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Combined Approach for Treating Stochastic Vector Optimization Problem

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Abstract

In this paper, a hybrid approach for treating Stochastic Vector Optimization Problem (SVOP) is suggested. This approach combines the characteristics of both the generalized Tchebycheff norm approach and the method of constraints.

In this approach the stochastic parameters is considered to be in the right hand of the constraints. This approach is deduced by combining the generalized Tchebycheff norm approach together with the constraint approach. The advantages of the suggested approach over the hybrid one lies in the fact that all its parameters can be included only in the constraints. In addition, it gathers the characteristics of both generalized Tchebycheff norm and constraint approaches.

Keywords: Stochastic Vector Optimization, Combined Approach, Chance-Constrained, Scalarization Techniques, Efficient Solutions.

1. INTRODUCTION

In the real life problem, we encounter many problems with uncertain parameters. Several combined problems are developed and formulated as approaches for characterizing the efficient solutions for VOP. A methods combining the weighting method and the ε −constraint method is called the Hybrid method. This method is described in Corley [1], and Wendell and Lee [2] in slightly different forms. The advantage of this approach is that it generates only the properly efficient solutions of VOP and there is no difficulty in solving problems.

Wendell [2] combined the weighting problem and the constraint problem to introduce another form of scalar optimization methods, which is combined different scalar optimization problems together to deduce new approaches for treating VOP which will be assumed to be rather simple than the available one's and/or with more weaker conditions than that imposed on the available approaches.

The ability of solving VOP relies completely on the success of solving the resulting SOP, Therefore, a great deal of work has been done to characterize the efficient solutions of VOP, let us mention that, Gass and Saaty [3] and Zadeh [4] solved VOP using the weighting problem, Chankong and Haimes [5] introduced other forms of scalarization which are the lagrangian problem and the constraint problem. Lin [6, 7] proposed another SOP to generate the efficient solutions of VOP which named by the proper equality constraint problem. Bowman [8] showed that the solutions of VOP can be characterized in terms of the generalized Tchebycheff norm problem and Choo and Atkins [9] gave an extension to the generalized Tchebycheff norm to characterize the proper efficient solutions of VOP even in the nonconvex cases.

Wendell and Lee [2] combined the characteristics of both the weighting problem and the constraint problem to introduce another form of SOP which called the hybrid approach problem.

[10] Proposed an efficient technique for stochastic bicriteria programming problem (SBCPP) with random variables in both the objective functions and in the right-hand side of the constraints. Widyan used mathematical and statistical tools to treat stochastic multicriterion programming problem (SMCPP) with random parameters in both the objective functions and the right hand side of the constraints [11].

In this work a new combined approach for characterizing the efficient solutions of stochastic vector optimization problems (SVOP) is presented. This approach is called a modified hybrid approach which combines the characteristics of both the generalized Tchebycheff norm and the stochastic constraint problems with random variable in the constraints.

This approach is rather simpler than the other scalarization method, since its parameters can be included only in the constraints instead of being included in both the objective function and the constraints.

There are two approaches to deal with SVOP through two phases, the first one, to transform the stochastic constraint problem into an equivalent deterministic model, then combined it with the deterministic Generalized Tchebycheff Norm. The second approach is to combine the stochastic constraint problem with deterministic Generalized Tchebycheff Norm problem, then transforming it to a deterministic one.

Our scope in this manuscript is concentrated on characterizing the efficient solutions of SVOP in terms of the first approach mentioned above.

2. VECTOR OPTIMIZATION PROBLEM FORMULATION

The Vector Optimization Problem (VOP) can be formulated as follows:

$$\min_{x\in M} \ F(x) \tag{1}$$

Where, $F(x) = (f_1(x), f_2(x), \cdots, f_m(x)): R^n \to R^m$ denotes the real-valued functions that represents the objective functions, and M is the decision space or the feasible region of the system which will be characterized by:

$$M = \{x \in R^n : G(x) \le 0\}$$

Where, $G(x) = \left(g_1(x), g_2(x), \cdots, g_k(x)\right): R^n \to R^k$ denotes the real-valued functions that represented the constraints.

There are several techniques for scalarization in order to characterize the efficient solutions of VOP. In our work, we will combined two of them, which are Generalized Tchebycheff Norm Problem[8] and ε −constraint problem [5]

2.1 Generalized Tchebycheff Norm Problem

Consider the following generalized Tchebycheff norm problem as:

$$\min_{x\in M} \ \max_j \beta_j \left|f_j(x) - u_j^*\right| \tag{2}$$

Where $\beta \in R_+^m$ (the positive orthant of R^m–space), $u^* \in R^m$ is an ideal target.

2.2 The Kth-Objective ε −Constraint Stochastic Problem

The Kth-Objective ε −Constraint stochastic Problem is defined by:

$$\min_{x\in M} f_k(x)$$

Subject to (3)

$$f_j(x) \le \varepsilon_j, \ \ j = 1, 2, \dots, m, \ \ \ j \ne k$$

Where, $\varepsilon_j, j = 1,2, \dots, m, j \ne k$ are stochastic parameters belonging to the probability distribution function.

The random Variables ε_j are assumed to be independently normally distributed with means $E(\varepsilon_j) = \mu_j$ and variances $Var(\varepsilon_j) = \sigma_j^2$, i.e. $\varepsilon_j : N(\mu_j, \sigma_j^2), j = 1, 2, 3, \dots, m$. The cumulative function of normal distribution is defined as

$$P(\varepsilon \le \varepsilon_j) = \int_{-\infty}^{\varepsilon_j} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\varepsilon_j-\mu_j)^2}{2\sigma_j^2}} d\varepsilon_j , \quad \varepsilon_j \in R,$$

$$\mu_j \in R, \sigma_j > 0$$

Let $Z_i = \frac{\varepsilon_j - \mu_j}{\sigma_j}, j = 1, 2, 3, \dots, m$ are assumed to be normally distributed with mean equal zero and variance equal one, i.e. $Z_j : N(0, 1), j = 1, 2, \dots, m$. Therefore,

$$\Phi(z) \ = P(Z \le z) \ge \alpha$$

Then, $z \ge \Phi^{-1}(\alpha)$, where, $\Phi^{-1}(.)$ the inverse distribution of the random variable.

Now, using chance constrained approach to transform problem (3) into an equivalent deterministic model as follows:

$$P(f_j(x) \le \varepsilon_j) \ge \alpha_j \quad , 0 \le \alpha_j \le 1, j = 1, 2, \dots, m$$

$$P\left(\frac{f_j(x) - \mu_j}{\sigma_j} \ge \frac{\varepsilon_j - \mu_j}{\sigma_j}\right) \le 1 - \alpha_j$$

Let $\gamma_j = 1 - \alpha_j$, then

$$f_j(x) \le \mu_j + \sigma_j \Phi^{-1}(\gamma_j \,)$$

3. THE MODIFIED HYBRID APPROACH FOR SOLVING SVOP PROBLEMS

This approach is deduced by combining the **generalized Tchebycheff norm** approach together with the **constraint approach**. The main advantage of the suggested approach over the hybrid one lies in the fact that all its parameters can be included only in the constraints.

The formulation of this approach can be states as:

$$\min_{x \in M} \quad \max_{j} \beta_j \left| f_j(x) - u_j^* \right|$$

Subject to 4)

$$f_j(x) \le \mu_j + \sigma_j \Phi^{-1}(\gamma_j), j = 1, 2, \dots, m$$

Where $\beta \in R_+^m, R_+^m$ is the positive orthant of R^m, u^* is an ideal target, and

The problem (4) can be reformulated to have the following equivalent form:

min z

Subject to (5)

$$N(\beta) = \{(z, x) \in R^{n+1} : \beta_j [f_j(x) + \delta - f_j^-] - z \le 0,\ f_j(x) \le \mu_j + \sigma_j \Phi^{-1}(\gamma_j), \quad j = 1, 2, \dots, m\}$$

and

$g_k(x) \le 0$, k=1,2,…,r

where $u^*$is taken as $u_j^* = \overline{f_j}$-δ, j=1,2,…,m,

$$\overline{f_j} = min_{x \in M} f_j(x)$$

and δ is a small positive number [12].

4. CHARACTERIZATION OF THE EFFICIENT SOLUTIONS FOR SVOP.

The following theorem which characterizes the efficient solutions of SVOP in terms of the modified hybrid approach and those which relate the hybrid and modified hybrid approaches is introduced.

Theorem 1. $x^*$is an efficient solution of problem (1) iff $x^*$is an optimal solution of problem (5) for any given $\beta > 0$ and for some γ_j for which problem (5) is feasible.

Proof: Necessity: Assume that for any given $\beta > 0$, x^* does not solve problem (5), hence

$\beta_j [f_j(\bar{x}) - \overline{u_j}] \le \beta_k [f_k(x^*) - u_k^*]$.

If $\beta > 0$ the above relation can be rewritten as

$f_j(\bar{x}) - \overline{u_j} \le f_k(x^*) - u_k^*$.

Hence x^* is not an efficient solution of SVOP.

Sufficiency: Suppose that x^* solves problem (4) for some β, γ_j, it must be also solve problem (4) for $\beta = \beta^\circ$, $\mu_j^* + \sigma_j^* \Phi^{-1}(\gamma_j^*) = f(x^*)$.

Suppose that x^* is not an efficient solution, this implies that there exist $x^\circ \in M$, such that $f_j(x^\circ) - u_j^\circ \le f_j(x^*) - u_j^*$.

Hence for any $\beta_j^\circ > 0$

$\beta^\circ [f_j(x^\circ) - u^\circ] < \beta^\circ [f_j(x^*) - u_j^*]$

This clearly contradicts the fact that x^* solves problem (4) for $\beta = \beta^\circ$and $\mu_j^* + \sigma_j^* \Phi^{-1}(\gamma_j^*)$ since x° is a feasible point of $\left(\beta^\circ, \mu_j^* + \sigma_j^* \Phi^{-1}(\gamma_j^*)\right)$, thus x^* must be an efficient solution of SVOP.

5. ILLUSTRATIVE EXAMPLE

This example can be demonstrated to show the validity of the proposed approach.

$$\min \ [x_1^2 + x_2^2, \qquad x_1^2 + x_2]$$

Subject to

$$M = \{(x_1, x_2) : x_1 + x_2 \le 1 \text{ and } x_1 \ge 0, x_2 \ge 0\}$$

It is clear that the point (0,0) is an efficient solution of the above problem, and then the problem can reformulated to take the following stochastic combined form:

min z

Subject to

N $(\beta, \varepsilon) = \{(z, x) : -z + \beta_1 (x_1^2 + x_2^2 + \delta) \le 0.$

$-z + \beta_2 (x_1^2 + x_2 + \delta) \le 0.$

$x_1^2 + x_2^2 \le \varepsilon_1,$

$x_1^2 + x_2 \le \varepsilon_2,$

$x_1 + x_2 \le 1.$

$x_1 \ge 0 \ and \ x_2 \ge 0\}$

Where, $\varepsilon_1, \varepsilon_2$ are random variables belonging to normal distribution with means $\mu_1 = 4, \mu_2 = 6$ and standard deviations $\sigma_1 = 0.5, \sigma_2 = 0.7$ respectively. Let the confidence levels $\gamma_1 = 0.99$, $\gamma_2 = 0.95$, and hence, $\Phi^{-1}(0.99) = 2.33$, $\Phi^{-1}(0.95) = 1.645$.Take $\delta = 0.1$.Using lingo package [13], for different values of β, to get the subset of efficient and non-dominated solutions as shown in Table1.

Table 1. The Subset of efficient and non-dominated solutions

β	x_1	x_2	z	f_1	f_2
0.1	2.671412E-04	2.672869E-04	9.000017E-02	8.100102E-03	1.620013E-02
0.3	3.093922E-04	3.092770E-04	7.000018E-02	4.900121E-03	9.800146E-03
0.5	1.524103E-04	1.528814E-04	5.000003E-02	2.500026E-03	5.000029E-03
0.7	1.206266E-04	1.208759E-04	7.000003E-02	4.900019E-03	9.800023E-03
0.9	1.143774E-04	1.147309E-04	9.000003E-02	8.100019E-03	1.620002E-02

6. CONCLUSION

In this work, a new combined approach for characterizing the efficient solutions of stochastic vector optimization problems (SVOP) is presented. This approach is called a modified hybrid approach which combines the characteristics of both the generalized Tchebycheff norm and the stochastic constraint problems with random variable in the right hand side of the constraints. This approach enables us to determine the efficient solutions for SVOP easier than the hybrid one.

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