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Single Finite Fourier Sine Integral Transform Method for the Determination of Natural Frequencies of Flexural Vibration of Kirchhoff Plates

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Abstract

This work presents the single finite Fourier sine integral transform method for finding the natural frequencies of flexural vibration of rectangular Kirchhoff plates with two opposite simply supported edges ($x = 0$, $x = a$) and two clamped edges ($y = 0$, $y = b$), where the origin is at a corner. For free harmonic vibrations, the problem is represented mathematically as a fourth order partial differential equation (PDE) over the domain, and boundary conditions along the edges. Application of the transform with respect to the x coordinate variable converts the Boundary Value Problem (BVP) to an integral equation which satisfies all the Dirichlet boundary conditions along $x = 0$, and $x = a$, due to the sinusoidal kernel function of the transform used. The integral equation is further simplified to a system of ordinary differential equations (ODEs), which is solved to obtain the unknown deflection in the transform space. Transforms of the boundary conditions are used in the solution of the ODE to generate a system of homogeneous equations in terms of the unknown integration constants. The condition for nontrivial solution is used to obtain the characteristic frequency equation which is solved by iteration methods to obtain the natural frequencies for any given mode of vibration. The results obtained are identical with results obtained by previous researchers who used Galerkin-Vlasov methods, Levy's single trigonometric series method, finite element method, and energy methods.

Keywords: Single finite Fourier sine integral transform method, Kirchhoff plate, characteristic frequency equation, dimensionless natural frequency parameters, vibration mode, harmonic vibration.

1. INTRODUCTION

The problem of determination of the natural frequencies of flexural vibrations of plates which is an eigenvalue problem of dynamics of plates have been extensively studied for various shapes (rectangular, trapezoidal, circular, skewed, elliptical, etc) forms, types, material properties (orthotropic, anisotropic) boundary and loading conditions [1 – 15].

The behaviour of plates – which are continuous elastic structures – under dynamic or time-dependent forces or displacements can be modelled mathematically by partial differential equations of motion derived from equilibrium considerations based on Newton's laws or D'Alembert's principle of dynamic equilibrium. Alternatively, the governing equations of dynamic plates could be derived from considerations of virtual work and energy principles as integral equations.

In most practical cases, only the lateral vibration is important, and the effects of extensional vibrations on the middle plane may be disregarded. Therefore, the inertial forces associated with the transverse flexural deflection of the plate are considered. In this study, only the flexural vibration of thin rectangular plate is considered. Damping effects caused either by internal friction or the surrounding media are not considered. Though damping is theoretically present in all flexural vibrations of Kirchhoff plates, it has usually insignificant effect on the natural frequencies and the steady state displacement amplitudes, and can consequently be safely disregarded in the analysis [2, 3, 6].

Hatiegan et al [8] determined the natural frequencies of thin rectangular plate with and without damages using the finite element method. Pouladkhan et al [16] presented a finite element model using ABAQUS (v.6.7) software for a simply supported rectangular plate; and obtained solutions for the natural frequencies and mode shapes, which were found to be comparable with the exact solution.

Lee et al [9] developed and used the Homotopy Perturbation Method (HPM) to solve the partial differential equation of flexural vibration of thin plates. They obtained the natural frequencies of a thin rectangular simply supported plate of constant thickness with minimal computational effort. Their solution is shown to converge rapidly to a combination of sine and cosine functions. The truncation of the series by using only the first three terms gave negligible error, and very accurate solutions. They also applied the HPM to solve the nonlinear problem of a rectangular plate of variable thickness, and obtained uncomplicated expression for the natural frequencies of the plate, whose solutions illustrated the efficiency and effectiveness of the HPM.

Studdert and O'Callaghan [17] investigated the transverse free vibrations of elastic plates of uniform thickness with rectangular orthotropy using the Edge Function Method and obtained accurate solutions for fundamental frequencies for a series of rectangular SCSC plates.

Mama et al [18] used the Galerkin-Vlasov variational method to study the dynamic characteristics of simply supported thin rectangular plates undergoing free flexural vibrations in harmonic motion. They found that the governing BVP was converted to an integral equation – Galerkin-Vlasov integral formulation of the BVP – which ultimately reduced to an algebraic eigenvalue problem. The algebraic eigenvalue problem was solved in the space domain to obtain the eigenfrequencies and modal shape functions of the vibrating Kirchhoff plate and their solutions were identical to solutions obtained previously by the classical methods of Navier's double trigonometric series and Levy's single Fourier series.

Ike and Nwoji [19] used the Kantorovich variational method to determine the natural frequencies of flexural vibrations of rectangular Kirchhoff plates with two opposite edges clamped and the other two edges simply supported. The variation integral formulation of the BVP was obtained using Galerkin and Kantorovich methods, and found to simplify to a system of fourth order ordinary differential equations which upon solution subject to the boundary conditions along the clamped ends, yielded the characteristic frequency equation as a transcendental equation. The roots of the transcendental equation obtained by computer based iterative methods were used to find the eigenfrequencies which were comparable to the results previously obtained for the solved problem by researchers who used Galerkin-Vlasov, Levy, Finite Element Method, Finite Difference Method, and Rayleigh-Ritz Methods.

Cho et al [20] obtained the approximate natural frequencies of rectangular plates with openings using an assumed mode method, where natural frequencies and vibration modes are determined by solving an eigenvalue problem of multi-degree-of-freedom system matrix equation derived by using Lagrange's equation of motion.

Eze et al [21] used the ordinary finite difference method (FDM) to perform free flexural vibration analysis of thin rectangular flat plates. Three basic types of boundary conditions, namely; all edges were clamped (CCCC), all edges were simply supported (SSSS) and two opposite edges were clamped while the other two opposite edges were simply supported (CSCS) were considered in their study. By expressing the governing partial differential equation in finite difference form and application of the appropriate boundary conditions, they obtained the fundamental natural frequencies which were good enough considering the reduction in computational rigor afforded by the FDM.

Misra [22, 23] used the Multiquadric Radial Basis Function Method for the static and dynamic analysis of rectangular isotropic plates. Werfalli and Karoud [24] used the Galerkin-based Finite Element Method to solve free vibration problems of rectangular plates. Njoku [25] used the beam analogy method for the dynamic analysis of rectangular thin isotropic plates and obtained satisfactory solutions using the Ritz formulation of the problem for different edge support conditions.

Jayasinghe Supum and Hashemi Sayed [26] presented a dynamic coefficient matrix method for the free vibration of thin rectangular plates. Eftekhari [27] presented a simple and accurate Ritz-DQM formulation for free vibrations of rectangular plates involving free corners. Ghashochi-Bargh and Ravazi [28] presented a simple analytical model for the natural vibrations of orthotropic and functionally graded plates. Kumar [29] presented a free transverse vibration analysis of rectangular Kirchhoff plates having arbitrarily varying non-homogeneity along two concurrent edge.

Lim et al [30] formulated a new symplectic elasticity approach and used it to obtain the exact mathematical solutions to the free vibration problems of rectangular Kirchhoff plates. Lim and Liew [31] presented a pb-2 Ritz formulation for the flexural vibrations of shallow cylindrical shells of rectangular plan form.

In this work, the single finite Fourier sine transform method is applied to determine the natural frequencies of flexural harmonic vibrations of Kirchhoff plates with two opposite simply supported edges and the other two opposite edges clamped.

2. THEORETICAL FRAMEWORK

The governing partial differential equation (PDE) of Kirchhoff plate of length a, and width b undergoing flexural vibration is:

$$D\nabla^4 w(x,y,t) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = p_z(x,y,t) \quad (1)$$

where $w(x, y, t)$ is the dynamic deflection, ρ is the density of the plate material, h is the plate thickness, $p_z(x, y, t)$ is the external excitation load, D is plate flexural rigidity, x, y are the in-plane Cartesian coordinates, t is the time, ∇^4 is the biharmonic operator.

∇^4 is given as:

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \quad (2)$$

For sinusoidal vibrations, the displacement response $w(x, y, t)$ is expectedly harmonic and can be expressed as:

$$w(x,y,t) = W(x,y)\sin(\omega_{mn}t - \phi) = F(x,y)\sin(\omega_{mn}t - \phi) \quad (3)$$

where $W(x, y) = F(x, y)$ is the dynamic modal displacement function, ω_{mn} are natural frequencies of flexural vibration, ϕ is the phase.

For free vibrations, there is no applied excitation, and

$$p_z(x,y,t) = 0 \quad (4)$$

The governing PDE then becomes:

$$\left(D\nabla^4 F(x,y) - \rho h\omega_{mn}^2 F(x,y)\right)\sin(\omega_n t - \phi) = 0 \quad (5)$$

$$\nabla^4 F(x,y) - \frac{\rho h\omega_{mn}^2}{D}F(x,y) = 0 \quad (6)$$

$$\frac{\partial^4 F(x,y)}{\partial x^4} + 2\frac{\partial^4 F(x,y)}{\partial x^2\partial y^2} + \frac{\partial^4 F(x,y)}{\partial y^4} - \frac{\rho h\omega_{mn}^2}{D}F(x,y) = 0 \quad (7)$$

$$\frac{\partial^4 F(x,y)}{\partial x^4} + 2\frac{\partial^4 F(x,y)}{\partial x^2\partial y^2} + \frac{\partial^4 F(x,y)}{\partial y^4} - \Omega_{mn}^2 F(x,y) = 0 \quad (8)$$

where $\Omega_{mn}^4 = \frac{\rho h\omega_{mn}^2}{D}$ (9)

Ω_{mn} is the dimensionless frequency parameter.

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3. METHODOLOGY

The problem considered is rectangular Kirchhoff plate simply supported along the edges $x = 0$, and $x = a$, and clamped along the edges $y = 0$, and $y = b$, as shown in Figure 1.

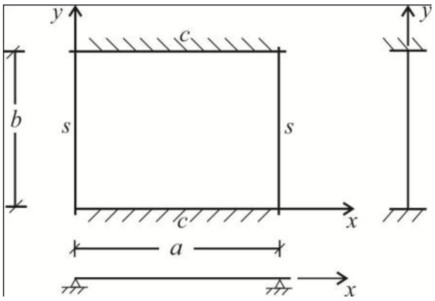

Figure 1: Kirchhoff plate with opposite edges clamped and the other edges simply supported

The domain PDE is given by Equation (8). The boundary conditions are:

$$w(x=0,y,t)=0 \qquad w(x,y=0,t)=0$$
$$w(x=a,y,t)=0 \qquad w(x,y=b,t)=0 \tag{9}$$

$$\frac{\partial^2 w}{\partial x^2}(x=0,y,t)=0$$
$$\frac{\partial^2 w}{\partial x^2}(x=a,y,t)=0$$
$$\frac{\partial w}{\partial y}(x,y=0,t)=0 \tag{9}$$
$$\frac{\partial w}{\partial y}(x,y=b,t)=0$$

Application of the single finite Fourier transform with respect to the x coordinate variable to the domain PDE yields:

$$\int_0^a \left\{ \frac{\partial^4 F(x,y)}{\partial x^4} + 2\frac{\partial^4 F(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 F(x,y)}{\partial y^4} - \Omega_{mn}^4 F(x,y) \right\} \sin\frac{m\pi x}{a}\,dx = 0 \tag{10}$$

4. RESULTS

By the linearity property of the finite Fourier sine transform, we obtain:

$$\int_0^a \frac{\partial^4 F(x,y)}{\partial x^4}\sin\frac{m\pi x}{a}dx + 2\frac{d^2}{dy^2}\int_0^a \frac{\partial^2 F(x,y)}{\partial x^2} \times \sin\frac{m\pi x}{a}dx + \frac{d^4}{dy^4}\int_0^a F(x,y)\sin\frac{m\pi x}{a}dx - \Omega_{mn}^4 \int_0^a F(x,y)\sin\frac{m\pi x}{a}dx = 0 \tag{11}$$

From integration by parts, and using the Dirichlet boundary conditions along $x = 0$ and $x = a$, the equation simplifies further as follows:

$$\left(\frac{m\pi}{a}\right)^4 F(m,y) - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 F(m,y)}{dy^2} + \frac{d^4 F(m,y)}{dy^4} - \Omega_{mn}^4 F(m,y) = 0 \tag{12}$$

$$\frac{d^4 F(m,y)}{dy^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 F(m,y)}{dy^2} + \left(\left(\frac{m\pi}{a}\right)^4 - \Omega_{mn}^4\right) F(m,y) = 0 \tag{13}$$

$F(m, y)$ is the single finite Fourier sine transform of $F(x, y)$

$$\text{where } F(m,y) = \int_0^a F(x,y)\sin\frac{m\pi x}{a}dx \tag{14}$$

$$\text{and } F(x,y) = \sum_{m=1}^{\infty} F(m,y)\sin\frac{m\pi x}{a} \tag{15}$$

The solution to the fourth order ordinary differential equation (ODE) can be assumed in the exponential form.

$$F(m,y) = B\exp(s\,y) \tag{16}$$

where s is a parameter we seek to find.

Then by substitution of Equation (16) into the ODE, we obtain

$$\left(s^4 - 2\left(\frac{m\pi}{a}\right)^2 s^2 + \left(\left(\frac{m\pi}{a}\right)^4 - \Omega_{mn}^4\right)\right) Be^{sy} = 0 \tag{17}$$

For nontrivial solutions, Be^{sy} is not equal to zero, and the following fourth degree auxiliary polynomial results:

$$s^4 - 2\left(\frac{m\pi}{a}\right)^2 s^2 + \left(\left(\frac{m\pi}{a}\right)^4 - \Omega_{mn}^4\right) = 0 \tag{18}$$

Solving,

$$s_{1,2} = \pm\alpha_1 = \pm\sqrt{\Omega_{mn}^2 + \left(\frac{m\pi}{a}\right)^2} \tag{19}$$

$$s_{2,3} = \pm i\alpha_2 = \pm i\sqrt{\Omega_{mn}^2 - \left(\frac{m\pi}{a}\right)^2} \tag{20}$$

$$\text{Hence, } \alpha_1 = \sqrt{\Omega_{mn}^2 + \left(\frac{m\pi}{a}\right)^2} \tag{21}$$

$$\alpha_2 = \sqrt{\Omega_{mn}^2 - \left(\frac{m\pi}{a}\right)^2} \tag{22}$$

The solution is then

$$F(m,y) = A_{1m}\cosh\alpha_1 y + A_{2m}\sinh\alpha_1 y + A_{3m}\cos\alpha_2 y + A_{4m}\sin\alpha_2 y \tag{23}$$

Application of the single finite Fourier sine integral transform to the boundary conditions at any time, t yield:

$$\int_0^a w(x,y=0)\sin\frac{m\pi x}{a}dx = F(m,y=0) = 0 \tag{24}$$

$$\int_0^a w(x,y=b)\sin\frac{m\pi x}{a}dx = F(m,y=b) = 0 \tag{25}$$

$$\int_0^a \frac{\partial w(x,y=0)}{\partial y}\sin\frac{m\pi x}{a}dx = \frac{d}{dy}F(m,y=0) = 0 \tag{26}$$

$$\int_0^a \frac{\partial w}{\partial y}(x, y=b)\sin\frac{m\pi x}{a}dx = \frac{d}{dy}F(m, y=b) = 0 \quad (27)$$

$$\frac{d}{dy}F(m,y) = A_{1m}\alpha_1 \sinh\alpha_1 y + A_{2m}\alpha_1\cosh\alpha_1 y - $$
$$-A_{3m}\alpha_2\sin\alpha_2 y + A_{4m}\alpha_2\cos\alpha_2 y \quad (28)$$

Then,

$$A_{1m} + A_{3m} = 0 \quad (29)$$

$$A_{2m}\alpha_1 + A_{4m}\alpha_2 = 0 \quad (30)$$

$$A_{1m}\cosh\alpha_1 b + A_{2m}\sinh\alpha_1 b +$$
$$A_{3m}\cos\alpha_2 b + A_{4m}\sin\alpha_2 b = 0 \quad (31)$$

$$A_{1m}\alpha_1\sinh\alpha_1 b + A_{2m}\alpha_1\cosh\alpha_1 b -$$
$$A_{3m}\alpha_2\sin\alpha_2 b + A_{4m}\alpha_2\cos\alpha_2 b = 0 \quad (32)$$

In matrix form, we have:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_1 & 0 & \alpha_2 \\ \cosh\alpha_1 b & \sinh\alpha_1 b & \cos\alpha_2 b & \sin\alpha_2 b \\ \alpha_1\sinh\alpha_1 b & \alpha_1\cosh\alpha_1 b & -\alpha_2\sin\alpha_2 b & \alpha_2\cos\alpha_2 b \end{pmatrix}\begin{pmatrix} A_{1m} \\ A_{2m} \\ A_{3m} \\ A_{4m}\end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0\end{pmatrix} \quad (33)$$

For nontrivial solutions, the characteristic equation is obtained as:

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_1 & 0 & \alpha_2 \\ \cosh\alpha_1 b & \sinh\alpha_1 b & \cos\alpha_2 b & \sin\alpha_2 b \\ \alpha_1\sinh\alpha_1 b & \alpha_1\cosh\alpha_1 b & -\alpha_2\sin\alpha_2 b & \alpha_2\cos\alpha_2 b \end{vmatrix} = 0 \quad (34)$$

Expansion and simplification gives the characteristic frequency equation as:

$$2\alpha_1\alpha_2(\cosh\alpha_1 b\cos\alpha_2 b - 1) +$$
$$(\alpha_2^2 - \alpha_1^2)\sinh\alpha_1 b\sin\alpha_2 b = 0 \quad (35)$$

The characteristic frequency equation is a transcendental equation which is solved for square plates ($a = b$) using computer based iteration methods to obtain the roots as follows:

for $m = 1$, $n = 1$,

$$\alpha_1 = \frac{6.2302}{a} \quad (36)$$

$$\alpha_2 = \frac{4.36765}{a} \quad (37)$$

$m = 2$, $n = 1$

$$\alpha_1 = \frac{8.0382}{a} \quad (38)$$

$$\alpha_2 = \frac{6.69876}{a} \quad (39)$$

$m = 1$, $n = 2$

$$\alpha_1 = \frac{8.89925}{a} \quad (40)$$

$$\alpha_2 = \frac{7.71086}{a} \quad (41)$$

Hence from Equation (9),

$$\omega_{mn} = \Omega_{mn}^2\sqrt{\frac{D}{\rho h}} \quad (42)$$

Then, $$\omega_{11} = \Omega_{11}^2\sqrt{\frac{D}{\rho h}} = \frac{28.946}{a^2}\sqrt{\frac{D}{\rho h}} \quad (43)$$

$$\omega_{21} = \Omega_{21}^2\sqrt{\frac{D}{\rho h}} = \frac{54.743}{a^2}\sqrt{\frac{D}{\rho h}} \quad (44)$$

$$\omega_{12} = \Omega_{12}^2\sqrt{\frac{D}{\rho h}} = \frac{69.327}{a^2}\sqrt{\frac{D}{\rho h}} \quad (45)$$

$$\omega_{22} = \Omega_{22}^2\sqrt{\frac{D}{\rho h}} = \frac{97}{a^2}\sqrt{\frac{D}{\rho h}} \quad (46)$$

The obtained natural frequencies of free flexural vibrations of square Kirchhoff plates clamped along $y = 0$ and $y = b$ and simply supported along $x = 0$, and $x = a$, where $a = b$ are presented in Table 1, together with the natural frequencies from previous works in the literature.

$$\omega_{mn} = \Omega_{mn}^2\sqrt{\frac{D}{\rho h}} = \frac{\lambda_{mn}^2}{a^2}\sqrt{\frac{D}{\rho h}} \quad (47)$$

Table 1: Dimensionless frequency parameters λ_{mn}^2 of square Kirchhoff plate simply supported on two opposite edges and clamped on the other two edges (CSCS plates) $\omega_{mn} = \frac{\lambda_{mn}^2}{a^2}\sqrt{\frac{D}{\rho h}}$

Eigenvalues / Method	λ_{11}^2	λ_{12}^2	λ_{13}^2	λ_{21}^2	λ_{22}^2
Single finite Fourier sine transform method (present study)	28.946	69.327		54.743	97
Galerkin-Vlasov, Fetea [32]	28.944	70.11	123.16	54.93	97.07
Levy method, Leissa [2]	28.946	69.32	129.086	54.743	94.584
Levy method, Lim and Liew [31]	28.951	69.327		54.743	94.585
Chakraverty [1]	28.95				
Gorman [10]	28.93				
FDM, Ezeh et al [21]	24.29				
Finite Element Method, Lim et al [30]	28.951	69.33		54.744	94.588
Ritz-DQM, Eftekhari [27]	28.9509	69.527		54.7431	94.5833

Ghashochi-Bargh and Ravazi [28]	28.835	69.345		54.906	94.834
Leissa and Qatu [6]	28.9509				
Kalita and Dutta [15]	28.9959				
Ritz method, Njoku [25]	28.95609				
Hearman [14]	28.95	69.32		54.75	94.59
Hamada [13]	28.90				
Warburton [11]	29.01	69.35		55.07	94.84
Das [12]	28.99			54.86	94.63
Kumar [29] (FDM) 12 grid points (12 by 12 grid points)	27.9509	68.327		53.7431	

5. DISCUSSION

In this work the single finite Fourier sine transform method has been successfully used to determine the natural frequencies of flexural vibration of Kirchhoff plates with two opposite edges ($x = 0$, and $x = a$) simply supported and the other two edges ($y = 0$, $y = b$) clamped. The plate is considered isotropic, homogeneous, flat and rectangular. The problem is represented mathematically as the BVP given by Equation (5) for the case of free sinusoidal vibrations, and boundary conditions given by Equation (9).

The finite sine transform is applied to the domain PDE with respect to the x coordinate variable yielding the integral equation – Equation (10). The sinusoidal kernel function satisfies all the Dirichlet boundary conditions along the simply supported edges $x = 0$, and $x = a$. The linearity property of the transformation is used together with the evaluation of the transforms of the derivatives to express the integral equation as the fourth order ordinary differential equation (ODE), Equation (13), which is in terms of $F(m, y)$ the single finite Fourier sine transform of the unknown function $F(x, y)$. The ODE is solved using methods for solving differential equations – trial functions method – to obtain the solution in the transform space as Equation (23). The transform of the boundary conditions along the clamped edges ($y = 0$, and $y = b$) are used to generate the system of homogeneous equations given in matrix form as Equation (33). The condition for nontrivial solution gives the characteristic frequency equation as the transcendental equation – Equation (35). The frequency equation is solved for square Kirchhoff plates to obtain the dimensionless natural frequency parameters λ_{mn}^2 presented for CSCS plates considered in Table 1, which also presents solutions previously obtained by Chakraverty [1], Gorman [10], Ezeh et al [21], Lim et al [30], Eftekhari [27], Fetea [32], Lim and Liew [31], Leissa [2], Leissa and Qatu [6], Kalita and Dutta [15], and Njoku [25]. Table 1 reveals that the present results are comparable to previous results by Fetea [32] who used the Galerkin-Vlasov method to solve the same problem. The present results are also comparable with previous results obtained by Lim et al [30] who used the Finite Element Method (FEM), Eftekhari [27] who used Ritz-DQM, and Ghasochi-Bergh and Ravazi [28].

6. CONCLUSION

In conclusion,

(i) The single finite Fourier sine integral transform method is a very good mathematical tool for solving the problem of finding the natural frequencies of flexural vibrations of Kirchhoff plates with two opposite simply supported edges ($x = 0$, $x = a$) and two opposite clamped edges ($y = 0$, $y = b$).

(ii) The integral Kernel function which is a sine function satisfies all the Dirichlet boundary conditions along the simply supported edges ($x = 0$, and $x = a$).

(iii) Application of the finite Fourier sine integral transform to the governing domain equation for harmonic vibrations converts the domain BVP to an integral equation.

(iv) The resulting integral equation is further simplified to a fourth order ODE in terms of the function $F(m, y)$ which is in the transform space.

(v) The enforcement of boundary conditions along the clamped edges $y = 0$, and $y = b$ results to a system of homogeneous equations in terms of the integration constants.

(vi) The system of homogeneous equations shows that the problem is basically an eigenvalue problem.

(vii) The transcendental equation is solved using the computer software based iteration methods to obtain the zeros or eigenvalues which are used to obtain the natural frequency parameters in terms of the material properties of plate (D, ρ, h, a).

(ix) Closed form expressions were obtained for the characteristic frequency equation from which the natural frequencies could be obtained for any mode of flexural vibration.

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