mutual coupling mechanisms in low-profile antennas. Section III presents the decoupling structure and overall antenna structure performance in terms of near-field scattering parameters and far-field radiation patterns. Moreover, a comparison summary with other recent decoupling structures for slot antennas coupling reduction is also presented. Finally, section IV concludes with a summary of the findings.

## II. SOURCES OF MUTUAL COUPLING IN PLANAR ANTENNAS

Mutual coupling in antenna systems is related to the electromagnetic interaction between antenna elements when in close-proximity. In addition to near-field coupling, there are two other main sources of mutual coupling that commonly take place in planar antennas, that are surface wave modes, and coupling via space waves [24]- [27]. Note that mutual coupling may result as simultaneous interaction of antenna elements in an array due to combination of the aforementioned sources of coupling, depending on the deployed substrate, the encountered losses and adopted antenna structure, among other factors [28]- [29].

Surface waves are known as the transverse magnetic (TM) and transverse electric (TE) modes of the associated dielectric substrate. Such waves propagate along dielectric/air interface and decay exponentially away from the interface. In principle, surface waves are mostly guided by the chosen substrate (controlled by its electrical properties and thickness) and the metallic ground structure. A good example for existence of surface wave modes in antennas is in printed patch/dipole structure (i.e. due to nature of using grounded dielectric slab) [24], [26]. In practice, surface waves are strongly excited when antennas are printed on thick substrates with high dielectric permittivity [4], [5]. As a result, surface waves result in strong mutual coupling between antenna elements in such substrates, due to the ability of this kind of wave modes to travel radially, even to several wavelengths [25]. The power that is carried by surface waves will be lost at end due to diffractions off substrate edges. Hence, this power loss can greatly deteriorate antenna's performance, including efficiency and gain, especially for thicker substrates [29].

The other source of coupling is space waves coupling. Space waves coupling is related to the energy that is coupled from one antenna to another from antenna's main beam power (i.e. space waves), after excluding power losses due to surface waves, if any [27], [29].

## III. MUTUAL COUPLING STUDY

In this section, the mutual coupling effect between two resonant slot antenna elements that are placed side-by-side on top of a grounded low-loss substrate is numerically investigated. In other words, the slot antennas are coexist within parallel-plate environment, due to presence of two metallic layers.

Figure 1 shows the numerical setup used to study the mutual coupling between the two slot antenna elements with the decoupling structure, the SCSRRs. Figure 2a shows a single complementary split-ring resonator (CSRR) unit cell and the employed decoupling unit cell in this work, the slotted complementary split-ring resonator (SCSRR) in Figure 2b, which is obtained by bridging a slotted line through two CSRR unit cells. More analysis and details of the decoupling structure that is adopted here (i.e., the SCSRR structure) can be found in [22]. The dimensions of the SCSRR

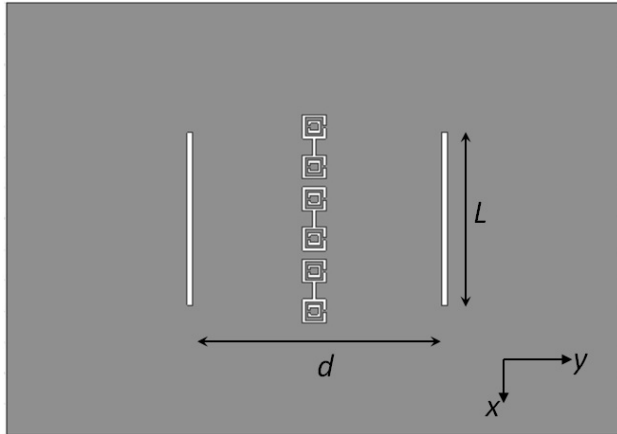


Fig. 1. Top view of the two slot antenna elements with the SCSRRs, decoupling structure. Note that one row of SCSRRs with three unit cells is shown here as one case study.

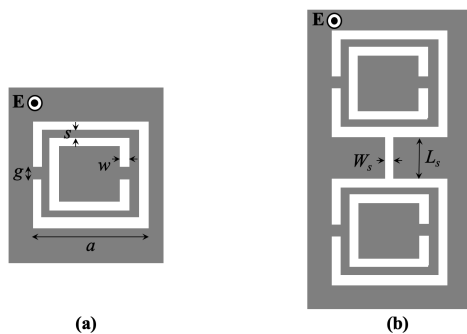


Fig. 2. (a) CSRR unit cell, and (b) the slotted CSRR unit cell. The gray shaded area represents metallization.

unit cell had been obtained through numerical computation of the transmission coefficient in order to identify the bandstop regime of the SCSRR unit cell using the microstrip transmission line technique, as outlined in [22]. The SCSRR is of dimension 3.9 mm x 10.4 mm, ring's cut gap  $g = 0.39$  mm,  $w = 0.45$  mm,  $s = 0.64$  mm, where the slotted line is of dimension  $L_s = 2.6$  mm and  $W_s = 0.455$  mm.

It is important to emphasize here that this kind of decoupling structure results in a convenient and low cost fabrication, since only etching of the structure is needed, which is placed in between the two antenna elements within the same layer. Hence, the overall antenna with the SCSRRs do not incur any additional layers to be embedded within the antenna elements and reflector layers, as compared against an earlier attempt with the use of embedded mushroom EBG structure in [10]. In this work, SCSRRs as a decoupling structure for mutual coupling reduction between two slot elements is numerically investigated. For convenience, a single row as well as 3 rows of SCSRRs between the antenna elements are considered, where each row comprises the use of 3 SCSRRs unit cells as shown in Fig. 1. The antenna system resides on a top board layer with 1.27 mm-thick grounded substrate with electric permittivity of 3.48, loss tangent of 0.0037 and dimension of 100 mm x 70 mm. The antennas are each of length  $L = 28$  mm and width of 1 mm, and spaced apart from the edges by a distance  $d = 0.7\lambda = 40.4$

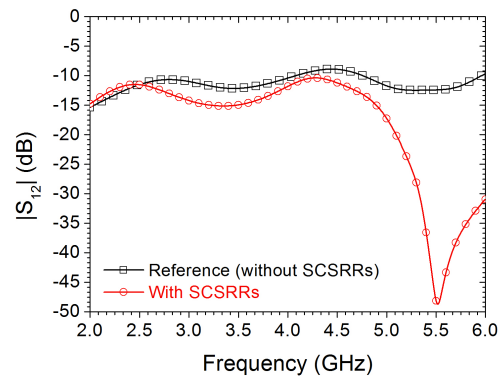


Fig. 3. Simulated transmission coefficient for the two antenna system with and without single row of SCSRRs for a spacing of  $0.7\lambda$  between the two antenna elements.

mm, where  $\lambda$  is the free-space wavelength at 5.2 GHz. Note that dimensions of the antenna elements and spacing are chosen as given in [9] for comparison purposes. The mutual coupling effect is studied through the computation of the magnitude of the transmission coefficient,  $|S_{21}|$ , where one of the two antennas is terminated with a  $50 \Omega$  impedance. For comparison purposes, the case of the two antennas without SCSRRs is also considered.

#### A. S-parameters results

Figure 3 shows the simulated transmission coefficient magnitude,  $|S_{21}|$ , for the two slot antenna elements with and without one row of SCSRRs. As can be seen from Figure 3, the case of two antenna elements without any decoupling structure shows a strong coupling, which is also attributed to the strong parallel-plate modes and space radiation. However, with only a single row of SCSRRs, more than 10 dB reduction in mutual coupling is achieved, in which SCSRRs fit well for slot antennas applications. A noticeable dip with coupling around -48 dB can be seen at 5.5 GHz.

Moreover, a numerical parametric study was conducted to investigate the effects of coupling reduction when spacing between the antenna elements is varied. Figure 4 depicts the coupling between the two antennas with and without a single row of SCSRRs, when spacing  $d$  is either  $0.3\lambda$  or  $0.7\lambda$ . As can be seen from Figure 4, more than 20 dB mutual coupling suppression was achieved when spacing of  $0.3\lambda$  was used, which shows the strong resonance behavior of the slotted CSRR resonators when the antenna elements are in close proximity. Figure 5 shows the reflection coefficient of the slot antennas for both cases. The matching of the slot antennas for the reference case is deteriorated, due to the strong coupling between antenna elements.

Next, the effect of number of SCSRRs rows that are placed in between the two slot antenna elements is discussed. Figure 6 shows the computed transmission coefficient between the two antenna elements with a single as well as three rows of SCSRRs. More than 25 dB reduction in mutual coupling is achieved as compared against the case without SCSRRs,

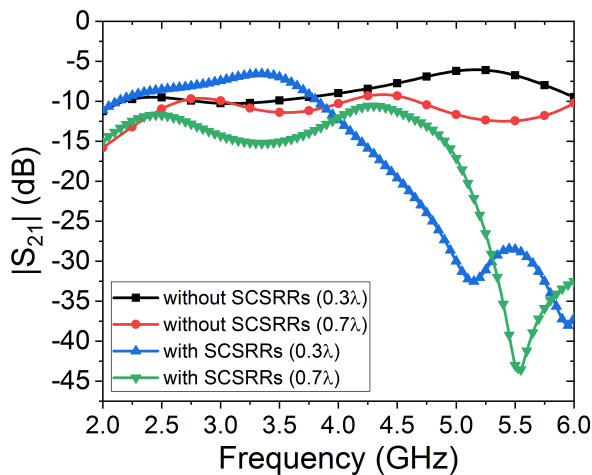


Fig. 4. Simulated transmission coefficient showing the coupling between the two antenna elements with and without single row of SCSRRs, for two different spacing,  $d$ .

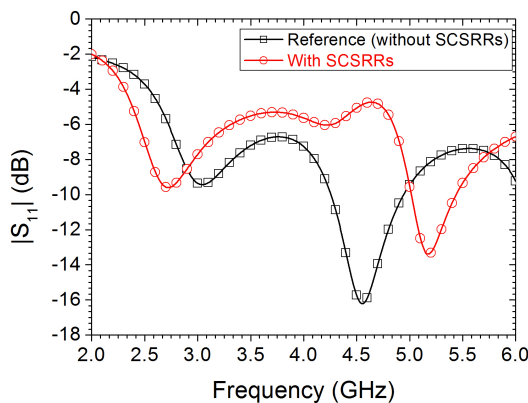


Fig. 5. Simulated reflection coefficient for the two antenna system with and without single row of SCSRRs for a spacing of  $0.7\lambda$  between the two antenna elements.

where a noticeable dip is observed this time at the resonance frequency of the antennas, 5.2 GHz, where mutual coupling of -45 dB was obtained. Although not shown here, the effect of deploying 5 rows of SCSRRs showed similar performance in coupling reduction as that case of 3 ribbons of SCSRRs. Hence, the case of 3-CSRR rows in this study can be considered as the optimal decoupling arrangement for high reduction of mutual coupling and radiation performance.

### B. Surface current distribution

Figures 7 - 9 depict the surface current distribution for the two slot antenna elements with and without the SCSRRs. A strong interaction between the antenna elements is observed and expected for the case without the SCSRRs, while the case of a single row of SCSRRs between the two slot antenna elements shows minimal current interaction between the antennas as shown in Figure 8. This mutual interaction is further mitigated in between the two antenna elements when

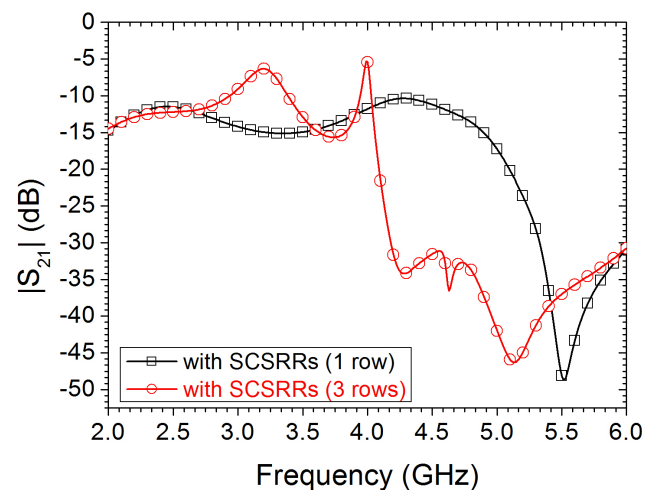


Fig. 6. Simulated transmission coefficient for the two antenna system with a single- and three- rows of SCSRRs for a spacing of  $0.7\lambda$  between the two antenna elements.

3 rows of SCSRRs are used, as seen from the surface current distribution shown in Figure 9.

In order to show the capability of the complementary-SRRs in mitigating space waves coupling, the magnitude of normal electric field strength was computed along the horizontal line joining the two antenna elements, i.e., along y-axis (see Figure 1). As can be seen from Figure 10, significant electric field was mitigated using the perforated CSRR rings for the

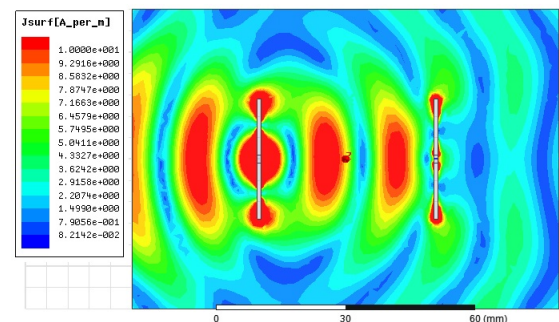


Fig. 7. Simulated magnitude of surface current distribution for the two antenna system without the SCSRRs at 5.2 GHz.

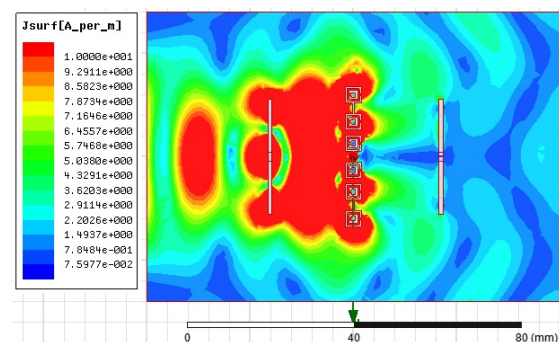


Fig. 8. Simulated magnitude of surface current distribution for the two antenna system with a single row of SCSRRs at 5.2 GHz.

case of 3-rows than the case of 1-row of CSRRs, since the filtering strength of the presented decoupling structure requires periodicity (i.e., more than one single row). The field strength for the two antennas case with solid metallic layer was minimal (see Figure 10) as compared against the other two cases of CSRRs, which is expected, since space waves coupling exists in the free-space region between the two antenna elements above the solid metallic layer. Hence, the existence of minimal field strength for the solid case.

### C. Radiation patterns

The radiation characteristics of the two antenna elements with and without the slotted-CSRRs was also investigated. One of the antenna elements was excited, while the other one was terminated with a  $50\ \Omega$  impedance. With the use of only a single row of SCSRRs, peak gain was observed to increase from 1.44 dBi at the resonance frequency of 5.2 GHz to nearly 4.80 dBi. As can be seen from Figure 11, the far-field co- and cross-polarization (x-pol) radiation pattern cuts for the two slot antennas with 1-row of SCSRRs showed almost similar scanning pattern in the E-plane cut to that of the two antennas without SCSRRs (in Figure 12), an indication that SCSRRs did not disturb the overall radiation pattern. Furthermore, the radiation efficiency of the antenna structure with CSRRs was similar to that one for the case without CSRRs, which was around 89%.

A comparison summary of the deployed decoupling structure for coupling reduction between slot antenna elements with other existing decoupling structures can be seen in Table. 1.

TABLE I  
COMPARISON OF MUTUAL COUPLING REDUCTION BETWEEN SLOT ANTENNA ELEMENTS WITH OTHER EXISTING DECOUPLING STRUCTURES.

Reference(s)	Decoupling structure	Operating freq.(GHz)	Separation distance	Coupling reduction	PCB layers
[6]	parasitic patch	12	$0.7\lambda$	12	3
[7]	parasitic monopole	2.1	$0.38\lambda$	12	2
[8]	parasitic meanderline	5.2	$0.17\lambda$	15	2
[10]	mushroom-EBGs	5.2	$0.7\lambda$	>20 dB	3
<b>Proposed</b>	<b>slotted-CSRRs</b>	<b>5.2</b>	<b><math>0.3\lambda</math></b>	<b>&gt;20 dB</b>	<b>2</b>

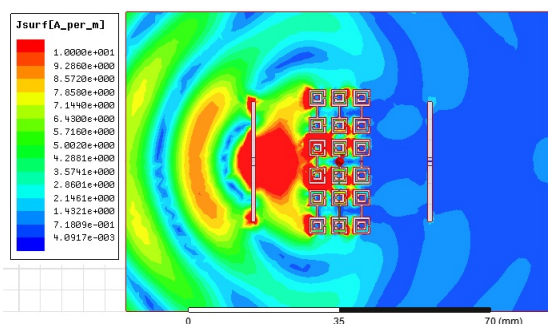


Fig. 9. Simulated magnitude of surface current distribution for the two antenna system with 3 rows of SCSRRs at 5.2 GHz.

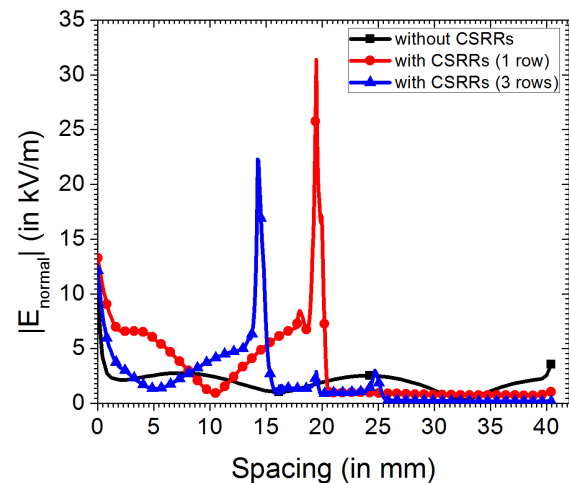


Fig. 10. Simulated magnitude of normal electric field strength along the line joining the two slot antennas with and without SCSRRs at 5.2 GHz.

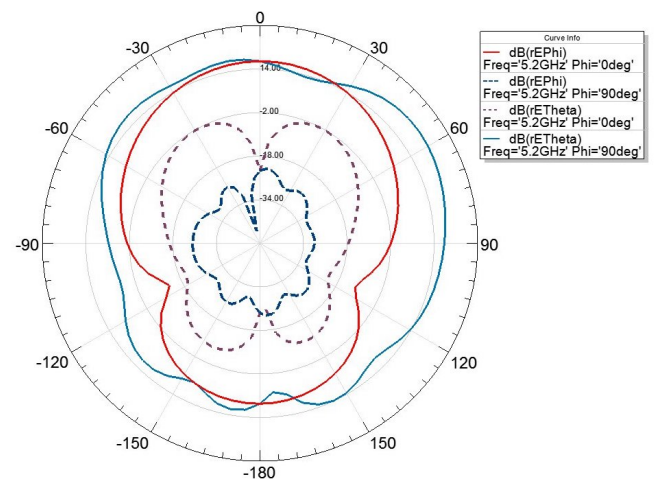


Fig. 11. Simulated co-pol and x-pol radiation patterns for the two slot antenna elements with 1-row of SCSRRs at 5.2 GHz for E-plane ( $\phi = 0^\circ$ ) and H-plane ( $\phi = 90^\circ$ ).

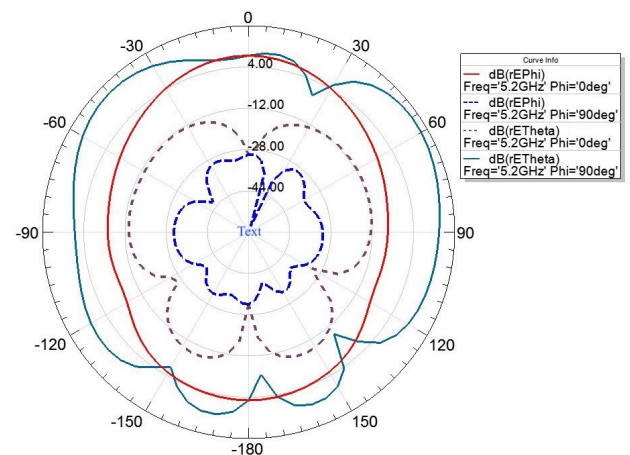


Fig. 12. Simulated co-pol and x-pol radiation patterns for the two slot antenna elements without SCSRRs at 5.2 GHz for E-plane ( $\phi = 0^\circ$ ) and H-plane ( $\phi = 90^\circ$ ) at 5.2 GHz.



#### IV. CONCLUSION

In this paper, mutual coupling reduction between two slot antenna elements was numerically investigated. Enhanced isolation between antenna elements' ports was achieved here using slotted complementary-split ring resonators. Based on the full-wave numerical simulations, more than 10 dB reduction of mutual coupling between two slot antenna elements was computed for the case of a single SCSRRs row at the antennas' resonance frequency of 5.2 GHz, while more than 25 dB reduction of mutual coupling between the elements was achieved when using 3 rows of SCSRRs as compared to the case without SCSRRs.

#### REFERENCES

- [1] S. Dumanli, C. J. Railton, and D. L. Paul, "A slot antenna array with low mutual coupling for use on small mobile terminals," *IEEE Trans. Antennas Propag.*, vol. 59, no. 5, pp. 1512–1520, May 2011.
- [2] M. A. Abou-Khousa, S. Kharkovshy, and R. Zoughi, "On the mutual coupling between circular resonant slots," in *3rd International Conference on Electromagnetic Near-Field Characterization and Imaging (ICONIC 2007)*, June 2007, pp. 117–122.
- [3] Kyohei Fujimoto, *Mobile Antenna Systems Handbook*, 3rd ed. Artech-House Inc., 2008.
- [4] R.P. Jedlicka, M.T. Poe and K.R. Carver, "Measured mutual coupling between microstrip antennas," *IEEE Transactions on Antennas and Propagation*, vol. 29, no. 1, pp. 147–149, Jan. 1981.
- [5] E. Penard and J.-P. Daniel, "Mutual coupling between microstrip antennas," *IET Electronics Letters*, vol. 18, no. 14, pp. 605–607, July 1982.
- [6] Q.-C. Zhang, J. Zhang, and W. Wu, "Reduction of mutual coupling between cavity-backed slot antenna elements," *Progress In Electromagnetics Research C*, vol. 53, pp. 27–34, 2014.
- [7] Z. Li, Z. Du, M. Takahashi, K. Saito, and K. Ito, "Reducing mutual coupling of mimo antennas with parasitic elements for mobile terminals," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 473–481, Feb 2012.
- [8] C. R. S. Dumanli and D. Paul, "A slot antenna array with low mutual coupling for use on small mobile terminals," *IEEE Trans. Antennas Propag.*, vol. 59, no. 5, pp. 1512–1520, May 2011.
- [9] K. Payandehjoo, and R. Abhari, "Suppression of substrate coupling between slot antennas using electromagnetic bandgap structures," in *2008 IEEE Antennas and Propagation Society International Symposium*, July 2008, pp. 1–4.
- [10] K. Payandehjoo and R. Abhari, "Employing ebg structures in multiantenna systems for improving isolation and diversity gain," *IEEE Antenna Wireless Propagat. Lett.*, vol. 8, pp. 1162–116, 2009.
- [11] M. Alibakhshikenari, B. S. Virdee, C. H. See, R. Abd-Alhameed, F. Falcone, and E. Limiti, "A new waveguide slot array antenna with high isolation and high antenna bandwidth operation on ku- and k-bands for radar and mimo systems," in *2018 48th European Microwave Conference (EuMC)*, Sep. 2018, pp. 1421–1424.
- [12] J. Ghosh, D. Mitra, and S. Das, "Mutual coupling reduction of slot antenna array by controlling surface wave propagation," *IEEE Trans. Antennas Propag.*, vol. 67, no. 2, pp. 1352–1357, Feb. 2019.
- [13] F. Yang and Y. Rahmat-Samii, "Microstrip antennas integrated with electromagnetic band-gap (EBG) structures: a low mutual coupling design for array applications," *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2936–2946, Oct. 2003.
- [14] D. Sievenpiper, L. Zhang, R. Broas, N. Alexopolous, and E. Yablonovitch, "High-impedance electromagnetic surfaces with a forbidden frequency band," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 11, pp. 2059–2074, Nov 1999.
- [15] R. Marques, F. Mesa, J. Martel, and F. Medina, "Comparative analysis of edge- and broadside- coupled split ring resonators for metamaterial design - theory and experiments," *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2572–2581, Oct. 2003.
- [16] I. Gil, R. Fernandez, Y. Vives, R. Jauregui, and F. Silva, "Characterization of EMI filters based on metamaterials," in *2009 International Symposium on Electromagn. Compatibility - EMC Europe*, June 2009, pp. 1–3.
- [17] L. Yousefi, B. Mohajer-Iravan, and O.M. Ramahi, "Enhanced bandwidth artificial magnetic ground plane for low-profile antennas," *IEEE Antenna Wireless Propagat. Lett.*, vol. 6, pp. 289–292, 2007.
- [18] M. Coulombe, S. Farzaneh Koodiani, and C. Caloz, "Compact Elongated Mushroom (EM)-EBG Structure for Enhancement of Patch Antenna Array Performances," *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1076–1086, April 2010.
- [19] J. Pendry, A. Holden, D. Robbins, and W. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 11, pp. 2075–2084, Nov 1999.
- [20] F. Falcone, T. Lopetegi, M. A. G. Laso, J. D. Baena, J. Bonache, M. Beruete, R. Marqués, F. Martín, and M. Sorolla, "Babinet principle applied to the design of metasurfaces and metamaterials," *Phys. Rev. Lett.*, vol. 93, no. 19, p. 197401, Nov 2004.
- [21] F. Falcone, T. Lopetegi, J. Baena, R. Marques, F. Martin, and M. Sorolla, "Effective negative-  $\epsilon$  stopband microstrip lines based on complementary split ring resonators," *IEEE Microwave Wireless Comp. Lett.*, vol. 14, no. 6, pp. 280–282, Jun. 2004.
- [22] M. M. Bait-Suwailam, O. F. Siddiqui, and O. M. Ramahi, "Mutual coupling reduction between microstrip patch antennas using slotted-complementary split-ring resonators," *IEEE Antenna Wireless Propagat. Lett.*, vol. 9, pp. 876–878, 2010.
- [23] Ansys Electromagnetics Suite, 17.2, <http://www.ansys.com>.
- [24] D. Pozar, "Input impedance and mutual coupling of rectangular microstrip antennas," *IEEE Trans. Antennas Propag.*, vol. 30, no. 6, pp. 1191–1196, Nov 1982.
- [25] A. Bhattacharyya, "Characteristics of space and surface waves in a multilayered structure," *IEEE Trans. Antennas Propag.*, vol. 38, no. 8, pp. 1231–1238, Aug 1990.
- [26] D. R. Jackson, J. T. Williams, A. K. Bhattacharyya, R. L. Smith, S. J. Buchheit, and S. A. Long, "Microstrip patch designs that do not excite surface waves," *IEEE Trans. Antennas Propag.*, vol. 41, no. 8, pp. 1026–1037, Aug 1993.
- [27] M. Khayat, J. Williams, D. Jackson, and S. Long, "Mutual coupling between reduced surface-wave microstrip antennas," *IEEE Trans. Antennas Propag.*, vol. 48, no. 10, pp. 1581–1593, Oct 2000.
- [28] E. van Lil and A. Van De Capelle, "Transmission line model for mutual coupling between microstrip antennas," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 8, pp. 816–821, August 1984.
- [29] D. Pozar, "Considerations for millimeter wave printed antennas," *IEEE Trans. Antennas Propag.*, vol. 31, no. 5, pp. 740–747, Sep 1983.