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Dual Scalar Aharonov-Bohm Effect and the Photon Mass

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Abstract
In this paper the dual scalar Aharonov-Bohm effect is explored in the context of massive electrodynamics. Specifically, the phase shift of a package of neutral particles with magnetic dipole moment (neutrons) is obtained in the presence of an internal magnetic field pulse from an infinite solenoid calculated with Proca's equations. The phase correction introduced for the Proca equations is compared with the experimental precision and a limit of the order of 2×10^{-44}g is obtained for the photon mass.

Keywords Photon mass, Proca's equations, Aharonoh-Bohm effect.

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I. INTRODUCCTION

The photon is the particle that mediates electromagnetic interaction. This particle appears as a consequence of the quantification of Maxwell's theory with the peculiarity that it has no mass [1]. On the other side, Proca [2] in 1936 introduced a relativistic generalization of Maxwell's equations which lead, through quantization, to a photon with mass [1]. Although Maxwell's scenario has been extensively tested experimentally, Proca's scenario does not fail to attract attention because it seems more intuitive that every particle in nature has mass. Consequently, the controversy of whether the photon has mass or not must be resolved only by means of experimental verification. A similar controversy arose with the neutrino, which is the particle that mediates strong nuclear interaction. This particle was considered massless like the photon, but recently, the oscillation of neutrinos has been experimentally observed, a property that is possible only if the neutrino has mass [3] [4]. Thus, the search for the photon mass (as opposed to the neutrino mass) remains an open and fundamental issue in particle physics. Recently,

Theoretical perspective: From the theoretical point of view the insertion of a massive term in the Lagrangian of quantum electrodynamics breaks its gauge invariance, meaning this, the theory cannot be renormalizable. But this is not the case with the Proca Lagrangian because it is the fixed gauge version of Stückelberg's Lagrangian [5] which restores gauge invariance. The above indicates that the theory can admit a massive photon, but finally it must be the experimental verification that has the last word. According to the uncertainty principle $\left(m_\gamma \approx \hbar / \left(\Delta t\right) c^2\right)$ the upper final limit that can be established for the photon mass is $m_\gamma \approx 10^{-66}$g, where the age of the universe which is around 10^{10} years has been used for Δt [6].

Methodology: In Proca's theory the electric and magnetic fields are modified, for example, the potential of a charged particle q has the shape of a Yukawa type potential [6], that is, $V\left(r\right) = \frac{q}{4\pi\varepsilon_0}\frac{e^{-\mu_\gamma r}}{r}$,which is reduced to the Coulomb potential at the limit $\mu_\gamma \to 0$. Being μ_γ (measured in m^{-1}) a parameter related to the mass of the photon through the relation $m_\gamma = \mu_\gamma \hbar / c$. A theorem due to Goldhaber and Nieto [7] states that: "*the fractional change due to the mass of the photon in the fields in a region of dimension D is of the order of* $\left(\mu_\gamma D\right)$". Therefore, if a certain phenomenon produces an observable intensity I in Maxwell's theory, the same intensity in Proca's theory will be $\sim I + I\left(\mu_\gamma D\right)^2$. The term $I\left(\mu_\gamma D\right)^2$ is the effect due to the mass of the photon, which must be very small and its effect must be, at most masked in the experimental precision, ΔI, of the studied phenomenon, thus, $I\left(\mu_\gamma D\right)^2 \leq \Delta I$. With this it is observed that $\mu_\gamma \leq D^{-1}\sqrt{\Delta I / I}$ which shows that the limit of the photon mass depends on the dimension D and the experimental precision ΔI ($\Delta I \propto I$). These considerations give rise to two types of phenomena that can be explored: 1) Type 1: phenomena of large length scales and / or low experimental precision, or 2) Type 2: phenomena with very small length scales and great experimental precision.

Among the type 1 phenomena are measurements of the deviation of the Earth's magnetic dipole field made by Fischbach et al. [8] ($m_\gamma \sim 1.0 \times 10^{-48}$g) and Goldhaber and Nieto [9] ($m_\gamma \sim 4.0 \times 10^{-48}$g), see also Xing *et al.* [10] ($m_\gamma \sim 5.1 \times 10^{-45}$g) and Wei *et al.* [11] ($m_\gamma \sim 1.5 \times 10^{-45}$g). A more extensive list of type 1 methods can be found in the review papers of Tu *et. al.* [6] and Goldhaber and Nieto [7][12]. Among the type 2 phenomena is the high precision test of Coulomb's law carried out by William *et al.* [13] ($m_\gamma \sim 1.4 \times 10^{-47}$g) and the cryogenic experiment by Ryan *et al.* [14] ($m_\gamma \sim 1.5 \times 10^{-42}$g). Since precision is a fundamental condition in type 2 phenomena, then it is reasonable to explore quantum phenomena in which this requirement is present. The first to explore this scenario were Boulware and Deser [15] studying the Aharonov-Bohm effect [16] with Proca's theory ($m_\gamma \sim 2.8 \times 10^{-45}$g). The Spavieri effect (AB to electron-positron effect) [17] and the Tackchuk effect [18] has been studied by Spavieri and Rodriguez [19] obtaining the limit ($m_\gamma \sim 2.8 \times 10^{-51}$g) and ($m_\gamma \sim 2.8 \times$

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10^{-41}g), respectively. The effect of type AB for neutrons proposed by Sangster *et. al.*[20] was studied by M. Rodriguez [21] ($m_\gamma \sim 1.1 \times 10^{-41}$g). Another quantum scenario that has been recently explored is that of atomic spectroscopy, specifically, Caccavano and Leung have explored the effect of the photon mass in the context of the hyperfine structure, finding an expression with the photon mass for the 21cm line of hydrogen [22]. To expand the discussion on the mass of the photon in quantum scenarios, see the article by Spavieri et. al. [23]. Following this methodology, in this work an experiment of type 2, specifically the Dual Scalar Aharonov-Bohm effect (DEAB) [24] is explored in the context of Proca's equations or massive electrodynamics. Consequently, the work is organized as follows: in section 2 Proca's theory is reviewed and the internal magnetic field of an infinite solenoid is derived, this result is an application of the procedure used by Boulware and Deser [15] to get the magnetic field in the context of the Aharonov-Bohm effect with photon mass. In section 3 we introduce the Aharonov-Bohm [16] scalar effect and its dual effect that will be analyzed in this work. With the results of section 2 and the DEAB effect in the section 4 we established a limit on the mass of the photon. Finally, in section 5 the conclusions are made.

II. PROCA'S EQUATIONS AND MAGNETIC FIELD OF AN INFINITE SOLENOID

A. Proca's equations

The set of electromagnetic field equations leading to the massive photon known as Proca's equations are as follows (in the international system of units, SI):

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} - \mu_\gamma^2 \phi \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \mu_\gamma^2 \mathbf{A} \tag{4}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ the magnetic field, ρ the charge density, $\mathbf{J}$ the current density, ϕ the scalar potential, $\mathbf{A}$ the vector potential, μ_0 and ε_0 are the permittivity and permeability of free space and μ_γ a characteristic length of the related theory (by means of quantization [1]) with the *photon mass*, that is:

$$m_\gamma = \frac{\mu_\gamma \hbar}{c} \tag{5}$$

Additionally, potentials maintain their standard form,

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{6}$$

and

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \tag{7}$$

along with the Lorentz condition

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \tag{8}$$

which is necessary for the validity of the conservation of the load. It should be noted that due to the parameter μ_γ present in the equations (1) and (4) the fields $\mathbf{E}$ and $\mathbf{B}$ acquire an exponential attenuation, in addition to the natural dependence on distance, hence Proca electrodynamics is also known as finite range electrodynamics.

B. Magnetic Field of an Infinite Solenoid

To find the static magnetic field inside a solenoid, you must first obtain the potential vector $\mathbf{A}$. To do this, substitute (6) in (4) together with Coulomb´s gauge $\nabla \cdot \mathbf{A} = 0$, this last condition is obtained from considering that the fields are static and thus the equation (8) gives the mentioned condition. So,

$$(-\nabla^2 + \mu_\gamma^2)\mathbf{A}_{AB} = \mu_0 \mathbf{J}_{AB}. \tag{9}$$

Now let $\mathbf{M}(z) = \bar{\mu}\hat{\mathbf{z}}$ be the magnetization per unit length of the solenoid so the associated surface current is $\mathbf{K} = \bar{\mu}\hat{\phi}$. Therefore, the equation (9) takes the following form:

$$(-\nabla^2 + \mu_\gamma^2)\mathbf{A}_{AB} = \hat{\phi}\,(\mu_0 \bar{\mu})\,\delta(\rho - a) = \nabla \times \{B_0 \hat{\mathbf{z}} \Theta(a-\rho)\}\,, \tag{10}$$

where $B_0 = \mu_0 \bar{\mu}$ is the internal standard magnetic field of an infinite solenoid in SI units, a is the radius of the solenoid, $(\hat{\rho}, \hat{\mathbf{z}})$ are unit vectors, Θ is the step function and ρ is the radius in the $x-y$ plane. If $\mathbf{A} = \hat{\mathbf{z}} \times \nabla \Pi(\rho)$ is taken, then the equation (10) after some manipulations takes the following form:

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Pi(\rho)}{\partial \rho}\right) - \mu_\gamma^2 \Pi(\rho) = B_0 \Theta(a-\rho). \tag{11}$$

The relevant homogeneous solutions of this equation are the modified Bessel functions I_0 and K_0 which are regular at the origin and at infinity, respectively:

$$x \to 0: \quad I_0(x) \sim 1 + \frac{x^2}{4}, \qquad K_0(x) \sim -\ln\left(\frac{x}{2}\right)$$
$$x \to \infty: \; I_0(x) \sim (2\pi x)^{-1/2} e^{x}, \; K_0(x) \sim -\left(\frac{\pi}{2}\right)^{1/2} e^{-x}.$$

Consequently, Green's function:

$$G(\rho, \rho') = I_0(\mu_\gamma \rho_<)\, K_0(\mu_\gamma \rho_>) \tag{12}$$

thus $\Pi(\rho)$ is given by:

$$\begin{aligned} \Pi(\rho) = {} & B_0 \Theta(a-\rho)[K_0(\mu_\gamma \rho) \int_0^\rho I_0(\mu_\gamma \rho')\rho' d\rho' \\ & + I_0(\mu_\gamma \rho) \int_\rho^a K_0(\mu_\gamma \rho')\rho' d\rho'] \\ & - B_0 \Theta(\rho - a) K_0(\mu_\gamma \rho) \int_0^a I_0(\mu_\gamma \rho')\rho' d\rho'. \end{aligned} \tag{13}$$

Therefore, the magnetic field is:

$$\mathbf{B} = \nabla \times \mathbf{A} = \hat{\mathbf{z}}\nabla^2\Pi = \hat{\mathbf{z}}B_0\Theta(a-\rho) + \hat{\mathbf{z}}\mu_\gamma^2\Pi(\rho). \quad (14)$$

The first term on the right side of (14) is, of course, the usual $\mathbf{B}_0$ usual magnetic field in Maxwell's theory. It is clear from (13) that $\mu_\gamma^2\Pi(\rho)$ cancels out at the limit $\mu_\gamma \to 0$ since I_0 is regular and K_0 is logarithmic in the arguments (for ρ fixed). The internal magnetic field of the solenoid which is of interest in this work will be given by the following expression:

$$\mathbf{B} = \hat{\mathbf{z}}B_0\Theta(a-\rho)\left\{1 + \mu_\gamma^2\left[C + D\right]\right\} \quad (15)$$

where,

$$C = -\ln\left(\frac{\mu_\gamma\rho}{2}\right)\int_0^\rho\left(1 + \frac{(\mu_\gamma\rho')^2}{4}\right)\rho' d\rho'$$

and

$$D = -\left(1 + \frac{(\mu_\gamma\rho)^2}{4}\right)\int_\rho^a \ln\left(\frac{\mu_\gamma\rho'}{2}\right)\rho' d\rho'$$

The integrals of C and D are as follows:

$$\int_0^\rho\left(1 + \frac{(\mu_\gamma\rho')^2}{4}\right)\rho' d\rho' = \frac{1}{\mu_\gamma^2}\left[\frac{(\mu_\gamma\rho)^4}{10} + \frac{(\mu_\gamma\rho)^2}{2}\right] \quad (16)$$

$$\int_\rho^a \ln\left(\frac{\mu_\gamma\rho'}{2}\right)\rho' d\rho' = \frac{4}{\mu_\gamma^2}\left[\begin{array}{c}\left[\frac{(\mu_\gamma a)^2}{2}\ln(\mu_\gamma a) - \frac{(\mu_\gamma a)^2}{4}\right] \\ -\left[\frac{(\mu_\gamma\rho)^2}{2}\ln(\mu_\gamma\rho) - \frac{(\mu_\gamma\rho)^2}{4}\right]\end{array}\right] \quad (17)$$

Substituting (16) and (17) in (15) we obtain:

$$\mathbf{B} = \hat{\mathbf{z}}B_0\left[1 + \begin{array}{c} 3\frac{(\mu_\gamma\rho)^2}{2}\ln(\mu_\gamma\rho) + 5\frac{(\mu_\gamma\rho)^4}{2}\ln(\mu_\gamma\rho) \\ -2(\mu_\gamma a)^2\ln(\mu_\gamma a) - \frac{(\mu_\gamma\rho)^2(\mu_\gamma a)^2}{2}\ln(\mu_\gamma a) \\ +(\mu_\gamma a)^2 - (\mu_\gamma\rho)^2 + \frac{(\mu_\gamma\rho)^2(\mu_\gamma a)^2}{4} - \frac{(\mu_\gamma\rho)^4}{4}\end{array}\right] \quad (18)$$

Now, although (18) is the expression of the magnetic field inside the solenoid, here you will be interested in the internal field near the origin, in this case the following approach is true:

$$\mu_\gamma\rho \ll \mu_\gamma a \ll 1 \quad (19)$$

Consequently, the field of interest in this work will have the following expression:

$$\mathbf{B} = \hat{\mathbf{z}}B_0\left[1 - 2(\mu_\gamma a)^2\ln(\mu_\gamma a)\right] \quad (20)$$

If the field is time dependent in the form of a pulse, then the field (20) take the form:

$$\mathbf{B}(t) = \hat{\mathbf{z}}B_0(t)\left[1 - 2(\mu_\gamma a)^2\ln(\mu_\gamma a)\right] \quad (21)$$

where

$$B_0(t) = \begin{cases} B_0 & \text{si } 0 \le t \le t_v \\ 0 & t > t_v \end{cases}$$

Where t_v is the time during which the pulse is applied.

III. AHARONOV-BOHM DUAL SCALAR EFFECT

A. Aharonov-Bohm scalar effect

Little mention is made in the literature of the Aharonov-Bohm (EAB) scalar (or electrical) effect for electrons proposed by Aharonov and Bohm in their famous 1959 paper [16] entitled "Importance of Electromagnetic Potentials in Quantum Theory" In this effect the shift of the interference pattern is caused by the presence of a scalar potential, $V = -eU$, in the path of the particles, although $\mathbf{E} = \mathbf{B} = 0$. Schrödinger's equation in this case takes the following form

$$(H_0 + V)\Psi = i\hbar\frac{\partial}{\partial t}\Psi \quad (22)$$

The figure (1) illustrates the Aharonov-Bohm scalar effect. In this effect, an electron package is divided into two packages that travel through two cylinders that act as Faraday boxes. During the transit of each electron packet through the cylinders a potential pulse is ignited in each cylinder for a time interval τ. The pulse is considered to be turned on when the electron pack is away from the edges of the cylinders. This condition allows a constant potential (force free effect) and also disregards edge effects. Now, according to the Schrödinger equation (22) the phase difference, $\Delta\phi_S$, that these two packages experience when they emerge from Faraday's boxes is:

$$\Delta\phi_S = \frac{1}{\hbar}\int_{t_1}^{t_2} e\left[U_2(t) - U_1(t)\right]dt = \frac{e}{\hbar}\left[U_2 - U_1\right]\tau \quad (23)$$

where it has been considered that:

$$U_2(t) = \begin{cases} U_2 & \text{si } t_1 \le t \le t_2 \\ 0 & t > t_1 \text{ y } t < t_2 \end{cases}$$

and

$$U_1(t) = \begin{cases} U_1 & \text{si } t_1 \le t \le t_2 \\ 0 & t > t_1 \text{ y } t < t_2 \end{cases}$$

where $\tau = t_2 - t_1$. A greater effect is achieved if $U_1 = -U_2$, so the phase would be double, that is:

$$\Delta\phi_S = 2\frac{e}{\hbar}U_2\tau \quad (24)$$

B. Aharonov-Bohm scalar dual effect

Allman *et. al.* [24] presented an experiment very similar to the EAB effect, but with neutrons. In this phase shift is produced the scalar potential, $V = -\mu\cdot\mathbf{B}$, in analogy with the scalar potential $V = -eU$. For this, the electron pack was replaced by a neutron package (Fig 2), and the Faraday cylinders or boxes with solenoids with pulses of current $\mathbf{i}_1$ and $\mathbf{i}_2$ that produce pulses of magnetic fields $B_2(t)$ and $B_1(t)$. Consequently, the phase difference, $\Delta\phi_N$, experienced by the neutron packets is:

$$\Delta\phi_N = \frac{1}{\hbar}\int_{t_1}^{t_2}\mu\left[B_2(t) - B_1(t)\right]dt \quad (25)$$

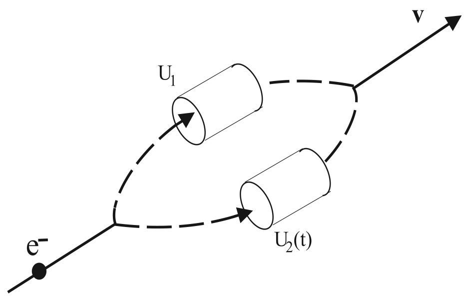

FIG. 1: Illustration of the scalar (or electric) AB effect for electrons. Two coherent electron beams travel through two held cylinders at different potentials, $U_1(t)$ and $U_2(t)$. Phase shift occurs when the potential in one of the cylinders is pulsed during the time it takes for the beam to pass through it. Inside the cylinders the electric and magnetic field is zero $E = B = 0$.

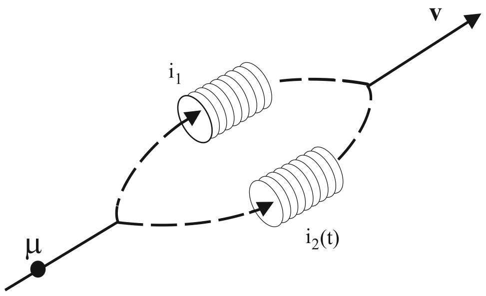

FIG. 2: Illustration of the scalar (or electrical) AB effect for neutrons. Two coherent neutron beams travel through two solenoids in which two currents circulate, $i_1(t)$ and $i_2(t)$. Phase shifting occurs when the current in each solenoid is pulsed for the time τ that the package remains within the solenoids.

Now, we consider that

$$B_2(t) = \begin{cases} B_0 & \text{si}\, t_1 \le t \le t_2 \\ 0 & t > t_1 \,\text{y}\, t < t_2 \end{cases}$$

and

$$B_1(t) = \begin{cases} -B_0 & \text{si}\, t_1 \le t \le t_2 \\ 0 & t > t_1 \,\text{y}\, t < t_2 \end{cases}$$

Then,

$$\Delta\phi_N = \frac{2}{\hbar}\int_{t_1}^{t_2} \mu B_2(t)\, dt = 2\frac{\mu B_0}{\hbar}\tau \tag{26}$$

where $\tau = t_2 - t_1$ and μ is the magnetic moment of the neutrons. Note that the edge effect is also not taken in this effect since the neutron packet is considered to be inside the cylinder and away from the edges when the current pulse at each solenoid is turned on.

IV. LIMIT ON THE MASS OF THE PHOTON

To establish a limit on the mass of the photon in the configuration proposed by Allman *et. al.* [24]. Replace the expression (21) in (26), thus obtaining the phase for the scalar effect for neutrons in massive electrodynamics , $\Delta\phi_{N,\gamma}$:

$$\Delta\phi_{N,\gamma} = \frac{2\mu B_0 \tau}{\hbar}\left[1 - 2(\mu_\gamma a)^2 \ln(\mu_\gamma a)\right]$$

It is convenient to separate this expression in the following way:

$$\Delta\phi_{N,\gamma} = \Delta\phi_N - \Delta\phi_N\left[2(\mu_\gamma a)^2 \ln(\mu_\gamma a)\right]$$

where $\Delta\phi_N = \frac{2\mu B_0}{\hbar}$. Therefore, the additional term in phase due to the mass of the photon is:

$$\Delta\phi_\gamma = \Delta\phi_N\left[2(\mu_\gamma a)^2 \ln(\mu_\gamma a)\right]$$

Considering the approximation (19) this last expression can be written as $2(\mu_\gamma a)^2 \ln(\mu_\gamma a) \simeq 2(\mu_\gamma a)^2$, whereby we are left with the correction to the phase due to the mass of the photon, $\Delta\phi_\gamma$, is as follows:

$$\Delta\phi_\gamma = 2\Delta\phi_N(\mu_\gamma a)^2$$

This agrees with the Goldhaber and Nieto theorem [7] mentioned in the introduction, i.e., the correction due to the photon mass is of the order of $(\mu_\gamma D)^2$ where D is the dimension of the interaction region, in this case $D$is the radius a of the solenoid. So,

$$2\Delta\phi_N(\mu_\gamma a)^2 \le \delta(\Delta\phi_N)$$

Thus,

$$\mu_\gamma = a^{-1}\sqrt{\frac{1}{2}\frac{\delta(\Delta\phi_N)}{\Delta\phi_N}} \tag{27}$$

where $\delta(\Delta\phi_N)$is the precision of the measurement in this scenario. The expression (27) is the result that allows obtaining a limit on the mass of the photon. As in this work the possibility of obtaining a limit m_γ is being explored, we proceed to test values of the parameters contained in (27) that are within the allowed experimental technology. For this purpose, it is necessary to consider ultra slow neutrons, according to Daum et. al. [25] neutrons can obtain low velocities, for example, $v = 10m/s$, this implies that the flight time of neutrons within a 1m long solenoid is $t_v = 10^{-1}m/s$. With this speed for the neutrons then the pulse of the magnetic field inside the solenoid can be activated for $\tau = 10^{-2}$s, this is equivalent to a flight distance of 10cm, which is necessary for the neutrons to be in the center of the solenoid and thus do not take into account the edge effects. According to Boulware and Deser [15] the magnetic field of the solenoid can be $B_0 = 10$T. Furthermore, the radius of the solenoid can be $a = 10^{-1}m$. Finally, the precision of the measurement that can be obtained in neutron interferometers is $\delta(\Delta\phi_N) = 5.2 \times 10^{-4}$rad, as reported by the work of Cimmino et. al. [26] in the measurement of the topological effect of the Aharonov-Bohm type for neutrons known as the Aharonov-Casher [27]. With all these data, the following limit is obtained for μ_γ:

$$\mu_\gamma = 10m^{-1}\sqrt{\frac{1}{2}\frac{5.2 \times 10^{-4}}{9.2 \times 10^{6}}} = 5,3 \times 10^{-5}m^{-1}$$

This implies a searched limit for m_γ according to (5) is:

$$m_\gamma = \frac{\mu_\gamma \hbar}{c} = 2 \times 10^{-44} g.$$

V. CONCLUSIONS

In this work, the internal field of a solenoid has been calculated with Proca's electrodynamics. This result together with the phase of the DEAB effect proposed by Allman et. al. [24] allows to obtain an expression for the phase of the DEAB effect in the context of the massive electrodynamics or mass of the photon. Under certain experimental considerations it is possible to obtain a limit on the mass of the photon of the order of $2 \times 10^{-44} g$. The obtained value is an order of magnitude greater than the limit reported by [15][10][11] and three orders of magnitude less than the result reported by M. Rodriguez [21] in the context of the Sangster et. al. [20]. It can be seen that type 1 tests compete or are comparable with the limits obtained in type 2 tests, such as the recent results of Xing *et al.* [10] and Wei *et al.* [11] through Fast radio bursts. Accordingly, with the advent of improvements in the precision of interferometers, it could be possible to obtain limits that are closer to the final limit of 10^{-66}g for the mass photon.

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