International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 13, Number 11 (2020), pp. 3950-3963
© International Research Publication House. https://dx.doi.org/10.37624/IJERT/13.11.2020.3950-3963

Truss Optimization with Natural Frequency Constraints Using Modified Social Engineering Optimizer

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Abstract

This paper presents a new enhanced version of the social engineering optimizer (SEO), named as MSEO, for size and shape optimization of truss structures considering dynamic constraints. The proposed MSEO introduces a concept drawn from artificial ecosystem-based optimization (AEO), known as decomposition operator, into the standard SEO to improve the exploitation capability. To validate the proposed algorithm thirty functions extracted from the CEC2014 database and six benchmark truss optimization problems with dynamic constraints are analyzed. Numerical results and comparison to other state-of-the-art metaheuristics optimization methods reveal the effectiveness of the proposed method.

Keywords: Social engineering optimizer; artificial ecosystem-based optimization; truss structures; size and shape optimization; dynamic constraints

I. INTRODUCTION

The natural frequencies of an engineering structure are parameters useful to avoid resonance phenomenon and keep the structural behavior desirable [1–3]. Additionally, engineering compositions should be as light as possible. However, weight reduction conflicts with frequency constraints. These constraints are highly non-linear, non-convex and implicit with respect to the design variables [4]. This has led to difficulty in the use of classical methods (gradient-based methods) owing to its dependence on derivatives, low convergence rate and its long runtime. Under such circumstances, the metaheuristic algorithms can serve as a valuable tool to solve this kind of problem because they do not suffer from the limitations mentioned above.

The first work that involves the use of metaheuristics to address the simultaneous shape and sizing optimization problem of truss structures subjected to dynamic constraints was performed by Lingyun et al. [5] using the genetic algorithm, since then several researchers have been employing different optimization algorithms. The most important works that involve metaheuristic algorithms to solve this problem are shown in Table 1.

Although several metaheuristics have been introduced to solve this problem, the No Free Lunch Theorem [6] states that no optimization method can perform the best for every optimization problem. This motivates the development of new and effective optimization methods. Therefore, this paper suggests the improve the performance of the recently proposed social engineering optimizer (SEO) and adapt it better for structural optimization. SEO was developed by Fathollahi-Fard et al. [7] and is a single-solution algorithm inspired by Social Engineering phenomenon, that is, the ability to obtain confidential information of people. This algorithm estimates the global optimum of a given problem in four steps: initialize the attacker and the defender, train and retrain, spot an attack and respond to attack. In the spot an attack phase, the SEO algorithm considers four different techniques to generate a new position for the defender and are as follows: obtaining, phishing, diversion theft and pretext. These techniques depend on a parameter named β as an input variable. According to [7], this phase executes the local search (exploitation). However, selecting the best among the four techniques to solve a given problem is a complicated task for the user which is often time-consuming; also, it is necessary to tune parameter β (rate of spotting an attack) for each problem, which affects the convergence speed. To overcome this drawback, the aim of this study is to investigate whether the basic concepts underlying artificial ecosystem-based optimization (AEO) [35] can be exported to enhance the SEO. This variation aims to improve the exploitative capability of the algorithm and that this phase does not depend on the setting of a parameter. The validity of the enhanced SEO (MSEO) is confirmed by testing for a diverse set of benchmark functions and applied to size and shape optimization problems of truss structures with dynamic constraints. The optimal results obtained by MSEO are compared with other solutions available in the literature.

The remainder of this paper is organized as follows. In Section 2, the SEO is briefly described. Section 3 describes the improvement in SEO. In Section 4, thirty benchmark functions proposed in the CEC2014 database [36] are utilized to test the validity of the proposed algorithm. Section 5 presents the general formulation of the size and shape of truss structures with multiple dynamic constraints. Section 6 presents the six most widely investigated benchmark truss optimization problems to illustrate the efficiency of the proposed approach. Finally, in Section 7, our conclusions are presented.

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II. SOCIAL ENGINEERING OPTIMIZER

The SEO [7] is a single-solution metaheuristic based on the Social Engineering (SE) phenomenon and its techniques. The following four steps describe the algorithm in detail:

(i) The algorithm starts with two randomly generated solutions in the search space. The best solution is called an attacker and the worst is called a defender.

(ii) Then, the defender is trained to improve his position. For this, the defender takes values of the attacker's variables randomly.

(iii) The defender changes his position to exploit the search space. For this, the user must choose 1 of the 4 established operators.

(iv) The positions of the defender are evaluated, and the best position is selected. If the position of the defender is better than that of the attacker, the defender takes the role of attacker and vice versa. Finally, the defender is replaced for a random solution in the search space.

Table 1. Main works in size and shape optimization of truss structures with dynamic constraints

Author	Metaheuristic
Wei et al. [8]	Parallel genetic algorithm
Gomes [9]	Particle swarm optimization (PSO)
Miguel and Fadel Miguel [10]	Harmony search (HS) and firefly algorithm (FA)
Kaveh and Zolghadr [11]	Hybrid charged system search and the big bang-big crunch algorithms (CSS-BBC)
Kaveh and Zolghadr [12]	Charged system search (CSS)
Kaveh and Zolghadr [13]	Democratic particle swarm optimization (DPSO)
Hosseinzadeh et al. [14]	Hybrid electromagnetism-like mechanism algorithm and migration strategy (EM–MS)
Kaveh and Mahdavi [15]	Colliding-bodies optimization (CBO)
Khatibinia and Naseralevi [16]	Orthogonal Multi-Gravitational Search Algorithm (OMGSA)
Kaveh and Ilchi Ghazaan [17]	Hybrid particle swarm optimization with an aging leader and challengers (ALC-PSO and HALC-PSO)
Farshchin and Camp [18]	School-based optimization (SBO)
Pham [19]	New differential evolution algorithm (ANDE)
Farshchin and Camp [20]	Multi-class teaching–learning-based optimization (MC-TLBO)
Dede and Ayvaz [21]	Teaching-learning-based optimization (TLBO)
Kaveh and Zolghadr [22]	Cyclical Parthenogenesis Algorithm (CPA)
Tejani et al. [23]	Modified sub-population teaching-learning-based optimization (MS-(TLBO)
Kaveh and Ilchi Ghazaan [24]	Vibrating particles system (VPS)
Kaveh and Zolghadr [25]	Tug of war optimization (TWO)
Cheng and Prayogo [26]	Fuzzy adaptive teaching–learning-based (FATLBO)
Ho-Huu et al. [27]	Improved differential evolution with roulette wheel selection (ReDE)
Tejani et al. [28,29]	Symbiotic organisms search (SOS) and improved symbiotic organisms search (ISOS)
Lieu et al. [30]	Adaptive hybrid evolutionary firefly algorithm (AHEFA)
Jalili and Hosseinzadeh [31]	Combined migration and differential evolution strategies (MS–DE)
Millan-Paramo and Abdalla Filho [32,33]	Modified simulated annealing algorithm (MSAA) and improved modified simulated annealing algorithm (IMSAA)
Kaveh and Mahjoubi [34]	Hypotrochoid spiral optimization algorithm (HSPO)

To perform the optimization process, SEO has three parameters to tune and they are: rate of training (α), rate of spotting an attack (β) and number of attacks (na). For more details, see [7]. The flowchart of SEO is illustrated in Fig. 1.

III. ENHANCED SOCIAL ENGINEERING OPTIMIZER

A metaheuristic algorithm is efficient when it can maintain a balance between intensification (exploitation) and diversification (exploration) during the optimization process. In the spot an attack phase is generated a new position for the defender using one operator among the four ones established.

These operators (obtaining, phishing, diversion theft and pretext) depend on the parameter β that must be tuned. Selecting the wrong operator to solve a given problem can cause to consume a large number of unused function evaluations (FEs). On the other hand, the parameter β must be tuned for each problem, affecting the convergence speed of the algorithm. Therefore, this paper proposes the MSEO to improve SEO algorithm's local search capabilities. The proposed algorithm introduces a concept drawn from artificial ecosystem-based optimization (AEO) [35] to substitute the spot an attack phase.

AEO was introduced by Zhao et al. [35] and is inspired by the flow of energy in an ecosystem on the earth. The AEO has three principal operators, including production, consumption, and decomposition. The production operator is to enhance the balance between exploration and exploitation. The consumption operator is utilized to improve the exploration of the algorithm. Finally, the decomposition operator is employed to promote the exploitation of the algorithm. As our interest is to improve the exploitation of the SEO, the decomposition operator used in AEO is implemented in the MSEO.

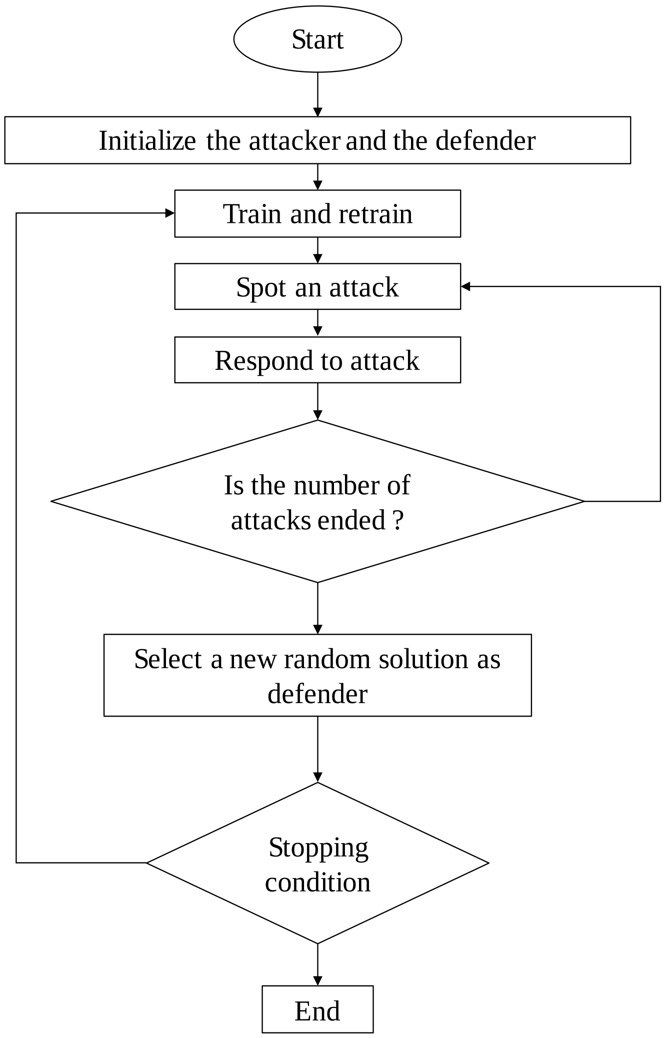

Figure 1. The SEO flowchart [7]

III.I Decomposition operator

According to Zhao et al. [35], decomposition is a vital activity for the ecosystem to work, and it provides essential nutrients for the growth of the producer. During the decomposition, when each individual in the population dies, the decomposer will decay or break down chemically its remains. In problem-solving process, the equation expressing this decomposition behavior is as follows:

$$\begin{gathered} x_i(t+1) = x_n(t) + D \cdot \left(e \cdot x_n(t) - h \cdot x_i(t)\right) \\ D = 3u, \;\; u \sim N(0,1) \\ e = r \cdot \mathrm{randi}([1\ 2]) - 1 \\ h = 2 \cdot r - 1 \end{gathered} \tag{1}$$

where x_n is the current position of the defender; x_i is the new position of the defender; N(0,1) a Gaussian random number with mean 0 and standard deviation 1 and r is a random number within the range of [0, 1].

This concept is exported to SEO. Thus, the spot an attack phase of the SEO is replaced by decomposition operator to generate the new position of the defender, improve the convergence rate and maintain a balance amid exploitation and exploration.

IV. THE 30 BENCHMARK FUNCTIONS OF THE CEC2014 DATABASE

The numerical efficiency of MSEO is analyzed on a set of 30 problems of CEC2014 database [36], as summarized in Table 2. For result validation, the comparison is made between several metaheuristics algorithms (WWO, BA, HuS, GSA, BBO, IWO, SOS, ISOS). In this study, 30D functions are used with search ranges as [− 100, 100] and set the FEmax to 150,000. All results are collected from 60 independent runs on each test function. In all examples, for the standard SEO, the rate of training (α), rate of spotting an attack (β) and number of attacks (na) are set as 0.2, 0.50 and 50, respectively. For the proposed algorithm, the rate of training (α) and number of attacks (na) are set as 0.2 and 50, respectively. Sensitivity analyses on these parameters are investigated in [7]. The algorithm is coded in Matlab program and executed using a machine with 2.4 GHz with 8 GB RAM. In order to analyze the experimental results, it was performed the Friedman rank test. The Friedman test is used to find the differences in treatments or algorithms across multiple test attempts. This test ranks the data within each row (or block) and then tests for a difference across columns. The test is performed on the average and standard deviation (SD) of functional values obtained.

The comparative average fitness value is presented in Table 3. From the table, it can be seen that the proposed MSEO has the best rank for unimodal, multimodal and composition functions. WWO and IWI give best rank than MSEO for hybrid functions. Finally, the proposed MSEO ranks first for overall performance. The comparative SD of fitness value is shown in Table 4. As can be seen, the proposed MSEO is the second best among the considered algorithms. These results confirm the merits of the proposed algorithm.

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Table 2. CEC 2014 benchmark functions [36].

Type	Function	Optimum
Unimodal	f1: Rotated high conditioned elliptic function	100
	f2: Rotated bent cigar function	200
	f3: Rotated discus function	300
Multimodal	f4: Shifted and rotated Rosenbrock function	400
	f5: Shifted and rotated Ackley's function	500
	f6: Shifted and rotated Weierstrass function	600
	f7: Shifted and rotated Griewank's function	700
	f8: Shifted Rastrigin's function	800
	f9: Shifted and rotated Rastrigin's function	900
	f10: Shifted Schwefel function	1000
	f11: Shifted and rotated Schwefel's function	1100
	f12: Shifted and rotated Katsuura function	1200
	f13: Shifted and rotated HappyCat function	1300
	f14: Shifted and rotated HGBat function	1400
	f15: Shifted and rotated Expanded Griewank's plus Rosenbrock's function	1500
	f16: Shifted and rotated Expanded Scaffe's f6 function	1600
Hybrid	f17: Hybrid function1 (f9, f8, f1)	1700
	f18: Hybrid function2 (f2, f12, f8)	1800
	f19: Hybrid function3 (f7, f6, f4, f14)	1900
	f20: Hybrid function4 (f12, f3, f13, f8)	2000
	f21: Hybrid function5 (f14, f12, f4, f9, f1)	2100
	f22: Hybrid function6 (f10, f11, f13, f9, f5)	2200
Composition	f23: Composition function1 (f4, f1, f2, f3, f1)	2300
	f24: Composition function2 (f10, f9, f14)	2400
	f25: Composition function3 (f11, f9, f1)	2500
	f26: Composition function4 (f11, f13, f1, f6, f7)	2600
	f27: Composition function5 (f14, f9, f11, f6, f1)	2700
	f28: Composition function6 (f15, f13, f11, f16, f1)	2800
	f29: Composition function7 (f17, f18, f9)	2900
	f30: Composition function8 (f20, f21, f22)	3000

V. TRUSS OPTIMIZATION PROBLEM FORMULATION WITH DYNAMIC CONSTRAINTS

The goal of the structural optimization problem is to minimize the weight of the structure while satisfying some constraints on the natural frequencies. The design variables include the cross-sectional areas of the members and the nodal coordinates. All six example problems solved in this work have been solved previously by other authors and are thus considered as benchmark problems. In all of those problems, lumped masses are added as external masses that are not an intrinsic part of the structure to be optimized. Therefore, such masses are not an integral part of the weight of the structure and are not included in the formulation. The mathematical formulation for this problem can be formulated as:

$$\text{Find,} \quad X = \{A, NC\}, \text{ where} \quad A = \{A_1, A_2, \ldots, A_n\} \text{ a} \quad (2)$$

$$\text{To minimize} \qquad W(X) = \sum_{i=1}^{n} \rho_i A_i L_i$$

$$\text{Subject to} \quad \begin{cases} f_q - f_q^{min} \geq 0 \\ f_r - f_r^{max} \leq 0 \\ A_i^{min} \leq A_i \leq A_i^{max} \\ NC_j^{min} \leq NC_j \leq NC_j^{max} \end{cases}$$

where W is the weight of the structure; n is the total number of members of the structure; ρ_i, A_i and L_i stand for the material density, the cross-sectional area and the length of the ith member, respectively; NC_j is a nodal coordinates (xj, yj, zj) of node jth of the truss; f_q and f_r are the qth and rth natural frequencies of the structure, respectively; the superscripts, "max" and "min" denote the maximum and minimum allowable limits respectively.

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Table 3. Comparative average of fitness values of the CEC2014 (The results of first eight algorithms are as per [29]).

Function	WWO	BA	Hus	GSA	BBO	IWO	SOS	ISOS	SEO	MSEO
f1	6.281E+05	3.166E+08	5.556E+06	1.441E+07	2.726E+07	1.463E+06	1.027E+06	9.822E+05	9.278E+05	7.846E+05
f2	3.304E+02	2.571E+10	1.007E+04	8.771E+03	4.012E+06	1.767E+04	2.132E+02	2.053E+02	2.842E+02	2.090E+02
f3	5.268E+02	7.200E+04	5.020E+02	4.538E+04	1.310E+04	8.167E+03	9.389E+02	7.790E+02	7.812E+02	5.283E+02
Friedman value of f1–f3	8	30	15	23	26	21	14	9	12	7
Friedman rank of f1–f3	2	10	6	8	9	7	5	3	4	1
f4	4.170E+02	3.698E+03	5.069E+02	6.764E+02	5.388E+02	5.003E+02	4.683E+02	4.598E+02	4.904E+02	4.216E+02
f5	5.200E+02	5.210E+02	5.207E+02	5.200E+02	5.202E+02	5.200E+02	5.206E+02	5.203E+02	5.209E+02	5.201E+02
f6	6.060E+02	6.364E+02	6.231E+02	6.196E+02	6.140E+02	6.022E+02	6.109E+02	6.105E+02	6.153E+02	6.108E+02
f7	7.000E+02	9.107E+02	7.000E+02	7.000E+02	7.010E+02	7.000E+02	7.000E+02	7.000E+02	7.001E+02	7.000E+02
f8	8.011E+02	1.070E+03	9.401E+02	8.005E+02	8.775E+02	8.437E+02	8.521E+02	8.147E+02	8.042E+02	8.023E+02
f9	9.611E+02	1.250E+03	1.012E+03	1.060E+03	9.514E+02	9.461E+02	9.705E+02	9.543E+02	9.569E+02	9.501E+02
f10	1.582E+03	6.426E+03	2.254E+03	4.392E+03	1.002E+03	2.565E+03	2.107E+03	1.157E+03	1.116E+03	1.006E+03
f11	3.349E+03	8.152E+03	3.303E+03	5.099E+03	3.247E+03	2.887E+03	4.017E+03	2.882E+03	3.122E+03	2.906E+03
f12	1.200E+03	1.203E+03	1.200E+03	1.200E+03	1.200E+03	1.200E+03	1.201E+03	1.200E+03	1.200E+03	1.200E+03
f13	1.300E+03	1.304E+03	1.300E+03	1.300E+03	1.301E+03	1.300E+03	1.300E+03	1.300E+03	1.300E+03	1.300E+03
f14	1.400E+03	1.473E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03
f15	1.503E+03	1.945E+05	1.517E+03	1.503E+03	1.515E+03	1.504E+03	1.518E+03	1.511E+03	1.510E+03	1.500E+03
f16	1.610E+03	1.613E+03	1.612E+03	1.614E+03	1.610E+03	1.610E+03	1.611E+03	1.609E+03	1.611E+03	1.610E+03
Friedman value of f4–f16	41	129	93	71	78	50	88	55	75	35
Friedman rank of f4–f16	2	10	9	5	7	3	8	4	6	1
f17	2.662E+04	4.641E+06	1.981E+05	5.786E+05	4.299E+06	8.644E+04	1.432E+05	1.767E+05	8.875E+04	3.652E+04
f18	2.026E+03	1.219E+08	3.781E+03	2.290E+03	2.842E+04	5.787E+03	8.320E+03	5.690E+03	5.903E+03	5.503E+03
f19	1.908E+03	2.005E+03	1.931E+03	1.995E+03	1.928E+03	1.908E+03	1.923E+03	1.908E+03	1.946E+03	1.908E+03
f20	5.364E+03	1.936E+04	3.866E+04	2.242E+04	3.141E+04	2.993E+03	5.770E+03	6.983E+03	5.700E+03	4.011E+03
f21	3.867E+04	1.095E+06	6.046E+04	1.706E+05	4.856E+05	3.907E+04	6.860E+04	9.112E+04	5.032E+04	3.903E+04
f22	2.482E+03	3.134E+03	3.073E+03	3.161E+03	2.723E+03	2.346E+03	2.496E+03	2.475E+03	2.729E+03	2.518E+03
Friedman value of f17–f22	10	56	40	45	48	17	33	28	34	19
Friedman rank of f17–f22	1	10	7	8	9	2	5	4	6	3
f23	2.615E+03	2.589E+03	2.616E+03	2.564E+03	2.617E+03	2.615E+03	2.615E+03	2.615E+03	2.616E+03	2.615E+03
f24	2.631E+03	2.601E+03	2.658E+03	2.600E+03	2.635E+03	2.618E+03	2.600E+03	2.600E+03	2.602E+03	2.600E+03
f25	2.708E+03	2.706E+03	2.725E+03	2.700E+03	2.712E+03	2.705E+03	2.700E+03	2.700E+03	2.701E+03	2.700E+03
f26	2.700E+03	2.703E+03	2.785E+03	2.800E+03	2.706E+03	2.700E+03	2.700E+03	2.700E+03	2.703E+03	2.700E+03
f27	3.093E+03	3.321E+03	4.965E+03	3.720E+03	3.397E+03	3.081E+03	3.266E+03	3.243E+03	3.325E+03	3.103E+03
f28	3.888E+03	4.535E+03	5.415E+03	5.282E+03	3.801E+03	3.693E+03	3.847E+03	3.768E+03	3.796E+03	3.771E+03
f29	4.094E+03	4.611E+06	2.078E+06	5.215E+04	1.492E+05	1.640E+04	1.724E+06	5.733E+05	5.022E+04	5.201E+03
f30	5.652E+03	1.981E+05	1.672E+04	1.914E+04	1.621E+04	9.196E+03	5.941E+03	5.377E+03	5.865E+03	5.658E+03
Friedman value of f23–f30	35	55	75	51	62	33	36	24	44	25
Friedman rank of f23–f30	4	8	10	7	9	3	5	2	6	1
Overall Friedman value	94	270	223	190	214	121	171	116	165	86
Overall Friedman rank	2	10	9	7	8	4	5	3	5	1

Table 4. Comparative SD of fitness values of the CEC2014 (The results of first eight algorithms are as per [29]).

Function	WWO	BA	Hus	GSA	BBO	IWO	SOS	ISOS	SEO	MSEO
f1	2.445E+05	1.047E+08	2.620E+06	1.319E+07	1.672E+07	5.717E+05	7.329E+05	7.055E+05	7.015E+05	3.010E+05
f2	2.022E+02	7.554E+09	6.013E+03	2.903E+03	1.549E+06	8.673E+03	2.028E+01	1.718E+01	4.599E+01	1.786E+01
f3	1.846E+02	1.755E+04	5.406E+02	1.043E+04	1.276E+04	2.693E+03	5.276E+02	6.239E+02	5.254E+02	1.863E+02
f4	3.636E+01	1.974E+03	3.662E+01	5.151E+01	3.835E+01	2.880E+01	3.175E+01	3.572E+01	3.758E+01	3.315E+01
f5	7.000E-04	4.810E-02	7.830E-02	6.000E-04	4.220E-02	3.800E-03	8.010E-02	6.670E-02	2.415E-02	1.952E-03
f6	2.620E+00	1.559E+00	2.178E+00	1.832E+00	2.354E+00	1.122E+00	2.568E+00	2.396E+00	2.427E+00	2.380E+00
f7	6.300E-03	3.232E+01	5.560E-02	1.000E-03	2.640E-02	1.210E-02	2.140E-02	1.830E-02	1.256E-01	9.410E-03
f8	2.336E+00	2.565E+01	1.273E+01	2.063E-01	2.069E+01	1.011E+01	1.232E+01	3.349E+00	9.465E+00	2.019E+00
f9	1.110E+01	4.413E+01	2.599E+01	1.743E+01	1.144E+01	1.139E+01	2.408E+01	1.344E+01	1.860E+01	1.326E+01
f10	3.616E+02	5.187E+02	4.332E+02	3.610E+02	6.800E-01	3.800E+02	3.441E+02	4.084E+01	1.740E+02	1.026E+02
f11	2.892E+02	3.622E+02	4.655E+02	5.673E+02	5.116E+02	4.477E+02	8.351E+02	4.541E+02	4.216E+02	2.959E+02
f12	5.610E-02	3.339E-01	7.770E-02	1.000E-03	5.620E-02	1.480E-02	1.833E-01	5.730E-02	1.847E-01	1.025E-01
f13	6.410E-02	5.483E-01	6.500E-02	6.650E-02	1.061E-01	6.500E-02	8.640E-02	7.100E-02	9.247E-02	7.352E-02
f14	4.410E-02	1.395E+01	4.740E-02	4.230E-02	1.992E-01	1.191E-01	1.296E-01	5.120E-02	1.092E-01	4.578E-02
f15	7.753E-01	1.403E+05	3.270E+00	7.297E-01	4.298E+00	8.484E-01	3.798E+00	3.717E+00	3.528E+00	7.686E-01
f16	4.667E-01	1.904E-01	7.249E-01	3.428E-01	5.923E-01	6.144E-01	6.059E-01	7.314E-01	5.784E-01	5.874E-01
f17	1.240E+04	1.790E+06	1.605E+05	2.199E+05	4.192E+06	6.847E+04	1.590E+05	1.645E+05	7.154E+04	1.846E+04
f18	1.252E+02	1.003E+08	2.247E+03	3.779E+02	1.967E+04	3.690E+03	1.031E+04	5.150E+03	7.412E+03	3.794E+02
f19	1.378E+00	2.032E+01	3.315E+01	3.432E+01	2.769E+01	1.655E+00	2.661E+01	1.787E+00	8.913E+00	1.408E+00
f20	3.177E+03	1.028E+04	8.493E+03	1.386E+04	1.760E+04	7.004E+02	3.295E+03	3.393E+03	4.128E+03	3.193E+03
f21	3.556E+04	7.507E+05	4.243E+04	6.529E+04	3.346E+05	2.301E+04	8.009E+04	1.078E+05	9.769E+04	4.022E+04
f22	1.429E+02	2.054E+02	2.673E+02	2.500E+02	2.344E+02	7.339E+01	1.515E+02	1.454E+02	2.158E+02	1.457E+02
f23	1.447E-01	1.284E+02	8.448E-01	6.450E+01	1.318E+00	7.950E-02	0.000E+00	0.000E+00	1.854E-01	9.010E-03
f24	6.885E+00	1.200E+00	1.249E+01	1.710E-02	5.974E+00	1.077E+01	1.300E-03	1.500E-03	6.336E-01	2.145E-03
f25	2.001E+00	1.498E+01	6.269E+00	1.319E+00	3.010E+00	8.076E-01	0.000E+00	0.000E+00	1.099E+00	0.000E+00
f26	6.500E-02	5.372E-01	3.533E+01	5.400E-03	2.202E+01	5.430E-02	8.620E-02	9.550E-02	9.245E-01	7.423E-02
f27	5.901E+01	6.462E+01	6.825E+02	3.505E+02	6.353E+01	3.503E+01	1.463E+02	1.363E+02	6.213E+01	5.918E+01
f28	3.607E+02	5.929E+02	4.613E+02	7.153E+02	9.334E+01	4.121E+01	1.901E+02	1.266E+02	1.749E+02	1.013E+02
f29	3.596E+02	2.831E+06	7.705E+06	3.781E+05	1.114E+06	5.140E+03	3.468E+06	2.146E+06	4.254E+05	3.256E+04
f30	7.381E+02	9.106E+04	6.583E+03	1.841E+04	6.076E+03	2.079E+03	3.249E+03	1.107E+03	1.025E+04	9.796E+02
Overall Friedman value	93	248	216.5	162	216	113.5	180.5	150.5	175	95
Overall Friedman rank	1	10	9	5	8	3	7	4	6	2

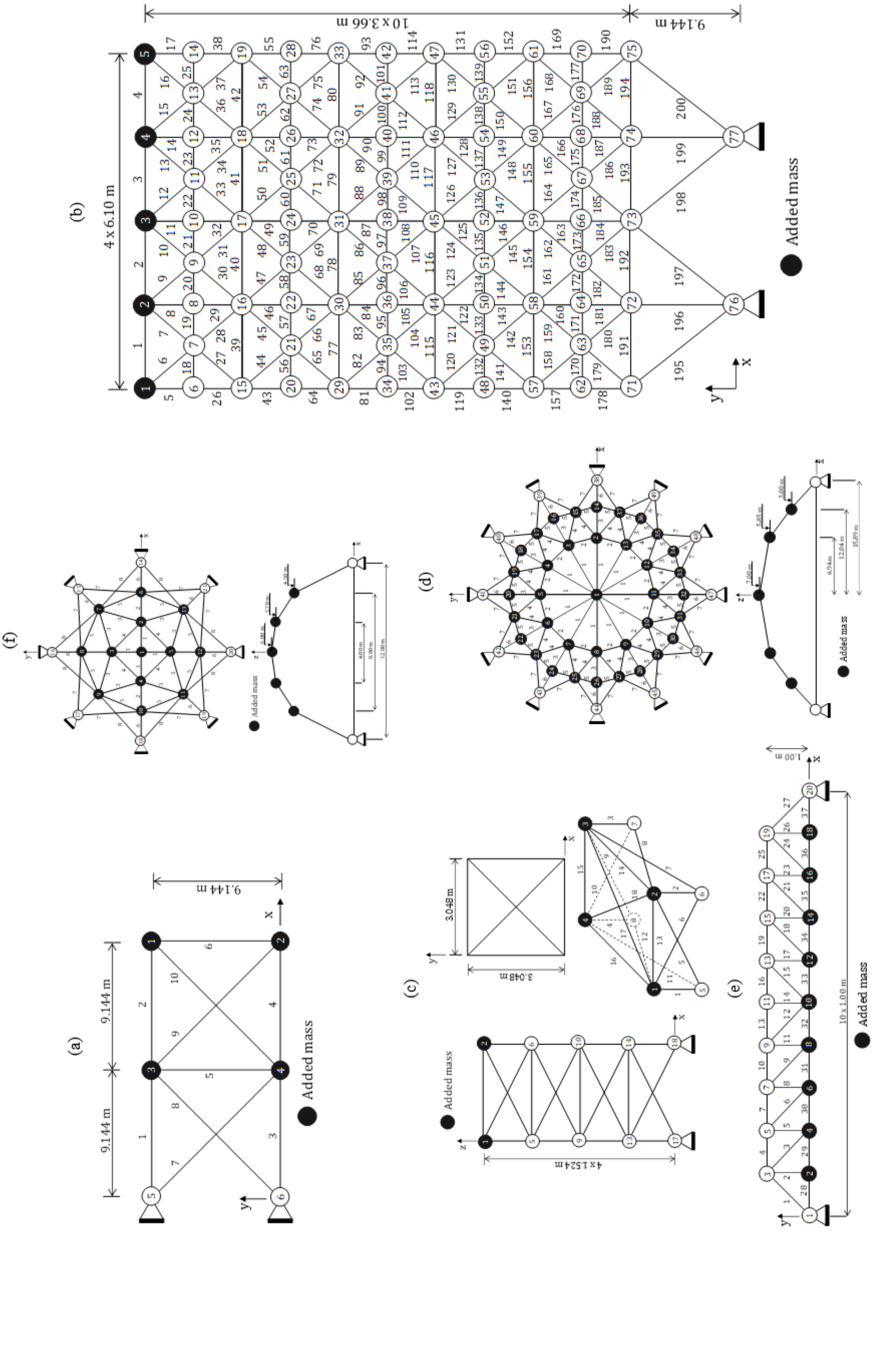

Figure 2. Truss optimization problems

VI. TRUSS PROBLEMS AND DISCUSSIONS

Six classical truss optimization problems (Fig. 2), including four ones regarding size optimization (10-bar planar truss, 200-bar planar truss, 72-bar space truss and 120-bar dome truss) and two concerning size and shape optimization (37-bar planar truss and 52-bar dome truss), are optimized to demonstrate the effectiveness and validity of the proposed MSEO. The design parameters of the problems are given in Table 5. The numerical results obtained by the MSEO are compared with those obtained by the SEO and other methods in the literature. In all numerical examples, the internal parameters of MSEO are: (i) according to [7], the rate of training (α) and number of attacks (na) are set as 0.2, and 50, respectively; (ii) the stopping condition is set to 5000 FE analyses. For each design example, the experiment is repeated for 100 times and the statistical information is reported in terms of the best weight, average weight, standard deviation (SD), the corresponding number of FE analyses (FEs) and frequency responses. The algorithm and the two-node linear bar element for FE analysis are coded in Matlab on a machine with 2.4 GHz and 8 of GB RAM.

Table 5. Design parameters of the truss problems

	Size optimization				Size and shape optimization	
	10-bar planar truss	200-bar planar truss	72-bar space truss	120-bar dome truss	37-bar planar truss	52-bar dome truss
Young's modulus E (N/m^2)	6.98×10^{10}	2.1×10^{11}	6.98×10^{10}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}
Material density ρ (kg/m^3)	2770	7860	2770	7971.81	7800	7800
Size variables (cm^2)	$0.645 \leq A \leq 50$	$0.1 \leq A \leq 30$	$0.645 \leq A \leq 30$	$1 \leq A \leq 129.3$	$1 \leq A \leq 10$	$1 \leq A \leq 10$
Shape variables (m)	–	–	–	–	$0.1 \leq y \leq 3$	all free nodes can displace ± 2 m in symmetrical manner
Frequency constraints (Hz)	$f_1 \geq 7$ $f_2 \geq 15$ $f_3 \geq 20$	$f_1 \geq 5$ $f_2 \geq 10$ $f_3 \geq 15$	$f_1 = 4$ $f_3 \geq 6$	$f_1 \geq 9$ $f_2 \geq 11$	$f_1 \geq 20$ $f_2 \geq 40$ $f_3 \geq 60$	$f_1 \leq 15.9155$ $f_2 \geq 28.6479$

VI.I. 10-bar planar truss structure

A 10-bar planar truss structure shown in Fig. 2a is the first investigated design example. A lumped mass of 454 kg is added at each of the free nodes. Table 6 comparatively summarizes the results found by the proposed algorithm and some other previous studies reported in the literature. It can be seen that MSEO had better performance in solving the problem in comparison with the standard SEO. The optimal weight achieved by the MSEO and the SEO are 529.96 kg and 532.09 kg, respectively. Furthermore, the SD obtained by MSEO (0.98 kg) is lower than the SEO (2.56 kg). It can also be seen that MSEO yields a relatively lighter design than PSO (537.98 kg) and HS (534.99 kg) but slight heavier design than ReDe (524.45 kg), SOS (525.28 kg), ISOS (524.73 kg), AHEFA (524.45 kg) and HSPO (524.40 kg). The convergence speed of the SOS (4000 FEs), ISOS (4000 FEs), and HSPO (3860 FEs) is faster than that of the MSEO (5000 FEs); however, MSEO is more stable than these methods with the smallest SD. Finally, regarding SD, MSEO ranks first among the considered metaheuristics. Natural frequencies optimal obtained by the MSEO show that none of the frequency constraints are violated.

VI.II. 200-bar planar truss structure

Figure 2b shows a planar 200-bar truss structure. At the top of the structure, a lumped mass of 100 kg is added at nodes 1 to 5. Elements are grouped into 29 groups by seeing symmetry of the structure. The results obtained are presented in Table 7. Comparing MSEO and SEO, it is observed that MSEO obtained best results in terms of optimal design (kg for MSEO and 2159.30 kg for SEO) and SD (0.15 kg for MSEO and 3.26 kg for SEO). As can be seen from Table 7, the optimum design achieved by MSEO is better than other considered metaheuristics. On the other hand, the proposed algorithm also required less structural analyses to converge to the optimal solution. Finally, with respect to the SD, MSEO ranks first among the considered metaheuristics. As shown in Table 7, none of the constraints are violated.

VI.III. 72- bar space truss structure

Figure 2c shows the schematic of the 72-bar space truss structure. There are 16 sizing variables and a lumped mass of 2770 kg is attached at all top nodes (nodes 1–4). The results obtained by different methods are tabulated in Table 8. The optimum design achieved by MSEO is better than the results reported by CSS-BBBC (327.51 kg), TLBO (327.57 kg), SOS (325.56 kg), ISOS (325.01 kg) and SEO (325.09 kg) but slightly heavier than that of HSPO (324.23 kg), ReDe (324.25 kg) and AHEFA (324.24 kg). However, the convergence speed of the MSEO is faster than these algorithms (8820 FEs for HSPO, 10840 FEs for ReDE, 8860 FEs for AHEFA and 5000 FEs for MSEO). Moreover, MSEO obtained the second SD (0.12 kg) overall, only being surpassed by ReDE (0.07 kg). Frequency values show that all constraints for the problem are satisfied.

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 13, Number 11 (2020), pp. 3950-3963
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Table 6. Optimal design results for the 10-bar planar truss by different algorithms

Variables (cm^2)	PSO	HS	ReDE	SOS	ISOS	AHEFA	HSPO	This study	
	[9]	[10]	[27]	[28]	[29]	[30]	[34]	SEO	MSEO
A_1	37.712	34.282	35.1565	35.3794	35.2654	35.1714	34.7531	32.9959	32.9885
A_2	9.959	15.653	14.7605	14.8826	14.6803	14.7203	15.3406	15.3753	15.5605
A_3	40.265	37.641	35.1187	35.7321	34.4273	35.1074	34.4741	32.6637	32.9867
A_4	16.788	16.058	14.7275	14.3069	14.9605	14.6986	15.0886	15.8709	15.3766
A_5	11.576	1.069	0.6450	0.6450	0.6450	0.6451	0.6450	0.6450	0.6450
A_6	3.955	4.740	4.5558	4.7142	4.5927	4.5593	4.5382	4.6561	4.6003
A_7	25.308	22.505	23.7199	24.1569	23.3417	23.7330	24.1727	25.5862	25.9534
A_8	21.613	24.603	23.6304	23.6047	23.8236	23.6795	23.6352	26.6018	25.5394
A_9	11.576	12.867	12.3827	12.1590	12.8497	12.3987	12.1966	12.2395	11.9845
A_{10}	11.186	12.099	12.4580	12.0061	12.5321	12.4231	12.2571	11.8438	12.2345
Best weight (kg)	537.98	534.99	524.45	525.28	524.73	524.45	524.40	532.09	529.96
f_1 (Hz)	7.000	7.0028	7.0000	7.0005	7.0001	7.0000	7.0000	7.0000	7.0000
f_2 (Hz)	17.786	16.7429	16.1924	16.2484	16.1703	16.1920	16.2356	15.8356	15.8136
f_3 (Hz)	20.000	20.0548	20.0000	19.9999	20.0024	20.0000	20.0000	20.0001	20.0000
Average weight (kg)	540.89	537.68	524.76	531.40	530.03	525.16	526.80	533.11	530.02
SD (kg)	6.84	2.49	1.11	4.22	3.48	1.92	3.50	2.56	0.98
FEs	–	20000	8300	4000	4000	5860	3860	5000	5000

Table 7. Optimal design results for the 200-bar planar truss by different algorithms

Variables (cm^2)	CSS-BBBC	HALC-PSO	HSPO	SOS	ISOS	AHEFA	This study	
	[11]	[17]	[34]	[28]	[29]	[30]	SEO	MSEO
A_1	0.2934	0.3072	0.3014	0.4781	0.3072	0.2993	0.3034	0.3119
A_2	0.5561	0.4545	0.4594	0.4481	0.5075	0.4508	0.5177	0.4544
A_3	0.2952	0.1000	0.0781	0.1049	0.1001	0.1001	0.1000	0.1002
A_4	0.1970	0.1000	0.0983	0.1045	0.1000	0.1000	0.1000	0.1002
A_5	0.8340	0.5080	0.5062	0.4875	0.5893	0.5123	0.5699	0.5398
A_6	0.6455	0.8276	0.8199	0.9353	0.8328	0.8205	0.8187	0.8203
A_7	0.1770	0.1023	0.1000	0.1200	0.1431	0.1011	0.1000	0.1000
A_8	1.4796	1.4357	1.3968	1.3236	1.3600	1.4156	1.4361	1.4500
A_9	0.4497	0.1007	0.1000	0.1015	0.1039	0.1000	0.1000	0.1000
A_{10}	1.4556	1.5528	1.5735	1.4827	1.5114	1.5742	1.4599	1.5865
A_{11}	1.2238	1.1529	1.1490	1.1384	1.3568	1.1597	1.1381	1.1613
A_{12}	0.2739	0.1522	0.1186	0.1020	0.1024	0.1338	0.1205	0.1231
A_{13}	1.9174	2.9564	3.10264	2.9943	2.9024	2.9672	2.9032	2.8850
A_{14}	0.1170	0.1003	0.1000	0.1562	0.1000	0.1000	0.1006	0.1002
A_{15}	3.5535	3.2242	3.2433	3.4330	3.4120	3.2722	3.7168	3.4419
A_{16}	1.3360	1.5839	1.5968	1.6816	1.4819	1.5762	1.5246	1.5595
A_{17}	0.6289	0.2818	0.2422	0.1026	0.2587	0.2562	0.2056	0.2560
A_{18}	4.8335	5.0696	5.3968	5.0739	4.8291	5.0956	5.1494	5.6099
A_{19}	0.6062	0.1033	0.1000	0.1068	0.1499	0.1001	0.1021	0.1375
A_{20}	5.4393	5.4657	5.2582	6.0176	5.5090	5.4546	5.3291	4.8664
A_{21}	1.8435	2.0975	2.1434	2.0340	2.2221	2.0933	1.9882	2.0335
A_{22}	0.8955	0.6598	0.8293	0.6595	0.6113	0.6737	0.6782	0.7302
A_{23}	8.1759	7.6585	7.3013	6.9003	7.3398	7.6498	7.9359	7.2405
A_{24}	0.3209	0.1444	0.1128	0.2020	0.1559	0.1178	0.3222	0.2424
A_{25}	10.9800	8.0520	7.9108	6.8356	8.6301	8.0682	8.9235	6.9521
A_{26}	2.9489	2.7889	2.8674	2.6644	2.8245	2.8025	2.5618	2.8343
A_{27}	10.5243	10.4770	10.8526	12.1430	10.8563	10.5040	10.4026	10.8478
A_{28}	20.4271	21.3257	20.8993	22.2484	20.9142	21.2935	21.3538	21.6867
A_{29}	19.0983	10.5111	10.7515	8.9378	10.5305	10.7410	10.6476	10.4182
Best weight (kg)	2298.61	2156.73	2157.77	2180.32	2169.46	2160.74	2159.30	2156.26
f_1 (Hz)	5.010	5.000	5.0000	5.0001	5.0000	5.0000	5.0000	5.0000
f_2 (Hz)	12.911	12.254	12.1499	13.4306	12.4477	12.1821	12.4003	12.3026
f_3 (Hz)	15.416	15.044	15.0004	15.2645	15.2332	15.0160	15.0002	15.0046
Average weight (kg)	–	2157.14	2169.05	2303.30	2244.64	2161.04	2162.19	2156.61
SD (kg)	–	0.24	10.82	83.59	43.48	0.18	3.26	0.15
FEs	–	13000	11640	10000	10000	11300	5000	5000

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 13, Number 11 (2020), pp. 3950-3963
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Table 8. Optimal design results for the 72-bar space truss by different algorithms

Variables (cm^2)	CSS-BBBC	TLBO	HSPO	SOS	ReDE	ISOS	AHEFA	This study	
	[11]	[20]	[34]	[28]	[27]	[29]	[30]	**SEO**	**MSEO**
A_1-A_4	2.854	3.5491	3.4315	3.6957	3.5327	3.3563	3.5612	3.6279	3.4335
A_5-A_{12}	8.301	7.9676	7.8436	7.1779	7.8303	7.8726	7.8736	7.8486	7.8749
A_{13}-A_{16}	0.645	0.6450	0.6450	0.6450	0.6453	0.6450	0.6450	0.6450	0.6450
A_{17}-A_{18}	0.645	0.6450	0.6450	0.6569	0.6459	0.6450	0.6451	0.6450	0.6450
A_{19}-A_{22}	8.202	8.1532	8.0390	7.7017	8.0029	8.5798	7.9710	8.5211	8.1347
A_{23}-A_{30}	7.043	7.9667	7.9306	7.9509	7.9135	7.6566	7.8928	7.9246	7.9170
A_{31}-A_{34}	0.645	0.6450	0.6450	0.6450	0.6451	0.7417	0.6450	0.6450	0.6450
A_{35}-A_{36}	0.645	0.6450	0.6450	0.6450	0.6451	0.6450	0.6451	0.6450	0.6463
A_{37}-A_{40}	16.328	12.9272	12.7040	12.3994	12.7626	13.0864	12.5404	11.9834	13.0359
A_{41}-A_{48}	8.299	8.1226	7.9684	8.6121	7.9657	8.0764	7.9639	8.0302	7.9889
A_{49}-A_{52}	0.645	0.6452	0.6451	0.6450	0.6452	0.6450	0.6459	0.6450	0.6461
A_{53}-A_{54}	0.645	0.6450	0.6450	0.6450	0.6450	0.6937	0.6462	0.6451	0.6463
A_{55}-A_{58}	15.048	17.0524	17.0169	17.4827	16.9041	16.2517	17.1323	17.0874	16.6600
A_{59}-A_{66}	8.268	8.0618	8.0127	8.1502	8.0434	8.1703	8.0216	8.0593	7.9739
A_{67}-A_{70}	0.645	0.6450	0.6450	0.6740	0.6451	0.6450	0.6450	0.6451	0.6450
A_{71}-A_{72}	0.645	0.6450	0.6450	0.6550	0.6473	0.6450	0.6451	0.6450	0.6460
Best weight (kg)	327.51	327.57	324.23	325.56	324.25	325.01	324.24	325.09	324.36
f_1 (Hz)	4.0000	4.000	4.0000	4.0023	4.0000	4.0000	4.0000	4.0000	4.0000
f_3 (Hz)	6.0040	6.000	6.0000	6.0020	6.0001	6.0008	6.0000	6.0000	6.0000
Average weight (kg)	–	328.68	325.42	331.12	324.32	329.47	324.41	326.11	324.39
SD (kg)	–	0.73	0.90	4.23	0.05	2.66	0.24	1.88	0.12
FEs	–	15000	8820	4000	10840	4000	8860	5000	5000

VI.IV. 120-bar dome truss structure

The fourth design example is the 120-bar dome truss structure shown in Fig. 2d. At the free nodes of the structure, a lumped mass is added as follows: 3000 kg at node one, 500 kg at the nodes 2 through 13 kg, and 100 kg at the rest of the nodes. The elements are categorized into seven groups using geometrical symmetry. Table 9 presents the comparison between the MSEO and other metaheuristics algorithms. It is observed that the optimum design obtained by the MSEO (8707.44 kg) is better than the results reported by CSS-BBBC (9046.34 kg), DPSO (8890.48 kg), CBO (8889.13 kg), HALC-PSO (8889.96 kg), MS-TLBO (8708.73 kg) ISOS (8710.06 kg) and SEO (8709.73 kg). Although the best weight provided by ReDE (8707.32 kg) is less heavy than that of the MSEO, the convergence speed (5080 FEs for ReDE and 5000 FEs for MSEO) and SD (0.15 kg for ReDE and 0.11 kg for MSEO) achieved with MSEO are better than this method. Regarding the SD, the MSEO is the first among the considered algorithms. Frequency values obtained by MSEO satisfy all allowable constraints.

Table 9. Optimal design results for the 120-bar dome truss by different algorithms

Variables (cm^2)	CSS-BBBC	DPSO	CBO	HALC-PSO	MS-TLBO	ReDE	ISOS	This study	
	[11]	[13]	[15]	[17]	[23]	[27]	[29]	**SEO**	**MSEO**
A_1	17.478	19.607	19.6917	19.8905	19.4486	19.5131	19.6662	19.4481	19.7843
A_2	49.076	41.290	41.1421	40.4045	40.3949	40.3914	39.8539	41.3771	40.4949
A_3	12.365	11.136	11.1550	11.2057	10.6921	10.6066	10.6127	13.7698	13.6093
A_4	21.979	21.025	21.3207	21.3768	21.3139	21.1415	21.2901	20.2608	20.1959
A_5	11.190	10.060	9.8330	9.8669	9.8943	9.8057	9.7911	8.9358	8.8850
A_6	12.590	12.758	12.8520	12.7200	11.7810	11.7781	11.7899	15.3452	15.4024
A_7	13.585	15.414	15.1602	15.2236	14.5979	14.8163	14.7437	12.9937	13.1873
Best weight (kg)	9046.34	8890.48	8889.13	8889.96	8708.73	8707.32	8710.06	8709.73	8707.44
f_1 (Hz)	9.000	9.0001	9.0000	9.0000	9.0000	9.0000	9.0001	9.0000	9.0000
f_2 (Hz)	11.007	11.0007	11.0000	11.0000	11.0000	11.0000	10.9998	11.0000	11.0000
Average weight (kg)	–	8895.99	8891.25	8900.39	8734.75	8707.52	8728.56	8712.51	8707.69
SD (kg)	–	4.26	1.79	6.38	27.05	0.15	14.23	4.15	0.11
FEs	–	6000	6000	17000	4000	5080	4000	5000	5000

International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 13, Number 11 (2020), pp. 3950-3963
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VI.V. 37-bar planar truss structure

Figure 2e illustrates a simply supported planar 37-bar truss. This problem considers simultaneous size and shape optimization. A lumped mass of 10 kg is added to each of the free nodes of the lower chord. The lower chord bars have fixed cross-section area of 0.4 cm2. All remaining structural members are clustered into 14 sizing variables, as shown in Fig. 6. Nodes on the upper chord are set as shape variables while the lower chord nodes are fixed. This optimization problem includes 14 size variables and 5 shape variables.

The optimization results obtained by the MSEO and other metaheuristics are given in Table 10. The best design obtained by MSEO (359.89 kg) is only surpassed by ReDE (359.81 kg) and AHEFA (359.81 kg). However, MSEO only requires 5000 FEs to obtain the optimal solution while ReDE and AHEFA require 13740 and 8640 FEs, respectively. Moreover, regarding the SD, MSEO (0.05 kg) rank first among considered metaheuristics. None of the frequency constraints are violated as shown in Table 9.

Table 10. Optimal design results for the 37-bar planar truss by different algorithms

Variables y_j (m); A_i (cm^2)	PSO	DPSO	HSPO	VPS	ReDE	ISOS	AHEFA	This study	
	[9]	[13]	[34]	[24]	[27]	[29]	[30]	SEO	MSEO
y_3, y_{19}	0.9637	0.9482	0.9606	0.9042	0.9533	0.9257	0.9589	0.8018	0.9199
y_5, y_{17}	1.3978	1.3439	1.3425	1.2850	1.3414	1.3188	1.3450	1.2249	1.3046
y_7, y_{15}	1.5929	1.5043	1.5219	1.5017	1.5319	1.4274	1.5355	1.4382	1.5103
y_9, y_{13}	1.8812	1.6350	1.6567	1.6509	1.6528	1.5806	1.6668	1.5154	1.6385
y_{11}	2.0856	1.7182	1.7330	1.7277	1.7280	1.6548	1.7397	1.6023	1.7199
A_1, A_{27}	2.6797	2.6208	3.0179	3.1306	2.9608	2.6549	2.8210	2.9609	3.0312
A_2, A_{26}	1.1568	1.0397	1.0000	1.0023	1.0052	1.0383	1.0019	1.1299	1.0013
A_3, A_{24}	2.3476	1.0464	1.0001	1.0001	1.0014	1.0000	1.0001	1.0674	1.0242
A_4, A_{25}	1.7182	2.7163	2.5470	2.5883	2.5994	3.0083	2.5308	2.6199	2.7509
A_5, A_{23}	1.2751	1.0252	1.2429	1.1119	1.1949	1.0024	1.2210	2.0352	1.1776
A_6, A_{21}	1.4819	1.5081	1.2679	1.2599	1.2165	1.4499	1.2429	1.5660	1.1731
A_7, A_{22}	4.6850	2.3750	2.5675	2.6743	2.4303	3.1724	2.4718	2.5157	2.4284
A_8, A_{20}	1.1246	1.4498	1.4142	1.3961	1.3644	1.2661	1.4018	1.1662	1.4156
A_9, A_{18}	2.1214	1.4499	1.5449	1.5036	1.5548	1.4659	1.5061	1.6066	1.5467
A_{10}, A_{27}	3.8600	2.5327	2.5457	2.4441	2.5247	2.9013	2.5604	4.3221	2.5364
A_{11}, A_{15}	2.9817	1.2358	1.2148	1.2977	1.1946	1.1537	1.2146	1.1679	1.2371
A_{12}, A_{15}	1.2021	1.3528	1.3371	1.3619	1.3163	1.3465	1.3605	1.6611	1.3093
A_{13}, A_{16}	1.2563	2.9144	2.3914	2.3500	2.4465	2.6850	2.3992	2.6949	2.4720
A_{14}	3.3276	1.0085	1.0000	1.0000	1.0003	1.0000	1.0000	1.0965	1.0004
Best weight (kg)	377.20	360.40	360.45	359.94	359.81	360.74	359.81	364.97	359.89
f_1 (Hz)	20.0001	20.0194	20.0000	20.0002	20.0005	20.0119	20.0000	20.0011	20.0000
f_2 (Hz)	40.0003	40.0113	40.0000	40.0005	40.0004	40.0964	40.0001	40.0015	40.0001
f_3 (Hz)	60.0001	60.0082	60.0000	60.0000	60.0022	60.0066	60.0002	60.0078	60.0001
Average weight (kg)	381.2	362.21	360.52	360.23	359.99	363.40	359.92	368.74	359.93
SD (kg)	4.26	1.68	0.16	0.22	0.15	1.57	0.09	2.86	0.05
FEs	12500	6000	9000	30000	13740	4000	8640	5000	5000

VI.VI. 52-bar dome truss structure

The 52-bar dome truss shown in Fig. 2f is the last optimization problem considered in this work. This problem is considered for simultaneous size and shape optimization. A lumped mass of 50 kg is attached to all the free nodes of the structure. The members are categorized into 8 groups, having in mind symmetry about the z-axis. All free nodes can shift ± 2 m in each direction of the vertical plane to maintain radial symmetry of the structure about node 1.

The optimal results obtained by MSEO and other metaheuristics algorithms are listed in Table 11. The optimal weight achieved by the proposed algorithm (193.52 kg) is better than PSO (228.38 kg), DPSO (195.35 kg), HALC-PSO (194.85 kg), HSPO (194.79 kg), ISOS (194.75 kg) and SEO

(197.84 kg). AHEFA reports better weight than MSEO, however, MSEO only requires 5000 FEs to obtain the optimal solution while AHEFA requires 12120 FEs. Moreover, regarding the SD, MSEO ranks first among considered metaheuristics. All this indicates the effectiveness and robustness of the MSEO. Finally, it can be seen that none of the frequency constraints are violated.

Table 11. Optimal design results for the 52-bar dome truss by different algorithms

Variables Z_j, X_j (m); A_i (cm^2)	**PSO**	**DPSO**	**HALC-PSO**	**HSPO**	**ISOS**	**AHEFA**	**This study**	
	[9]	[13]	[17]	[34]	[29]	[30]	**SEO**	**MSEO**
Z_1	5.5344	6.1123	5.9362	5.9330	6.1631	5.9953	6.0590	6.1501
X_2	2.0885	2.2343	2.2416	2.2950	2.4224	2.3062	2.2512	2.2495
Z_2	3.9283	3.8321	3.7309	3.7080	3.8086	3.7308	3.7649	3.8669
X_6	4.0255	4.0316	3.9630	2.8102	4.1080	4.0000	3.9944	4.1006
Z_6	2.4575	2.5036	2.5000	2.5000	2.5018	2.5000	2.5007	2.5001
A_1	0.3696	1.0001	1.0001	1.0000	1.0074	1.0000	1.0040	1.0178
A_2	4.1912	1.1397	1.1654	1.1175	1.0003	1.0832	1.1419	1.1307
A_3	1.5123	1.2263	1.2323	1.2160	1.1982	1.2014	1.3114	1.2242
A_4	1.5620	1.3335	1.4323	1.4802	1.2787	1.4527	1.3730	1.4577
A_5	1.9154	1.4161	1.3901	1.4087	1.4421	1.4212	1.3901	1.4451
A_6	1.1315	1.0001	1.0001	1.0000	1.0000	1.0000	1.3883	1.0000
A_7	1.8233	1.5750	1.6024	1.6064	1.4886	1.5570	1.3164	1.5266
A_8	1.0904	1.4357	1.4131	1.3916	1.4990	1.3904	1.5171	1.3851
Best weight (kg)	228.38	195.35	194.85	194.79	194.75	193.20	197.84	193.52
f_1 (Hz)	12.751	11.315	11.4339	11.6528	12.5459	11.6629	11.5001	11.3405
f_2 (Hz)	28.649	28.648	28.6480	28.6480	28.6518	28.6480	28.6480	28.6476
Average weight (kg)	234.30	198.71	196.85	204.10	207.55	198.73	200.77	195.11
SD (kg)	5.22	13.85	2.38	7.81	8.74	4.41	4.68	2.21
FEs	11270	6000	7500	4000	4000	12120	5000	5000

VI. CONCLUSIONS

This paper proposes a new enhanced SEO (MSEO) for size and shape optimization of truss structures subjected to multiple dynamic constraints. MSEO introduces a decomposition operator, a concept borrowed from artificial ecosystem-based optimization (AEO), to improve exploitation capability of the standard SEO. Thirty benchmark functions of the CEC2014 database and six well-known truss optimization problems are tested to verify the effectiveness and robustness of the proposed algorithm. In the benchmark functions, the results indicate the superiority of the MSEO compared to the standard SEO and other metaheuristics. In the design problems, the performance of the MSEO is comparable to the other state-of-the-art methods in terms of the best weight, average weight, standard deviation (SD) and FEs required by the optimization process. Regarding SD, MSEO rank first among the metaheuristics considered, and in one example the SD is only slightly heavier than the best one found. MSEO is a simple algorithm to implement and could be applied to various engineering optimization problems such as reliability-based design optimization, topology optimization and laminated composite optimization problems where the computational cost is significant

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