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Topology Optimization of Automotive Body Structures: A review*

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Abstract

This paper reviews and briefly presents some of the pertinent works concerning the application of structural topology optimization techniques in the development of optimal automotive body structures. The survey starts with a brief introduction to the challenges faced by the automotive industry due to higher demands from customers and regulatory legislation. Then the survey presents a brief introduction to the general concept of optimization and narrows down the concept to structural optimization related to the automotive industry today. Thereafter, the three categories of structural optimization, namely, sizing, shape, and topology parameterization are discussed, and a brief formulation of the topology optimization problem presented. Further, applications of optimization, particularly structural topology optimization techniques are presented with short summaries of the referenced papers. The focus here is on the application of topology optimization in the development of new vehicle body structures. Structural topology optimization techniques are increasingly used to develop lightweight body structures during the early stages of the design process. While certain structural optimization techniques such as sizing and shape parameterizations are generally used during the advanced stages of the design process to fine tune the product geometry.

Keywords: Automotive body structures, lightweight, optimization, structural optimization, sizing, shape, topology.

1. INTRODUCTION

Higher demands from customers and regulatory legislations means that the automotive industry is currently faced with the challenging task of developing a broad spectra of vehicle body structures that are lightweight, comfortable, safe, and cost effective – in short development cycles (Yildiz, et al., 2004; Kabir, et al., 2017). While higher demands from customers suggest that the automotive industry should develop a broad spectra of vehicle body structures that are cost effective and comfortable with excellent performance, regulatory legislation on the other hand imposes tough emission targets as well as high safety standards which in turn necessitates the development of efficient vehicles (Vivian & Held, 2014). These demands and legislation have not only led to the development of powertrains such as electric or hybrid drives and down-sized IC-engines but also to a strong motivation for developing lightweight body structures (Barton & Fieldhouse, 2018). Lightweight body structures will help to reduce inertia (Campbell, 1955) thereby improving fuel efficiency, and, consequently lowering the emissions of pollutants (Kaščák & Spišák, 2013; Ibrahim, 2009; Bjelkengren, 2008). It was found that a 10% reduction in the weight of a vehicle can result into a 6-7% reduction in the consumption of fuel (Ghassemieh, 2011). Additionally, reduced inertia gives enhanced acceleration, deceleration as well as handling (Barton & Fieldhouse, 2018), which in turn helps to reduce wear on components such as tyres, engines, brakes, suspensions and transmission systems because they are under less stress and strain (Hillier & Coombes, 2004). This, in addition, enhances road safety – both to the occupants of vehicles, as well as, the road users and can also help to reduce damage to road surfaces since the kinetic energy of vehicles is reduced (Barton & Fieldhouse, 2018).

This foregoing shows that the objective of a vehicle structural design should be to develop body structures that are as light as possible (Campbell, 1955) in order to reduce the consumption of fuel, and therefore, lower the emissions of pollutants (Ibrahim, 2009). Moreover, they must have sufficient levels of strength and stiffness (Brown, et al., 2002; Matsimbi, et al., 2020), to offer maximum resistance to deflections (Hillier & Coombes, 2004; Costin & Phipps, 1965), and maximum resistance to the deterioration of mechanical properties of joints and components (Masini, et al., 2004). Moreover, the stiffness of a body structure has an important influence on the performance characteristics of the vehicle, through vehicle dynamics and ride conform (Matsimbi, et al., 2020). Therefore, a body structure must fulfil several requirements, some of which have conflicting objectives. Thus for instance, a stronger and stiffer body structure would be able to withstand general use more reliably (Hillier & Coombes, 2004) and also provide a safer environment for the occupants (Masini, et al., 2004). However, stronger structures mean more weight and expense on materials, thereby representing a conflict for which designers as well as manufacturers alike must find a compromise (Hillier & Coombes, 2004). This compromise can be achieved more easily now than in the past by making use of different materials (Glennan, 2007; Bjelkengren, 2008; Hillier & Coombes, 2004) that are now available at realistic costs than was the case a few years ago (Hillier & Coombes, 2004). However, every material has certain properties, which make it more suitable for some applications than others (Happian-Smith, 2002). The use of different lightweight materials in body structures may result in additional costs to the automaker (Bjelkengren, 2008), and therefore an increase in the overall cost of the vehicle. Therefore, the approach to achieving lightweight body structures should not only focus on making use of different materials on the existing body structures. Rather, a systematic design approach that focuses on reducing the overall weight of vehicles taking advantage of the unique attributes of specific lightweight materials (Lotus Engineering, 2010; U.S. DOE, 2012).

Increasingly, researchers are employing design approaches such as structural topology optimization techniques (Bendsøe & Kikuchi, 1988) during the conceptual stages of the design process to search for and determine optimum conceptual models of new body structures (Reed, 2002; Cavazzuti, et al., 2010; Cavazzuti, et al., 2011; Quinn, 2010). These conceptual models can provide the designer with greater flexibility to make structures lighter, stronger and less expensive (Hillier & Coombes, 2004; Cavazzuti, et al., 2011). Modelling of body structures is the process of developing strategic representations of body structures using available information (Brown, et al., 2002; Leckie & Bello, 2009). Such models of the body structures are used to predict what will happen in the real situation and are simple enough to support making of decisions at low cost and within reasonable time frames (Leckie & Bello, 2009; Brown, et al., 2002; Happian-Smith, 2002). The models are also used to investigate the structural response of different geometries or topologies to the applied road loads. The body structure usually accounts for a large proportion of the development as well as manufacturing cost in a new vehicle program (Brown, et al., 2002). Therefore, it is important to ensure that the most efficient or optimal model is determined during the early stages of the design process to minimize changes and excessive costs during the advanced stages of the design process. This explains the rapid development and use of structural topology optimization techniques in the automotive industry (Fredricson, 2005).

The objective of this paper is to review and briefly present some of the pertinent works concerning the use of structural topology optimization techniques in the development of optimal automotive body structures. Initially, the general concept of optimization is introduced. Then, structural optimization is defined and three of its categories, namely; sizing, shape, and topology optimization are presented, and thereafter, some pertinent works that employ structural topology optimization in the development of automotive body structures are examined. The following section, section 2, introduces the general concept of optimization.

2. OPTIMIZATION: INTRODUCTION

Optimization is a process of finding the conditions that give the maximum or minimum of a function. Most real-life problems are non-linear. A non-linear constrained optimization problem with a single-objective function is concerned with finding the values of the design variables in the domain x that will minimize the function $f(\boldsymbol{x})$ given by Equation 1 (Vanderplaats, 2006). Design variables are all the quantities that can be treated as variables and are collected in the design vector or domain x.

$$\begin{cases} \text{Minimize:} & f(\boldsymbol{x}) & \\ \text{Subject to:} & g_i(\boldsymbol{x}) \leq 0 & j = 1, \dots, m \\ & h_k(\boldsymbol{x}) = 0 & k = 1, \dots, p \\ & x_i^L \leq x_i \leq x_i^U & i = 1, \dots, n \end{cases} \tag{1}$$

where the symbol $f(\boldsymbol{x})$ is the objective function to be minimized, $g_i(\boldsymbol{x})$ the inequality constraints function, $h_k(\boldsymbol{x})$ the equality constraints function and, $\boldsymbol{x}$ the independent design variable for all the foregoing functions.

There are many optimization techniques that can be used to solve such a problem. These techniques can be classified broadly as either local or global optimization algorithms (Venter, 2010). The local optimization algorithms are concerned with finding different optimal solutions, particularly local optima in a multimodal function. Figure 1 shows a multimodal two-dimensional function with three minima points. A function is said to be multimodal when there is more than one minimum (or maximum) within the domain. The function $f(x)$ has local minima at $x = \pm 1$ and has a global minimum at $x = 0$.

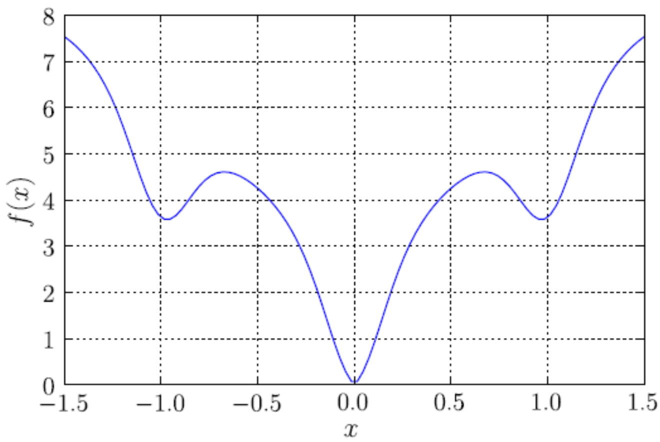

Figure 1. Multimodal two-dimensional function (Venter, 2010)

Most local optimization algorithms are gradient based, that is, they make use of information of gradient to find the optimum solution to Equation 1 (Venter, 2010). Gradient-based methods can be divided into constrained or unconstrained optimization. Constrained optimization techniques include the Steepest Descent (SD), Conjugate Gradient (CG), Quasi-Newton (QN) as well as Newton's method. The unconstrained optimization techniques include the Simplex and Sequential Linear Programming (SLP), Sequential Quadratic Programming (SQP), Exterior Penalty (EP), Interior Penalty (IP), Generalized Reduced Gradient (GRG) as well as the Method of Feasible Directions (MFD) (de Weck & Kim, 2004; Venter, 2010). One disadvantage with local optimization algorithms is the fact that they can converge at any one of the minima or maxima points (local or global) depending on which point is encountered first. That is, it is often not possible with the local optimization algorithms to determine whether the best currently known solution is a local or global optimum (Weise, 2009). A diversification or a multi-start approach is often used to overcome local optimality. A multi-start approach simply finds the local optima and then re-starts the procedure from a new starting point once an optimum has been found (Venter, 2010).

The global optimization algorithms incorporate randomization and they are much suited for finding the global or near global optimum solutions (de Weck & Kim, 2004; Venter, 2010). The global optimization methods can be classified as deterministic or evolutionary algorithms (de Weck & Kim, 2004; Weise, 2009; Venter, 2010). Most of the deterministic algorithms are only developed to solve a narrow class of problems. The most popular deterministic algorithm is the direct method (Venter, 2010). The three most common evolutionary techniques include the Genetic Algorithms (GA), the Simulated Annealing (SA) and the Particle Swarm Optimization (PSO) methods, but other techniques such as the Tabu Search (TS), Genetic Programming (GP), Harmony Search (HS) as well as Colony Optimization (CO) are also found in practice (de Weck & Kim, 2004; Venter, 2010). The areas of application of global optimization algorithms include but is not limited to Structural Optimization, Engineering and Design, Optics, Economics and Finance, Biology, Chemistry, Constraint Satisfaction Problems (CSP), Networking and Communication as well as Operations Research (Weise, 2009). Although there are many areas where global optimization algorithms can be applied, structural optimization is one branch that deals with optimization of load carrying structures such as automotive body structures. The following section, section 3, defines structural optimization and three of its categories, namely; sizing, shape, and topology optimization are discussed further.

3. STRUCTURAL OPTIMIZATION

Structural Optimization is a discipline that deals with optimal design of load-carrying structures. Gordon (1978) defines a structure in mechanics as an assemblage of materials to create an object that is suitable of sustaining loads. Optimization on the other hand, can be defined as an act of making things the best. Thus, alternatively, structural optimization can be said to be a discipline of assemblage of materials to create objects that are suitable of sustaining loads (Gordon, 1978) in the best way possible (Christensen & Klarbring, 2009). The main objective of structural optimization is to find an optimal structural design that, in terms of weight versus cost, is capable of fulfilling performance requirements (constraints) given by the deformations, stresses, eigenfrequencies, and or geometry of the structure, under load conditions. Constraints here are defined as the restrictions or limitations of the amount of materials that must be used to create the structure, since, without restrictions, the structure could be made stiff without limitation, making the problem of optimization to be ill-defined (Christensen & Klarbring, 2009). Of course, a stronger and stiffer structure would provide a safer environment (Masini, et al., 2004) and would be more able to withstand general use more reliably (Hillier & Coombes, 2004). However, stronger and stiffer structures mean more weight and expense of the materials used, thereby representing a conflict for which designers as well as manufactures alike must find a compromise (Hillier & Coombes, 2004).

Optimization of structures in terms of strength and stiffness can be achieved by ensuring that assemblage of the individual elements that make up a structure have the correct load-paths – that is, by ensuring that the geometry of the structure is connected in the best way possible (Brown, et al., 2002; Barton & Fieldhouse, 2018; Eschenauer & Olhoff, 2001). More often, determining the optimal geometry of a new structure is rather a challenging task due to the many constraints that the structure must satisfy. These constraints include cost, manufacturability, packaging requirements as well as impact safety (Baskin, 2016). Traditionally, and still dominant, the best geometry of structures is in most cases inspired by existing designs (Eschenauer & Olhoff, 2001; Brown, et al., 2002). While in some cases, several design concepts with several performance requirements based on the function of the structure are investigated (Christensen & Klarbring, 2009), after which one of them is chosen as a final concept for which a detailed design can start (Yildiz, et al., 2004). However, this is a trial and error approach and depends on the selection of the initial design – guided by the designers' creativity as well as prior experience (Cavazzuti & Splendi, 2012; Yildiz, et al., 2004). Approaches such as these are time-consuming and may result in uneconomical use of material which translate into high manufacturing costs of structures (Sudin, et al., 2014) and therefore, do not guarantee reaching the optimal design (Sudin, et al., 2014; Cavazzuti & Splendi, 2012; Yildiz, et al., 2004).

The mathematical design optimization approach offers a conceptually different formulation of the problem from the iterative-intuitive one in the sense that, when the problem is formulated, the performance requirements of the function of the structure act as constraints of the problem and the optimal solution is given a precise mathematical form (Christensen & Klarbring, 2009). This approach is more automated than the iterative-intuitive one and can be used to determine the optimal solutions of mechanical structures whose main task is to carry loads safely and economically (Christensen & Klarbring, 2009). This approach is a subset termed as structural optimization. Structural optimization can be categorized into three geometric design parameterizations, namely; shape, sizing, and topology. Figure 2 shows the three categories of structural optimization. The three categories shown address different aspects of structural design (Bendsøe & Sigmund, 2003), and are discussed briefly hereunder:

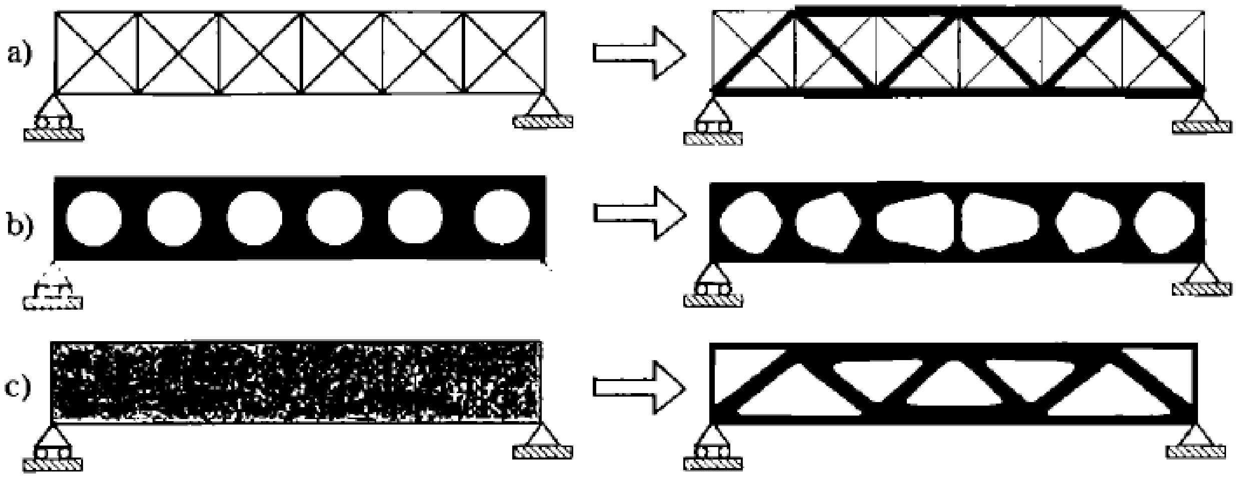

Figure 2. The three categories of SO (a) Sizing optimization, (b) Shape optimization, and (c) topology optimization, original problems on the left and the optimal solutions on the right (Bendsøe & Sigmund, 2003)

Sizing optimization – This is the simplest form of structural optimization (Srivastava, et al., 2017). The goal of sizing optimization is to find the optimal distribution of thickness of a sheet or the optimal cross-sectional areas of the truss members of a structure (Bendsøe & Sigmund, 2003; Christensen & Klarbring, 2009; Petrucci, 2009). In sizing parameterization, the thickness of a sheet or plate or the diameter of the truss members are used as design variables (Srivastava, et al., 2017; Haftka & Grandhi, 1986). A typical sizing optimization problem is shown in Figure 2(a) with the original problem shown on the left and the optimal solution shown on the right. It can be seen in Figure 2(a) that only the size rather than the number of truss members were altered. Sizing optimization is typically performed during the detailing stages of the design process where only a fine tuning of product geometry is necessary (Saitou, et al., 2005; Aulig, 2017).

Shape optimization - The goal of a typical shape optimization is to find an optimal shape of the structure (Bendsøe & Sigmund, 2003; Atrek & Agarwal, 1992). Figure 2(b) shows a shape optimization problem, where the original non-optimized problem is shown on the left and the optimal solution is shown on the right. Figure 2(b) shows that only the shapes rather than the number of shapes were altered between the original problem and the optimal solution. In shape optimization problems, the shape of the domain becomes a design variable (Petrucci, 2009). Structural optimization problems with shape design variables are more complex than structural optimization problems with sizing design variables (Haftka & Grandhi, 1986). This is because shape optimization controls the geometry of the structure (Haftka & Grandhi, 1986) and changes the product geometry during the optimization process (Saitou, et al., 2005). It has been found that for many problems, the shape optimization is more effective than the sizing optimization (Ding, 1986). A typical example where sizing and shape optimizations can be compared is that of a stress concentrations near the hole of a panel, a typical sizing optimization would only change the size of the thickness of the panel near the hole while shape optimization would find the suitable shape that optimizes the stress concertation on the boundary of the hole (Haftka & Grandhi, 1986). Similar to sizing optimization, shape optimization is also performed relatively late during the design process where the required geometry is already available, and optimization is only applied as a fine tuning step (Aulig, 2017). Usual shape and sizing optimization techniques do not change the connectivity or topology of the structure during the solution process, so the solution obtained will have the same topology as that of the initial problem (Eschenauer & Olhoff, 2001). The topology of a structure refers to how the members are connected.

Topology optimization – The is the most general form of structural optimization (Christensen & Klarbring, 2009) and is mainly performed during the conceptual stages of the design process (Srivastava, et al., 2017). The word topology is derived from the Greek noun *topos* which means, place, location, domain or space (Eschenauer & Olhoff, 2001). Topology optimization can be differentiated into two cases, namely; a discrete case and a continuum case. In a discrete case, topology optimization is concerned with the problem of finding a discrete set of members that must be present on a design such as the required truss members that will make up a truss (Aulig, 2017; Christensen & Klarbring, 2009). The truss members are taken as design variables and the variables are allowed to take a value of zero. The value of zero indicates that the members are removed from the truss. This allows the connectivity of the nodes to change and this means that the topology of the truss changes (Christensen & Klarbring, 2009). In contrast to the discrete case, continuum topology optimization is concerned with the distribution of material on a given domain or design space, this results in a geometry layout that is defined by the shape of voids and material regions (Aulig, 2017; Christensen & Klarbring, 2009). The shape of voids indicates that the thickness of shapes takes the value of zero, indicating that material is removed from the design space. The regions of materials indicate that the thickness of the shapes takes the maximum value, indicating that material is present in the design space. In practice, continuum topology optimization is addressed in discretized form, where the material distribution within the domain or design space is represented by the value

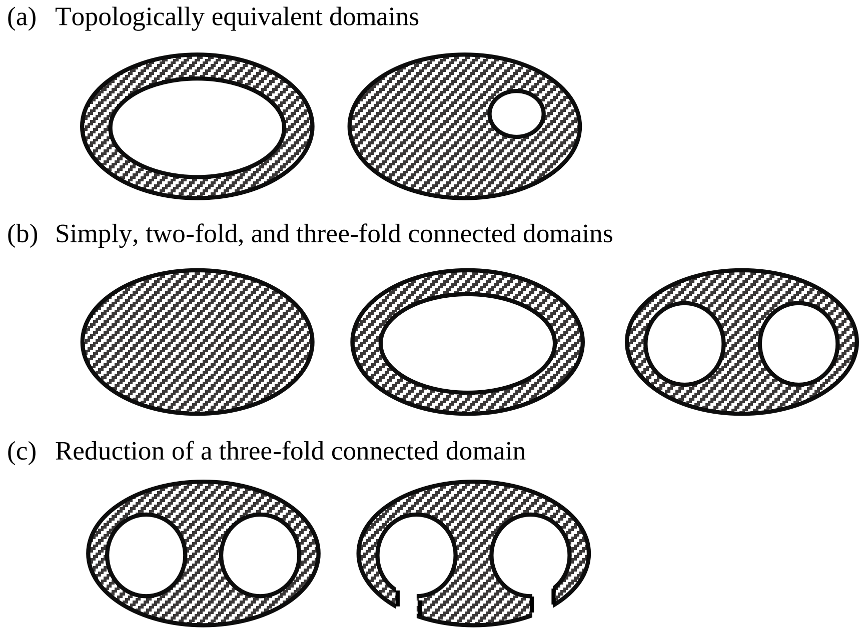

Figure 3 Two-dimensional classes of topological domains (Eschenauer & Olhoff, 2001)

of zero indicating the absence of the material and the value of one indicating the presence of the material within the given design space (Aulig, 2017; Christensen & Klarbring, 2009). Topology optimization of continuum structures is concerned with finding the shape, number, and position of holes as well as their connectivity on a given domain (Petrucci, 2009).

In mathematical language, topology is concerned with objects that are deformable in the elastic range. Deformations are also known as unique mappings or transformations. Topology mapping or transformation is defined as the transformation of one topological domain into another that does not generate new neighbourhoods or destroy the existing relations (Eschenauer & Olhoff, 2001). Therefore, topology can be generally referred to as the invariable theory of topological domains. Put in another way, topological mappings or transformations are the continuous transformations whose transformations are also continuous (Eschenauer & Olhoff, 2001).

There are three different classes of topology, namely; topologically equivalent domains as shown in Figure 3(a), degree of connection demonstrated in Figure 3(b), and n-fold connected class shown in Figure 3(c). The n-fold connected class exists if (n-1) cuts can be performed on a boundary of a domain in order to transform the multiply connected domain into a simple domain (Eschenauer & Olhoff, 2001). A class is generally defined by the degree of connection of the domains.

As noted earlier in this paper, topology mapping is defined as the transformations of one topological domain into another without generating new neighbourhoods or destroying the existing relations. The classical shape and sizing optimization does not generate new neighbourhood relationships as shown in Figure 3(a), where only the shape or size of the domain has been changed without changing the connection of the domains. However, topology optimization seeks to change the neighbourhood relations and therefore the degree of connectivity of the domains as illustrated in Figure 3(c), where a three-fold connected domain is reduced into a simply connected domain. It is for this reason that topology optimization is often considered as a pre-process for the classical shape and sizing optimization (Eschenauer & Olhoff, 2001). Topology optimization of solid structures involves the determination of features such as the number, size, position, shape and location of holes and connectivity of the domains (Bendsøe & Sigmund, 2003).

The goal of topology optimization is to find the optimal lay-out (connectivity of elements) of the structure within a specified design domain. In Figure 2(c), the topology, that is, the shape (geometry) of the structure has been altered by removing material from the original problem shown on the left which resulted in the optimal solution shown on the right. This is the goal of topology optimization. However, for a typical topology optimization problem, the only known quantities in the problem definition are the external loads, the possible support conditions, the design domain (the volume of the structure) to be constructed and possibly some additional design restrictions such as the location and size of prescribed holes or solid areas. Since the design domain has to be given as the entire solid volume of the structure, it then follows that in topology optimization problems, the physical size and shape and connectivity of the structure are unknown (Bendsøe & Sigmund, 2003). Figure 2 also shows that the structural optimization algorithm is based on the concept of gradually removing unnecessary or inefficient material from a structure to achieve an optimal design. The following subsection, subsection 3.1. further discusses the formulation of topology optimization problems.

3.1. FORMULATION OF TOPOLOGY OPTIMIZATION PROBLEMS

A topology optimization problem with the objective of minimizing compliance (C) is given by Equation 2 (Sigmund, 2001; Meijboom, 2003). Compliance is the inverse of stiffness, thus:

$$\min_{x} C(x) = \boldsymbol{U}^T\boldsymbol{K}\boldsymbol{U} = \sum_{e=1}^{N}(x_e)^p \boldsymbol{u}_e^T\boldsymbol{k}_0\boldsymbol{u}_e,$$

$$subject\ to: \frac{V(x)}{V_0} \leq f, \tag{2}$$

$$\boldsymbol{KU} = \boldsymbol{F},$$

$$\mathbf{0} < x_{min} \leq x \leq x_{max} = \mathbf{1}.$$

Here the symbol **U** stands for the global displacement vector, **F** global force vector, **K** global stiffness matrix, $\mathbf{u}_e$ local (element) displacement vector, $\mathbf{k}_0$ local (element) stiffness matrix, x_{min} vector of minimum relative densities (non-zero to avoid singularity), x_{max} vector of maximum relative densities (always greater than x_{min} but up to 1), x_e relative density of the material in the element and can vary between x_{min} and x_{max}, x vector of design variables (vector with the densities of all the elements), N number of finite elements in the domain, p penalty factor (typically = 3) and its roles is to make intermediate densities unfavourable in the optimized solution, $V(x)$ material volume, V_0 design domain (solid volume of the structure), and f allowable volume fraction.

There are several techniques that can be used to solve Equation 2. Amongst the popular ones are the Homogenization, Solid Isotropic Material with Penalization (SIMP), Sequential Linear Programming (SLP) (Sigmund, 2001; Meijboom, 2003), and the Method of Moving Asymptotes (MMA) (Svanberg, 1987). The key to solving this problem is to use the approach of density function. The density function represents the relationship between the Young's modulus (material stiffness) and the density of the material (Gea, 1996). According to the SIMP method, the density function can be expressed as given in Equation 3, thus:

$$\frac{E_e}{E_0} = x_e^p \tag{3}$$

where the symbols E_e and E_0 are the intermediate and the original Young's moduli or moduli of elasticity of the isotropic material, respectively. Another simple approach is to use the Optimality Criteria (OC) method. According to (Bendsøe, 1995), the heuristic updating scheme for the design domains can be expressed as given in Equation 4, thus:

$$x_e^{new} = \begin{cases} \max(x_{min}, x_e - m) & if \quad x_e B_e^{\eta} \leq \max(x_{min}, x_e - m), \\ x_e B_e^{\eta} & if \ \max(x_{min}, x_e - m) \leq x_e B_e^{\eta} < \min(1, x_e + m), \\ \min(1, x_e + m) & if \ \min(1, x_e + m) \leq x_e B_e^{\eta}, \end{cases} \tag{4}$$

where the parameter m is a positive moving limit, $\eta \left(= \frac{1}{2}\right)$ a numerical damping coefficient to stabilize the iteration, and B_e is found from the optimality condition as given in Equation 5, thus:

$$B_e = \frac{-\frac{\partial c}{\partial x_e}}{\lambda \frac{\partial V}{\partial x_e}}, \tag{5}$$

where the symbol λ is a Lagrangian multiplier and that can be found by a bisection algorithm. Topology derivatives often cause the level set function to have several one-node hinges and checkerboard patterns and can lead to numerical errors. A one-node hinge is a node that has two opposite solid elements and two opposite void elements in its neighbourhood and a checkerboard is an area with artificial stiffness (Meijboom, 2003). It is therefore mandatory to use a filtering technique to prevent these sensitivities (checkerboard patterns). The sensitivity of the objective function with respect to the design variables is found by calculating the derivative of the minimum compliance c with respect to the relative density of the material in the element, x_e (Sigmund, 2001) as given in Equation 6, thus:

$$\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1}\boldsymbol{u}_e^T\boldsymbol{k}_0\boldsymbol{u}_e \tag{6}$$

The sensitivities are weighted according to the difference between a chosen averaging radius $R \geq 1$ and the distance r_j from the centre of element i to the centre of element j times the density of the element x_e.

The following subsection, subsection 3.2. presents some pertinent works concerning the application of optimization, particularly the structural topology optimization technique, in the automotive industry.

3.2. OPTIMIZATION OF AUTOMOTIVE BODY STRUCTURES

The search of lightweight but stiff body structures can be found as early as the 1930s, by Swallow (1939), where at least 16% in structural weight saving was achieved while at the same time the torsion stiffness was increased by at least 50%. This was achieved by substituting the separate chassis-frame construction of a body structure by a fully unitary construction, while the vehicles were still largely identical. Although no specific optimization technique was applied, the study by the author demonstrated the use of structural optimization and showed how a lightweight but stiff body structure can be achieved by changing the connectivity of the individual elements of the body structure. A study by Fenyes (1981) used structural optimization to investigate the potential reduction of mass by using various materials on a simplified model of the body structure shown in Figure 4. The author used an optimization program to design the body structure and to find the best material for each component of the body structure. The

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thickness or the gauge of the components was taken as the variable, which led to sizing optimization. The materials that were considered were aluminium, glass/epoxy, graphite/epoxy, mild steel, and HSLA steel. The author found that it was possible to develop a body structure that weighed 80 kg by combing steel and 27 kg of aluminium materials. This weight was only 2 kg heavier than the all-aluminium design of the body structure. Although the combination of steel and aluminium resulted in a significant weight savings compared to an all steel body, the challenge with the optimization technique used by the author is that the technique can only be used on a body structure that has a known geometry or topology. This is consistent with the structural sizing optimization, since this technique can only be used later during the design process for fine tuning of the final geometry.

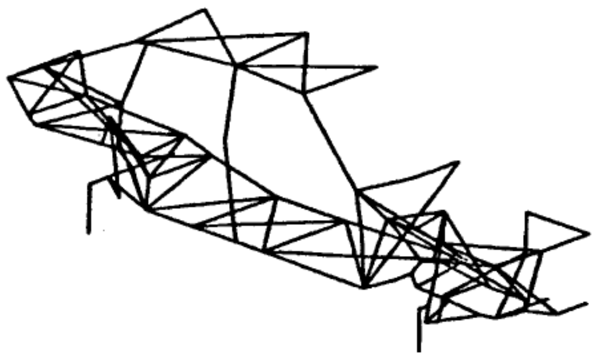

Figure 4. Simplified beam model of a body structure (Fenyes, 1981)

A study by Lotus Engineering (2010) showed that 42% in weight savings can be achieved at a cost that is 35% higher when the all steel construction of the body structure is replaced by a body structure that utilizes aluminium, magnesium and composite materials and a 2% contribution from high strength steel. This, shows that substituting steel with lightweight materials will result in additional costs to the automaker (Bjelkengren, 2008), and therefore, an increase in the overall cost of a vehicle, which will then have an impact on the customers. However, the study by (Lotus Engineering, 2010) further showed that 38% of the total weight of the vehicle can be reduced at an extra cost of just 3% by using synergistic, total vehicle approach to reducing weight, even if all steel construction of the body structure is replaced by a body structure that utilizes lightweight materials. Garud *et al.* (2018) studied the deflections, stresses and modal behaviour of a ladder frame chassis using five different materials. The materials in consideration were steel, advanced high strength steel (AHSS), aluminium, titanium as well as a carbon fibre reinforced polymer. The wall thickness was varied from 3 mm to 12 mm while the geometry was varied to accommodate between 4 and 5 cross members. The study found that a ladder frame chassis that was formed out of AHSS performed better than its counterparts. The authors further showed that the weight of the chassis frame can be reduced from 173.3 kg to 162.5 kg by changing the cross-sections of side cross members to C-sections and T-sections. The challenge with the work by Garud *et al.* (2018) is similar to that of Fenyes (1981) in that, the connectivity or actual geometry of the body structure should be known before such optimization steps can be undertaken.

Sakurada *et al* (1993) performed structural optimization of the model of the underbody of the body structure shown in Figure 5. The objective of this study was to minimize the weight of the underbody structure while improving its torsion stiffness. The authors managed to reduce the weight of the underbody structure by 6% and the torsion stiffness was improved by 10%. The authors used sizing optimization to achieve these results and the thickness of the panels was taken as the design variable. Although an improvement of 10% on the torsion stiffness was achieved, this type of optimization is often applied on existing structures or during the later stages of the design process to fine tune the product geometry.

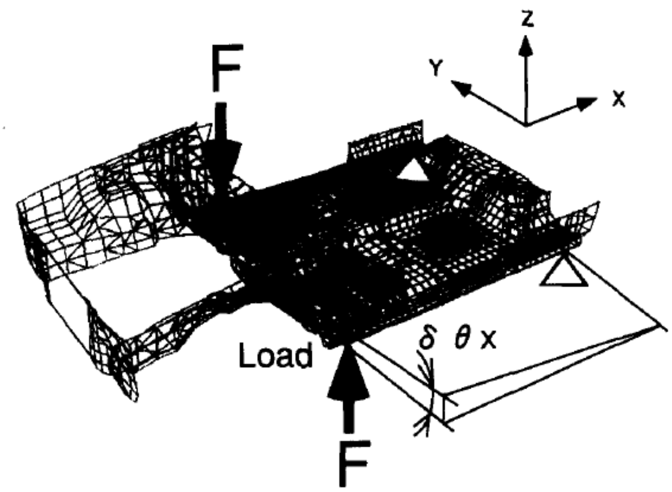

Figure 5. Model of an underbody structure (Sakurada, et al., 1994)

The ensuing material focuses on topology optimizations of body structures that were undertaken by Fukushima, et al., (1992); Reed, (2002); Quinn, (2010); Cavazzuti, et al., (2010); Cavazzuti, et al., (2011); Yang, et al., (2012); Bastien, et al., (2012); and Tian & Gao, (2016) for the purpose of minimising the mass of these body structures. Figure 6 shows the design space with the applied loads (a) and the optimized frame structure (b) used by Fukushima *et al*. (1992). This figure, Figure 6, shows a two-dimensional underbody structure of passenger vehicles that was optimized under six different crash load conditions.

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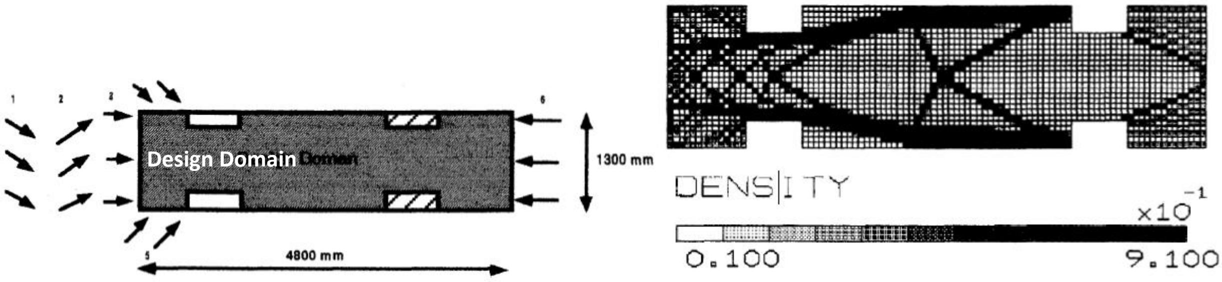
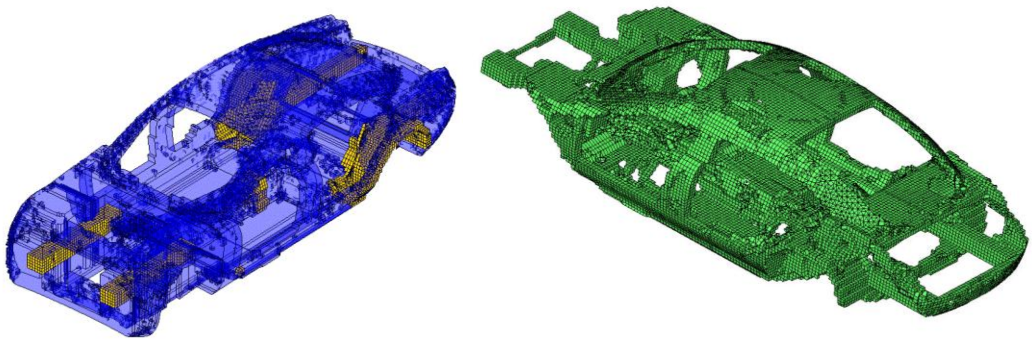


(a) Design space and applied loads (b) Optimized frame structure

Figure 6. Configuration of the (a) design domain and applied loads, and (b) optimized frame structure (Fukushima, et al., 1992)

Although Fukushima *et al.* (1992) concluded that this frame structure has a larger bending stiffness than the usual conventional structure, the topology optimized structure was not interpreted and analysed to support this conclusion. However, the study demonstrated how the topology optimization technique that is based on the homogenization method can be applied to achieve lightweight designs of two-dimensional optimization body structures.

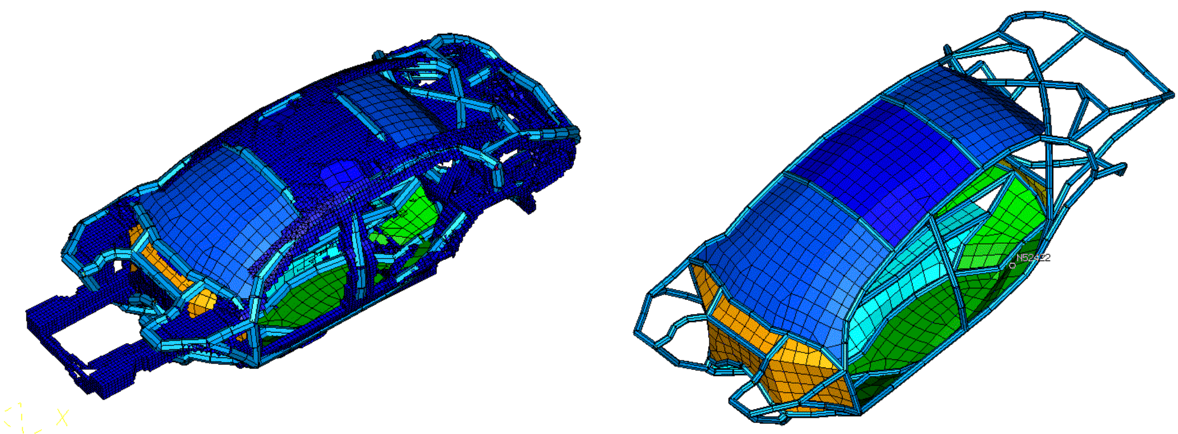
Reed (2002) undertook topology optimization on the body structure shown in Figure 7(a) with eight different loading conditions considered in this study.



(a) Design space (b) Optimized structure

Figure 7. Configuration of the (a) design space, and (b) optimized body structure (Reed, 2002)

The optimized structure in Figure 7(a) was interpreted into the model that was made of beam and shell elements shown in Figure 8(a). The model was further refined to the beam and shell model as shown in Figure 8(b).

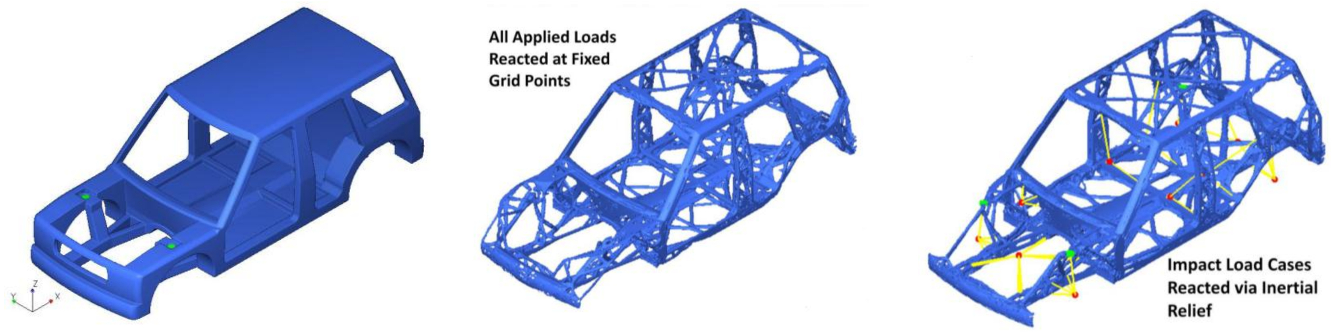


(a) Beam and shell topology results (b) Beam and shell model

Figure 8. Configuration of (a) Beam and shell topology results, and (b) beam and shell model (Reed, 2002)

Reed (2002) further performed section and gauge optimization of the beam and shell model that is shown in Figure 8(b). It was concluded in the study that the stiffness of the final optimized structure weighed 100 kg and was 35000 Nm/deg. This value of stiffness is extremely high for the given mass unless the body structure was made of very strong lightweight materials. Typical values for modern unitized body structures are approximately 8000 to 10000 Nm/deg for typical passenger vehicles and higher around 12000 to 20000 Nm/deg. for luxury vehicles and these vehicles have structural mass combined with the mass of the body and subsystems of approximately 870 kg (Brown, et al., 2002; Happian-Smith, 2002; Pang, 2019).

A study by Quinn (2010) used topology optimization on a full vehicle FEM to determine the alternative critical load paths on the body structure shown in Figure 9(a). Here, ten different static load cases with different boundary conditions were applied simultaneously. The analysis was carried out without (Figure 9b) and with (Figure 9c) inertial relief. Inertia relief is an option that is used to perform static FEA on the FE models that are not constrained. This problem took approximately two days to solve. Although the purpose of the study was to determine the critical load paths on the body structure, the study made no mention of the stiffness and the mass of the optimized body structure, even though the objective of structural optimization problems is to determine the structure that has the best possible stiffness with as little mass as possible. However, the results from this study demonstrated how a body-in-white (BIW) conceptual model can be obtained using topology optimization. The study further showed how static analysis can be performed using inertia relief to obtain a stiffer design when compared to the results of static analysis performed under contrived constraints.



(a) Design space (b) Without inertia relief (c) With inertia relief

Figure 9. Configuration of the (a) design space, (b) optimized body structure without inertia relief, and (c) optimized body structure with inertia relief (Quinn, 2010)

Studies by Cavazzuti *et al.* (2010) and Cavazzuti *et al.* (2011) used topology optimization in succession with topometry and size optimization to find the most efficient material layout of the chassis framework of the design space shown in Figure 10.

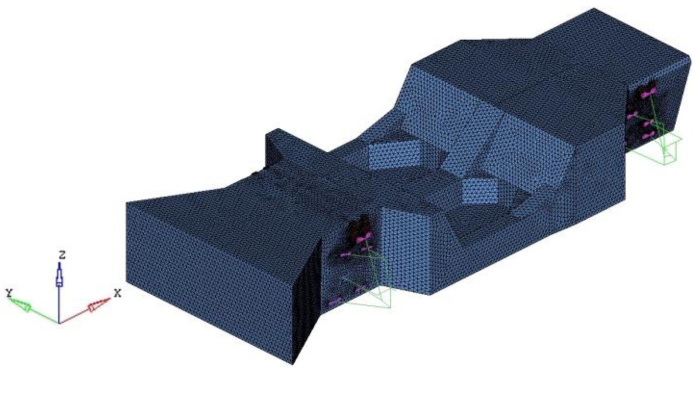
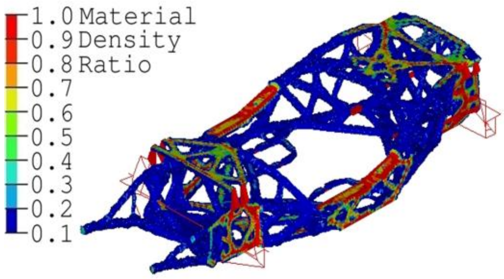
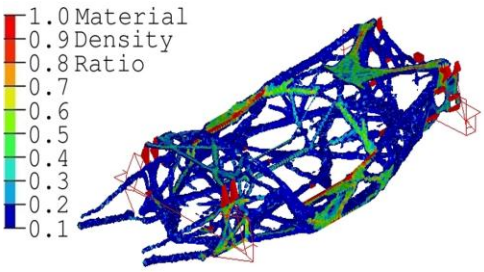


Figure 10. Design space of the topology optimization (Cavazzuti, et al., 2010)



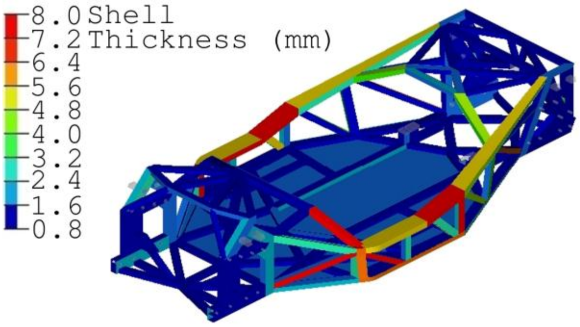
(a) With clamped sills



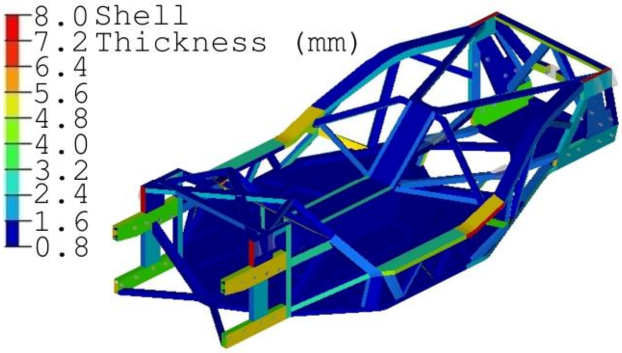
(b) With inertia relief

Figure 11. Configuration of the optimized chassis frames (a) with clamped sills and (b) with inertial relief (Cavazzuti, et al., 2010)

The optimization results of this work shown in Figure 11 were first manually interpreted into CAD models before performing topometry and sizing optimization, consecutively. The optimized chassis frames after topometry and size optimization are shown in Figure 12.



(a) With clamped sills



(b) With inertia relief

Figure 12. Size optimization results of the model with (a) clamped sills and (b) inertia relief (Cavazzuti, et al., 2010)

Five different load cases were considered during the optimization process and two separate optimization processes undertaken under these load cases. One optimization was with inertia relief while the other was with clamped sills. The results of topology optimization are shown in Figure 11. The authors concluded that the material layout on this study resulted in a structure that had a significant weight reduction when compared to the existing chassis frame of a Ferrari F458 Italia model (Cavazzuti, et al., 2010; Cavazzuti, et al., 2011). However, the stiffness and stiffness to weight ratio of these structures were not studied even though the stiffness to weight ratio is a parameter that is of paramount importance during the preliminary stages of the design process and determines the weigh penalty that can result in increasing or reducing the stiffness of body structures (Matsimbi, et al., 2020).

The study by Yang *et al.* (2012) used topology optimization to design a body structure of a parallel hybrid electric vehicle (HEV). Three load cases were considered in this study. The design space of this study is shown in Figure 13 and the three optimized models developed shown in Figure 14.

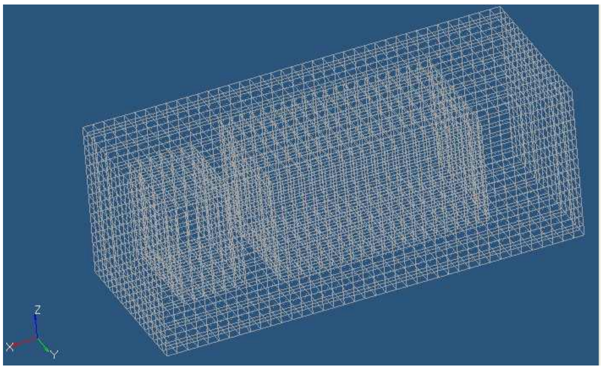
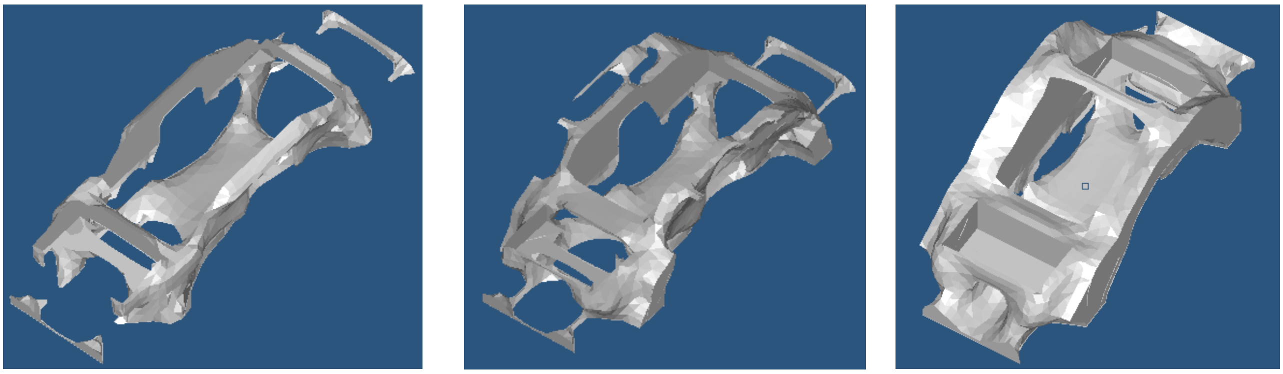


Figure 13. Topology design space (Yang, et al., 2012)

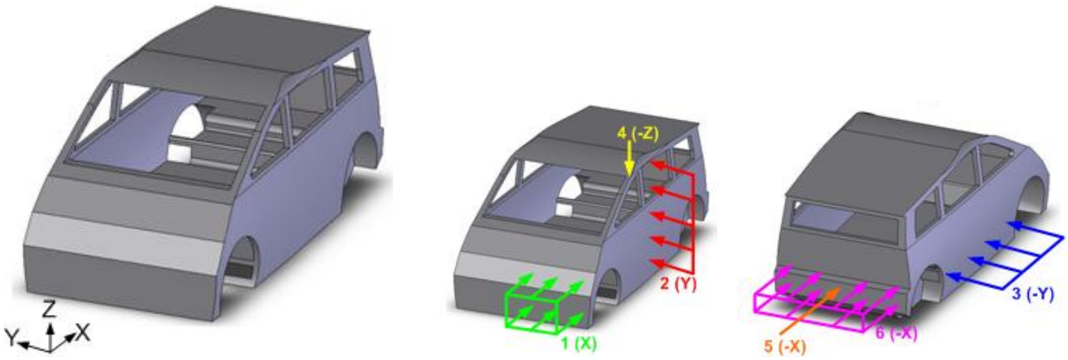


(a) vol = 20% (b) vol = 30% (c) vol = 40%

Figure 14. Topology optimization results with different volume constraints (Yang, et al., 2012)

The objective of the study was to develop a parallel BIW of a HEV under different load cases as well as various volumetric constraints. The volumetric constraints were varied between 20% and 40%. The study considered the static loads, torsion load during turning as well as the moment load during braking of the vehicle. Although the study made mention of the bending and torsion stiffness, these results were not interpreted into a CAD model that can be used to perform further studies using the FEA.

Topology, shape and sizing optimization were used by Bastien *et al.* (2012) to design an optimal body structure. In this study, six different load cases were considered, concurrently. The material that was used in this study was a standard steel grade with a Young's modulus or modulus of elasticity of 210 GPa, a Poisson's ratio of 0.3 and a mass density of 7850 kg/m^3. The design space as well as the corresponding load scenarios in this case are as shown in Figure 15. The optimized body structure is shown in Figure 16. No further studies on the body structure were performed in this study. However, further optimization studies were conducted on the front crash structure though there was no study conducted to interpret or determine the performance behaviour of the body structure.



(a) Design space (b) Load cases (c) Load cases

Figure 15. Configuration of the (a) design space, and load cases (b) and (c) (Bastien, et al., 2012)

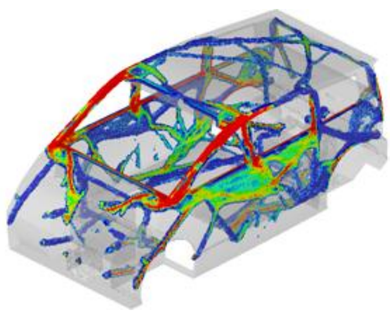
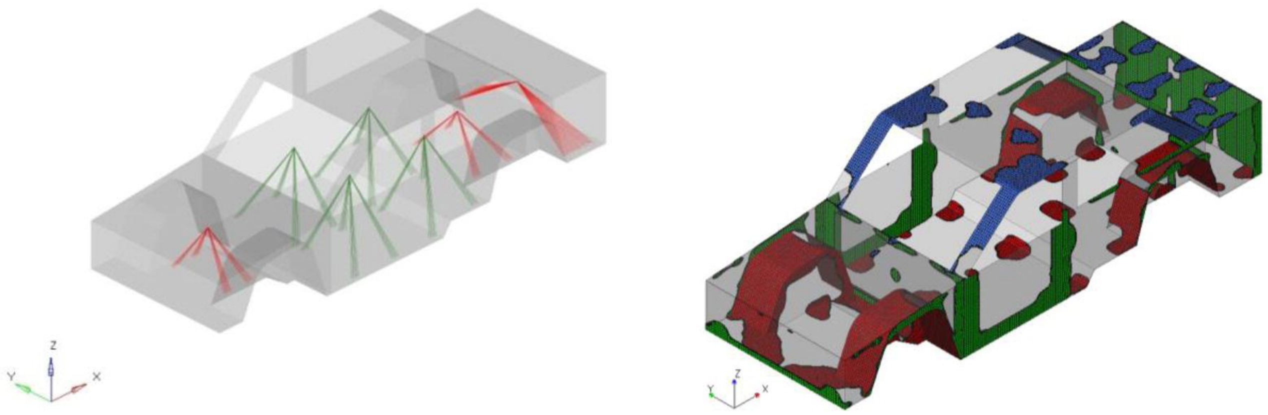


Figure 16. Topology optimisation results (Bastien, et al., 2012)

The topology optimization results from the studies by Bastien *et al.* (2012) are unconventional when compared to the modern vehicle structures due to the roof panel that was replaced by a triangulated lattice like structure. However, the results can be used to give recommendations of other panels, such as bracings that are required on the panels of doors.

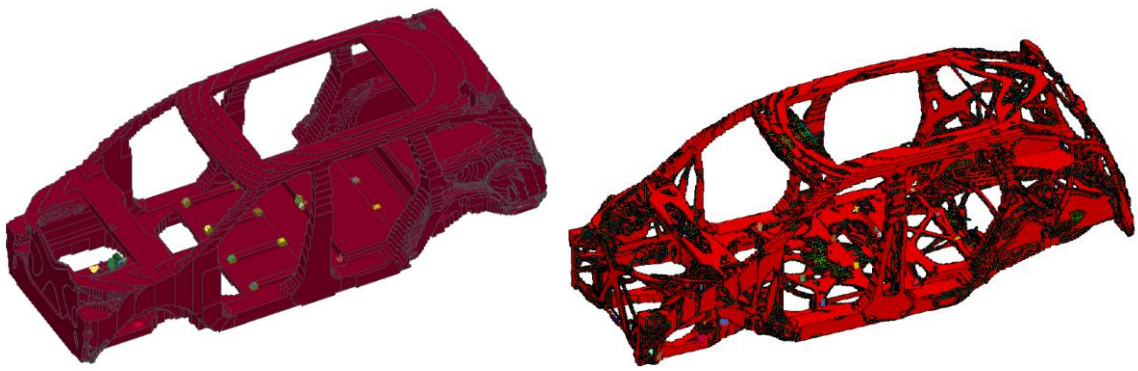
Topology optimization was used to design a body structure for crashworthiness by Tian and Gao (2016). Four crash load cases, frontal, side, and rear impact as well as roof crush were considered in this study. The study was carried out in two parts, the first part considered each load case separately, and the other part considered multiple load cases concurrently. Figure 17 shows both the design space and optimized body structure. The design space is shown in Figure 17(a). The objective of the study was to determine the most efficient load paths under crash conditions. The optimized body structure is shown in Figure 17(b). The authors concluded that the of results of the optimized structure under a single load case could seldomly satisfy the other load cases. Therefore, it is important to carry out topology optimization under multiple load cases in order to satisfy all load cases. Although the optimization results of a body structure carried out under multiple load cases can be used to provide the conceptual BIW model of the vehicle that satisfy all load cases. The authors did not interpret the optimal structure into a useful geometry that can be used to perform FEA.



(a) Design space (b) Optimized structure

Figure 17. Configuration of the (a) design space and (b) optimized structure (Tian & Gao, 2016)

Aulig *et al.* (2016) applied topology optimization to a realistic body structure with the objective of improving the stiffness and the crashworthiness of the structure. Eleven load cases were considered in this study. Initially, two crash load cases namely; front and rear crash loads were considered, concurrently. Thereafter, nine static load cases, divided between the seat, front, and rear were considered, simultaneously. Figure 18 shows both the design space and the optimized body structure. The design space is shown in Figure 18(a) and the corresponding optimized structure as shown in Figure 18(b). The purpose of the study was to investigate the scalability of the Hybrid Cellular Automata (HCA) method by evaluating the optimization of a body structure and showing the possibility of finding trade-off when considering the loads of each element of the body structure.



(a) Design space (b) Optimized structure

Figure 18. Configuration of the (a) design space and (b) optimized structure (Aulig, et al., 2016)

Although the study successfully applied the HCA topology optimization technique to propose the conceptual layout of a body structure under both stiffness and crash load cases. This study did not interpret the results of topology optimization into a CAD model that can be used for further analysis of the body structure. However, the study demonstrated that the best trade-off of optimization results can be achieved by considering both stiffness and crash load cases concurrently rather than considering either stiffness or crash load cases, separately.

4. DISCUSSION AND SUMMARY

One of the major challenges in the automotive industry is to find a compromise between the weight and stiffness of body structures. The weight of a vehicle is a major determinant of fuel economy and also affects the performance characteristics of vehicles such as acceleration, deceleration as well as handling and has a direct influence on the wearing of components such as tyres, brakes, suspensions, engines, and transmission systems due to inertia. The stiffness on the other hand, is a major determinant of the safety and comfort as well as performance characteristics of vehicles. However, weight and stiffness are not mutually exclusive, since reducing the weight of a body structure is most likely to be accompanied by the reduction in its stiffness, which in turn compromises the structural integrity of the body structure. This is a contradicting objective that both designers and manufacturers alike must find a compromise for. This compromise can be achieved more easily now than in the past by making use of different materials that are now available at realistic costs than was the case a few years ago. However, different materials have different material properties, making each material more suitable for certain applications than others. This then necessitates the need to use optimization techniques to find the optimal material for each subassembly of body structures. The sizing optimization technique can be used to find the optimal size of each subassembly for a given material. However, the challenge with sizing optimization is the fact that this technique cannot be used during the conceptual stages of the design process since it is often applied during the advanced stages of the design process to fine tune the product geometry. The development of body structures usually accounts for a large proportion of the development time as well as manufacturing cost in a new vehicle program. Therefore, it is essential to ensure that optimal body structures are determined during the early stages of the design process to minimise changes and cost during the advanced stages of the design process and to ensure short overall turnaround time.

Increasingly, researchers are employing structural topology optimization techniques during the early stages of the design process to find optimal concepts of vehicle body structures. However, there is still a very limited amount of literature on the development of new vehicle body structures using topology optimization. It is evident in all the studies reviewed here that; the load cases applied during the topology optimization process were more than one. It is not clear, therefore, what differences in the optimised structure would arise when each one of the single load cases was considered separately. Moreover, though two of the eight reviewed articles interpreted the results of topology optimization into CAD geometries that can be used for further Finite Element Analysis (FEA), no FEA studies were conducted for these two, which minimise their utility. The observations by Cavazzuti *et al.* (2010) and Bastien *et al.* (2012) that the interpretation of the topology results is crucial and that it can be challenging and requires experience is underscored. It cannot, from the review conducted here, be determined if other modelling techniques such as the simple structural surfaces method can be used to guide the interpretation of the results of topology optimization.

5. CONCLUSION

In search of methods such as topology optimization used to develop vehicle body structurers that are lightweight and stiff enough in the automotive industry, the following conclusion arise from the current survey:

(1) Different materials provide alternative options of developing lightweight body structures. However, every material has certain properties, which make it more suitable for some applications than others. Moreover, the use of different lightweight materials in body structures may result in additional costs to the automaker and therefore an increase in the overall cost of the vehicle. Therefore, the approach to achieving lightweight body structures should not only focus on making use of different materials on the existing body structures. Rather, a systematic design approach that focuses on reducing the overall weight of vehicles taking advantage of the unique attributes of specific lightweight materials should be adopted.

(2) Structural optimization techniques are increasingly used to develop lightweight body structures. However, there are certain structural optimization techniques such as sizing and shape parameterizations that are only used during the advanced stages of the design process for fine tuning the product geometry. It is noted though that product geometries are not available during the initial stages of the development of new vehicle structures. This reduces the usefulness of these methods in proposing alternative concepts of new vehicle structures. Structural topology optimization techniques are currently used generally during the conceptual stages of the design process to search and propose alternative structural layouts of body structures. There is a limited amount of literature available that studies the use of structural topology optimization in the development of new vehicle body structures. Furthermore, all studies reviewed here showed that the load cases applied in the topology optimization design were more than one. There is, however, no clarity on what differences of the optimised structure would arise when each one of the single load cases was considered separately.

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