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Rough Set Approach for Analyzing the Effect of Viscoelastic and Micropolar Parameters on Hiemenz Flow in Hydromagnetics

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ABSTRACT

This research describes the hydromagnetic problem of two dimensional Hiemenz flow for a micropolar, viscoelastic, incompressible, viscous, electrically conducting fluid, impinging perpendicularly onto a plane in the presence of a transverse magnetic field. An approach based on the rough set theory is introduced where the mathematical model which describes the problem is first transformed into a dimensionless form. Then it is solved by using the Runge–Kutta numerical integration procedure in conjunction with shooting technique. Finally a set of maximally generalized decision rules (classification rules) are generated by using rough sets methodology.

Keywords: Viscoelastic fluids; Micropolar fluids; Hiemenz Flow; Rough set theory; feature selection; Hydromagnetics.

NOMENCLATURE

$\bar{u}$ and $\bar{v}$	Velocity components along x and y axes
X and y	Dimensionless velocity component in the x- and y-direction
N	angular velocity
ν	Kinematic viscosity
σ	electrical conductivity of fluid
K^*	weissenberg number
B	induced magnetic field
a	constant
μ	Dynamic viscosity
K	vortex viscosity
γ	spin gradient viscosity
j	microinertia per unit mass
k_0	viscoelastic parameter
M	Hartman Number

1) INTRODUCTION

In recent past the attention of many scientists was attracted to viscoelastic fluids due to application of this kind of fluids in industrial engineering, chemical industries, biomedical engineering, paints, polymers and technological applications since this type of fluids retains old distortions and its new behavior depends on previous distortions due to its "elastic" nature. Beard and Walters [1] developed the first model which describe and simulate the viscous fluids then many scientists and engineers studied and analyzed the flow and heat transfer characteristics of viscoelastic fluids as a type of non-Newtonian fluids [2-10].

Hiemenz was the first one who studied the two dimensional flow of a fluid near a stagnation point and show that the governing equations which describe the flow can be reduced to an ordinary differential equation with the aid of similarity transformation [11] then Several studies were elaborated by researchers to study Hiemenz flow in different ways to include various physical effects in hydromagnetics [12-14]. Also, Micropolar fluids are introduced by Eringen [15] and he characterized the structure of these fluids and define it physically as it consist of rigid, randomly oriented (or spherical) particles suspended in a viscous medium and theses particles can rotate with their own spins and microrotations, since the deformation of fluid particles is ignored. Then Eringen [16] extended his investigation of micropolar elasticity and many researchers and engineers focus their efforts in studying micropolar fluids as it has a great role in industrial applications Examples include exotic lubricants, food industry, biological and bio-medical sciences. For excellent review see [17-19].

The problem of reducing has been investigated for many numerous applications in different fields, since the irrelevant and redundant features in the dataset lead to low accuracy. There are two main approaches to reduce the input dimensionality, namely feature extraction and feature selection. Rough set theory was used as a tool to reduce the dimensionality as well as dealing with uncertainty in datasets. Many heuristic algorithms are proposed based on rough set theory, also numerous approached based on rough set theory and other theory are investigated to extract decision rules and reduce the dimensionality of dataset [20-30].

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In this paper, we consider the effect of a transverse magnetic field on the Hiemenz flow (the two dimensional flow near a stagnation point) of micropolar viscoelastic fluids. The governing Equations are solved by using the Runge–Kutta numerical integration procedure in conjunction with shooting technique. We present numerical results for a range of values of the Hartman number, of the viscoelastic parameter, and of the material properties of the fluid. Besides, the outcomes are elaborated graphically for involved variables.

2) MATHEMATICAL FLOW MODEL

Let us consider two-dimensional flow of a viscous, incompressible, electrically conducting, micropolar, viscoelastic fluid impinging perpendicularly onto a plane directed along the *x*-axis, as shown in Fig. 1. The flow is embedded in a uniform magnetic field of constant strength H_0.

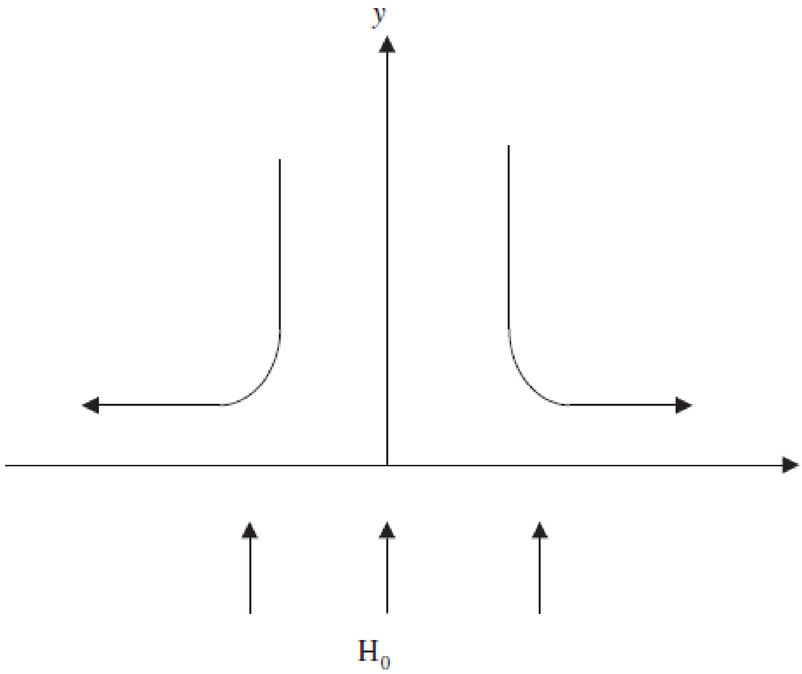

Fig. 1. Flow model and coordinate system

The governing equations which describe the mathematical model for this problem take the form [33]:

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}=a^2x+\left(\nu+\frac{k}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}+ \frac{\sigma B^2}{\rho}\left(ax-\bar{u}\right)-k^*\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial x\,\partial y^2}+\bar{v}\frac{\partial^3 \bar{u}}{\partial y^3}+\frac{\partial \bar{u}}{\partial x}\frac{\partial^2 \bar{u}}{\partial y^2}-\frac{\partial \bar{u}}{\partial y}\frac{\partial^2 \bar{u}}{\partial x\,\partial y}\right) \tag{2}$$

$$\bar{u}\frac{\partial N}{\partial x}+\bar{v}\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{k}{\rho j}\left(2N+\frac{\partial \bar{u}}{\partial y}\right), \tag{3}$$

According to [33] Boundary conditions for the for stagnation-point flow are as follows:

$$\begin{aligned} &y=0: \quad \bar{u}=0, \quad \bar{v}=0, \quad N=-m\frac{\partial \bar{u}}{\partial y} \\ &y\to\infty: \quad \bar{u}=ax, \quad N=0 \end{aligned} \tag{4}$$

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The partial differential conservation equations (1)-(4) are thereby converted into dimensionless system by defining the stream function ψ as:

$$\bar{u} = \frac{\partial \psi}{\partial y} \qquad \bar{v} = -\frac{\partial \psi}{\partial x} \tag{5}$$

And the following non-dimensional variables are introduced:

$$\eta = \sqrt{\frac{a}{\bar{u}}}\, y, \qquad \psi = \sqrt{a\bar{v}}\, x f\left(\eta\right), \qquad N = \sqrt{\frac{a}{\bar{v}}}\, a x g\left(\eta\right) \tag{6}$$

Also to facilitate numerical solutions for low values of the viscoelastic parameter k_0, the following formalization expressions are used:

$$\begin{aligned} f &= f_0 + k_0 f_1 + k_0^2 f_2 + \ldots\ldots\ldots\ldots\ldots \\ g &= g_0 + k_0 g_1 + k_0^2 g_2 + \ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{7}$$

The final mathematical model was obtained as:

$$\left(1+\Delta\right) f_0''' + \Delta g_0' + f_0 f_0'' + 1 - f_0'^2 + M^2\left(1 - f_0'\right) = 0 \tag{8}$$

$$\lambda_0 g_0'' - \Delta.B_1\left(2g_0 + f_0''\right) - f_0' g_0 + g_0' f_0 = 0 \tag{9}$$

$$\left(1+\Delta\right) f_1''' + \Delta g_1' + f_0 f_1'' + f_1 f_0'' - 2f_0' f_1' - M^2 f_1' - \left(2f_0' f_1''' - f_0 f_0^{iv} - f_0''^2\right) = 0 \tag{10}$$

$$\lambda g_1'' - \Delta.B_1\left(2g_1 + f_1''\right) - f_0' g_1 - f_1' g_0 + g_0' f_1 + g_1' f_0 = 0 \tag{11}$$

$$\begin{aligned} &\left(1+\Delta\right) f_2''' + \Delta g_2' + f_0 f_2'' + f_2 f_0'' + f_1 f_1'' - 2f_0' f_2' - f_1'^2 - M^2 f_2' \\ &\qquad - \left(2f_0' f_1''' + 2f_1' f_0''' - f_0 f_1^{iv} - f_1 f_0^{iv} - 2f_0'' f_1''\right) = 0 \end{aligned} \tag{12}$$

$$\lambda g_2'' - \Delta.B_1\left(2g_2 + f_2''\right) - f_0' g_2 - f_1' g_1 - f_2' g_0 + g_0' f_2 + g_1' f_1 + g_2' f_0 = 0 \tag{13}$$

Subject to the boundary conditions:

$$\begin{aligned} &f_0(0) = f_1(0) = f_2(0) = f_0'(0) = f_1'(0) = f_2'(0) = 0 \\ &f_0'(\infty) = 1, \quad f_1'(\infty) = f_2'(\infty) = 0 \\ &g_0(0) = -m f_0''(0), \qquad g_1(0) = -m f_1''(0), \qquad g_2(0) = -m f_0''(0) \\ &g_0(\infty) = g_1(\infty) = g_2(\infty) = 0 \end{aligned} \tag{14}$$

3) ANALYSIS

The system of equations (8)-(13) subject to the boundary condition in (4) was solved by the aid of Runge–Kutta numerical-integration procedure in conjunction with a shooting technique. The results of these calculations are divided into two parts. The first part represents the case of viscoelastic fluids where the surface values $f''\left(0\right)$ of the velocity gradient at $\Delta = 0$ are shown in table 1. While the

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second part represents the case of micropolar viscoelastic fluid ,where the surface values $f''(0)$ of the velocity gradient and the surface values $g'(0)$ of the microrotation gradients for various nonzero values of Δ are shown in table 2.

Table 1. Values of $f''(0)$ at $\Delta = 0$ for various values of M and k0, for a second-order viscoelastic fluid.

U	M	K_0	$f''(0)$
X1	0.0	0	1.23259
X2	0.0	0.025	1.26297
X3	0.0	0.05	1.29336
X4	0.2	0	1.24857
X5	0.2	0.025	1.27933
X6	0.2	0.05	1.31009
X7	0.4	0	1.29537
X8	0.4	0.025	1.32725
X9	0.4	0.05	1.35913
X10	0.6	0	1.36988
X11	0.6	0.025	1.40360
X12	0.6	0.05	1.43732
X13	0.8	0	1.46798
X14	0.8	0.025	1.50423
X15	0.8	0.05	1.54048
X16	1.0	0	1.58533
X17	1.0	0.025	1.62475
X18	1.0	0.05	1.66418
X19	1.2	0	1.71804
X20	1.2	0.025	1.76124
X21	1.2	0.05	1.80444
X22	1.4	0	1.86285
X23	1.4	0.025	1.91038
X24	1.4	0.05	1.95790
X25	1.6	0	2.01715
X26	1.6	0.025	2.06952
X27	1.6	0.05	2.12189
X28	2.0	0	2.34666
X29	2.0	0.025	2.41013
X30	2.0	0.05	2.47361
X31	3.0	0	3.24095
X32	3.0	0.025	3.33951
X33	3.0	0.05	3.43806
X34	5.0	0	5.14796
X35	5.0	0.025	5.34429
X36	5.0	0.05	5.54062
X37	10.0	0	10.07474
X38	10.0	0.025	10.63314
X39	10.0	0.05	11.25077

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Table 2. Values of $f''(0)$ and $g'(0)$ for various values of M , k0 and Δ , for a second-order viscoelastic fluid.

U	M	K_0	Δ	$f''(0)$	$-g'(0)$
X1	0.0	0	0.5	1.00365	0.04813
X2	0.0	0.025	0.5	1.01735	0.04844
X3	0.0	0.05	0.5	1.03106	0.044876
X4	0.2	0	0.5	1.01669	0.04876
X5	0.2	0.025	0.5	1.03056	0.04834
X6	0.2	0.05	0.5	1.04444	0.04865
X7	0.4	0	0.5	1.05489	0.04897
X8	0.4	0.025	0.5	1.06927	0.04895
X9	0.4	0.05	0.5	1.08365	0.04927
X10	0.6	0	0.5	1.11573	0.04959
X11	0.6	0.025	0.5	1.13094	0.04989
X12	0.6	0.05	0.5	1.14615	0.05021
X13	0.8	0	0.5	1.19581	0.05054
X14	0.8	0.025	0.5	1.21216	0.05107
X15	0.8	0.05	0.5	1.22851	0.05140
X16	1.0	0	0.5	1.29164	0.05173
X17	1.0	0.025	0.5	1.30942	0.05241
X18	1.0	0.05	0.5	1.32720	0.05308
X19	1.2	0	0.5	1.40001	0.05383
X20	1.2	0.025	0.5	1.41949	0.05418
X21	1.2	0.05	0.5	1.43897	0.05452
X22	1.4	0	0.5	1.51827	0.05529
X23	1.4	0.025	0.5	1.53969	0.05564
X24	1.4	0.05	0.5	1.56112	0.05599
X25	1.6	0	0.5	1.64429	0.0673
X26	1.6	0.025	0.5	1.66789	0.05710
X27	1.6	0.05	0.5	1.69148	0.05746
X28	2.0	0	0.5	1.91341	0.05950
X29	2.0	0.025	0.5	1.94199	0.05989
X30	2.0	0.05	0.5	1.97057	0.06028
X31	3.0	0	0.5	2.64380	0.06544
X32	3.0	0.025	0.5	2.68810	0.06588
X33	3.0	0.05	0.5	2.73241	0.06633
X34	5.0	0	0.5	4.20127	0.07352
X35	5.0	0.025	0.5	4.28933	0.07406
X36	5.0	0.05	0.5	4.37739	0.07459
X37	10.0	0	0.5	8.22464	0.08223
X38	10.0	0.025	0.5	8.48811	0.08299
X39	10.0	0.05	0.5	8.75158	0.08375

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U	M	K_0	Δ	$f''(0)$	$-g'(0)$
X40	0.0	0	1.5	0.76688	0.12087
X41	0.0	0.025	1.5	0.77215	0.12129
X42	0.0	0.05	1.5	0.77741	0.12172
X43	0.2	0	1.5	0.77691	0.12151
X44	0.2	0.025	1.5	0.78224	0.12194
X45	0.2	0.05	1.5	0.78757	0.12236
X46	0.4	0	1.5	0.80632	0.12336
X47	0.4	0.025	1.5	0.81185	0.12379
X48	0.4	0.05	1.5	0.81737	0.12422
X49	0.6	0	1.5	0.85321	0.12622
X50	0.6	0.025	1.5	0.85906	0.12666
X51	0.6	0.05	1.5	0.86490	0.12710
X52	0.8	0	1.5	0.91502	0.12984
X53	0.8	0.025	1.5	0.92130	0.13029
X54	0.8	0.05	1.5	0.92758	0.13074
X55	1.0	0	1.5	0.98905	0.13397
X56	1.0	0.025	1.5	0.99587	0.13444
X57	1.0	0.05	1.5	1.00269	0.13490
X58	1.2	0	1.5	1.07286	0.13838
X59	1.2	0.025	1.5	1.08032	0.13886
X60	1.2	0.05	1.5	1.08778	0.13934
X61	1.4	0	1.5	1.16438	0.14291
X62	1.4	0.025	1.5	1.17258	0.14341
X63	1.4	0.05	1.5	1.18077	0.14391
X64	1.6	0	1.5	1.26195	0.14745
X65	1.6	0.025	1.5	1.27096	0.14797
X66	1.6	0.05	1.5	1.27995	0.14848
X67	2.0	0	1.5	1.47042	0.15624
X68	2.0	0.025	1.5	1.48129	0.15679
X69	2.0	0.05	1.5	1.49217	0.15735
X70	3.0	0	1.5	2.03655	0.17598
X71	3.0	0.025	1.5	2.05329	0.17604
X72	3.0	0.05	1.5	2.07002	0.17669
X73	5.0	0	1.5	3.24417	0.20237
X74	5.0	0.025	1.5	3.27708	0.20318
X75	5.0	0.05	1.5	3.30999	0.20400
X76	10.0	0	1.5	6.36472	0.23245
X77	10.0	0.025	1.5	6.46101	0.23370
X78	10.0	0.05	1.5	6.55830	0.23494

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U	M	K_0	Δ	$f''(0)$	$-g'(0)$
X79	0.0	0	5	0.47168	0.25896
X80	0.0	0.025	5	0.47284	0.25933
X81	0.0	0.05	5	0.47400	0.25970
X82	0.2	0	5	0.47765	0.26071
X83	0.2	0.025	5	0.47882	0.26108
X84	0.2	0.05	5	0.48000	0.26146
X85	0.4	0	5	0.49521	0.26581
X86	0.4	0.025	5	0.49642	0.26619
X87	0.4	0.05	5	0.49764	0.26657
X88	0.6	0	5	0.52346	0.27384
X89	0.6	0.025	5	0.52474	0.27423
X90	0.6	0.05	5	0.52602	0.27462
X91	0.8	0	5	0.56108	0.28425
X92	0.8	0.025	5	0.56244	0.28465
X93	0.8	0.05	5	0.56381	0.28506
X94	1.0	0	5	0.60662	0.29642
X95	1.0	0.025	5	0.60809	0.29684
X96	1.0	0.05	5	0.60957	0.29726
X97	1.2	0	5	0.65865	0.30977
X98	1.2	0.025	5	0.66025	0.31021
X99	1.2	0.05	5	0.66186	0.31065
X100	1.4	0	5	0.71593	0.32383
X101	1.4	0.025	5	0.71768	0.32429
X102	1.4	0.05	5	0.71943	0.32475
X103	1.6	0	5	0.77738	0.33821
X104	1.6	0.025	5	0.77929	0.33869
X105	1.6	0.05	5	0.78121	0.33917
X106	2.0	0	5	0.90960	0.36692
X107	2.0	0.025	5	0.91188	0.36745
X108	2.0	0.05	5	0.91417	0.36797
X109	3.0	0	5	1.27182	0.43286
X110	3.0	0.025	5	1.27525	0.43350
X111	3.0	0.05	5	1.27868	0.43414
X112	5.0	0	5	2.05015	0.53342
X113	5.0	0.025	5	2.05667	0.53428
X114	5.0	0.05	5	2.06318	0.53514
X115	10.0	0	5	4.06866	0.67589
X116	10.0	0.025	5	4.08709	0.67718
X117	10.0	0.05	5	4.10552	0.67848

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Then applying the proposed approach based on rough set theory which summaries as:

Step 1: discretize the decision table by using the Boolean reasoning algorithm.

Step 2: compute the reduct of the discretized decision table with the aid of genetic algorithm.

Step 3: generate a set of maximally generalized decision rules (classification rules).

The following flowchart represents the complete steps to extract the set of classification rules.

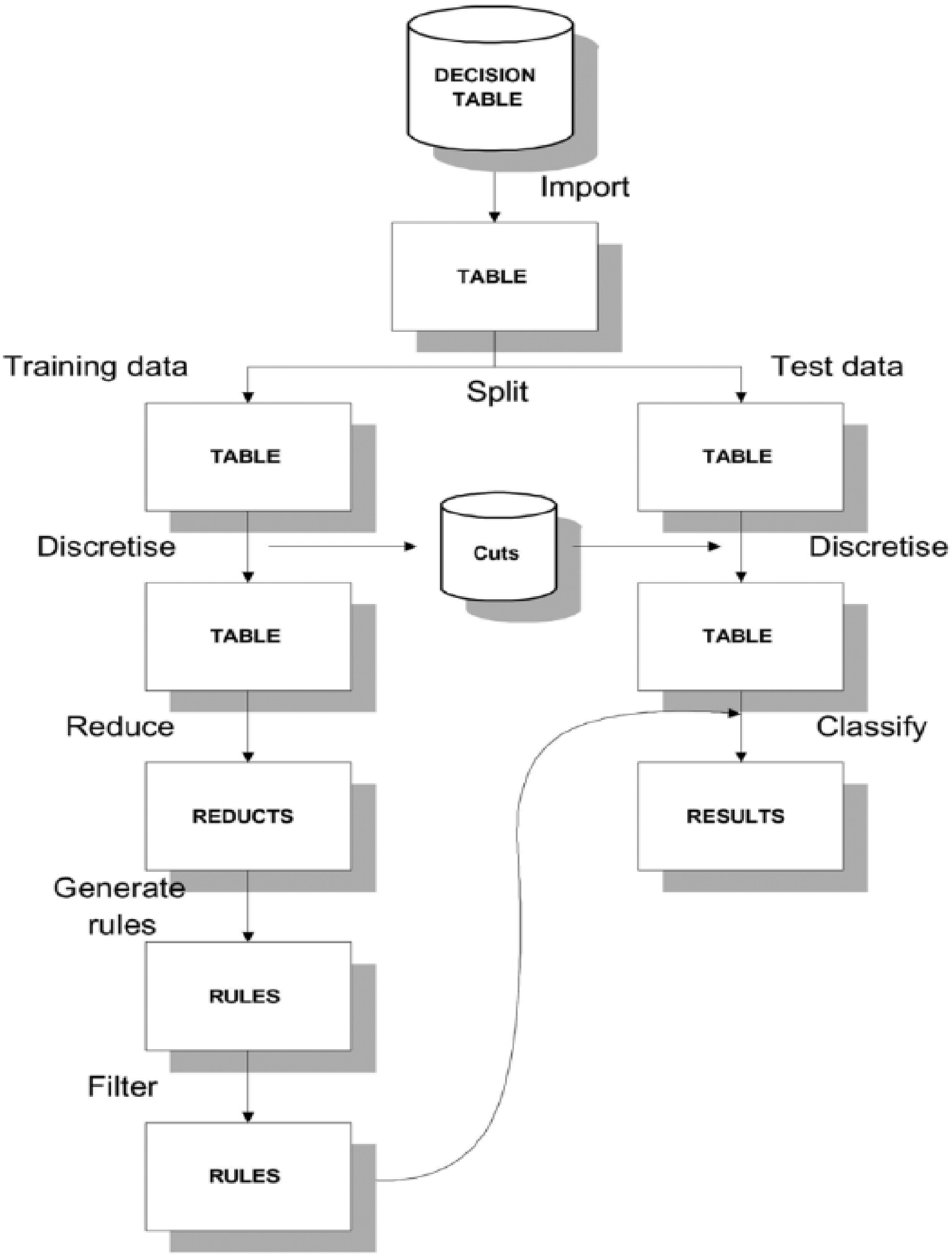

Fig. 2: Complete Steps to Extract Decision Rules

It worth noting that in this stage we will use software called ROSETTA which is an RST analysis toolkit. Table 3 shows part of the rule set extracted by using rough set methodology which explained in fig. 2.

Table 3: Part of the Generated Rule Set

Rules
⋮
IF $(M = [0.7, 0.9) \wedge K_0 = [*, 0.013))$ THEN $(f''(0) = \{0.49265\})$
IF $(M = [2.5, 4.0) \wedge K_0 = [0.013, 0.038))$ THEN $(f''(0) = \{3.33951\})$
IF $(M = [4.0, 7.5) \wedge K_0 = [*, 0.013))$ THEN $(f''(0) = \{5.14796\})$
IF $(M = [7.5, *) \wedge K_0 = [0.013, 0.038))$ THEN $(f''(0) = \{10.63314\})$
IF $(M = [7.5, *) \wedge K_0 = [0.038, *))$ THEN $(f''(0) = \{11.25077\})$
IF $(M = [0.5, 0.7) \wedge K_0 = [*, 0.013) \wedge \Delta = [*, 1.0))$ THEN $(-g'(0) = \{0.04959\})$
IF $(M = [0.7, 0.9) \wedge K_0 = [0.038, *) \wedge \Delta = [*, 1.0))$ THEN $(-g'(0) = \{0.05140\})$
IF $(M = [0.5, 0.7) \wedge K_0 = [0.038, *) \wedge \Delta = [3.3, *))$ THEN $(-g'(0) = \{0.27462\})$
IF $(M = [7.5, *) \wedge K_0 = [0.013, 0.038) \wedge \Delta = [3.3, *))$ THEN $(-g'(0) = \{0.67718\})$
IF $(M = [0.1, 0.3) \wedge K_0 = [0.013, 0.038) \wedge \Delta = [*, 1.0))$ THEN $(f''(0) = \{1.03056\})$
IF $(M = [0.1, 0.3) \wedge K_0 = [0.038, *) \wedge \Delta = [3.3, *))$ THEN $(f''(0) = \{0.48000\})$
⋮

It is noted that In the case of viscoelastic fluids $f''(0)$ is proportional to the friction factor. And In the case of micropolar viscoelastic fluids $f''(0)$ is proportional to the friction factor and $g'(0)$ is proportional to the wall couple stress.

CONCLUSION

This paper suggests the use of rough set theory to process and extract rules for Analyzing the Effect of Viscoelastic and Micropolar Parameters on Hiemenz Flow in Hydromagnetics. The results of this study indicate that as the micropolar parameter Δ increases, the friction factor decreases. A similar behaviour is noted when the Hartman number and the viscoelastic parameter increase. The absolute value of the microrotation gradient is found to increase with increasing Hartman number, micropolar parameter, and viscoelastic parameter. Also the obtained results are in good agreement with previous studies. The technique has been simplified logic-based rules, reduces the time and resources required to building knowledge.

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