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A NOTE ON AN ORDER BETWEEN OBJECT-ORIENTED SOFT CONCEPTS IN A SOFT CONTEXT

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Abstract:
The purpose of this work is to study the algebraic structure in the family of all the m-concepts, so we introduce the notion of an order in the set of all m-concepts and show that the ordered set is a complete lattice. And, we discover what is the condition for the isomorphic relation between two m-concept lattices in a given soft context.

1. INTRODUCTION

Formal concept analysis [10] was introduced by Wille, which is an important theory for the research of information structures induced by a binary relation between the set of attributes and objects attributes. The three basic notions of FCA are formal context, formal concept, and concept lattice. A formal context is a kind of information system, which is a tabular form of an object-attribute value relationship [2, 3, 9]. A formal concept is a pair of a set of objects as called the extent and a set of attributes as called the intent. The set of all formal concepts together with the order relation forms a complete lattice called the concept lattice [9,10]. In order to deal complicated problems, Molodtsov introduced the concept of soft set in [8]. The operations for the soft set theory was introduced by Maji et al. in [4]. In [1], Ali et al. proposed new operations modified some concepts introduced by Maji. We have formed a soft context by combining the concepts of the formal context and the soft set defined by the set-valued mapping in [6]. Additionally, we introduced and studied the new concepts named soft concepts and soft concepts lattices.
In [11], Yao introduced a new concept called *an object oriented formal concept* in a formal context by using the notion of approximation operations.
And also, by using the two operation, we investigated the new concept of m-concepts related closely the object oriented concept in formal context in [7].
In this paper, we introduce the notion of an order in the set of all m-concepts and show that the ordered set is a complete lattice. And, we discover what is the condition for the isomorphic relation between two m-concept lattices in a given soft context.

2. PRELIMINARIES

A formal context is a triplet (U, V, I), where U is a non-empty finite set of objects, V is a nonempty finite set of attributes, and I is a relation between U and V. Let (U, V, I) be a formal context. For a pair of elements $x \in U$ and $y \in V$, if $(x, y) \in I$, then it means that object x has attribute y and we write xIy. The set of all attributes with a given object $x \in U$ and the set of all objects with a a given attribute $y \in V$ are denoted as the following [9,10]:

$$x^* = \{y \in V | xIy\};\ y^* = \{x \in U | xIy\}.$$

And, the operations for the subsets $X \subseteq U$ and $Y \subseteq V$ are defined as:

$$X^* = \{y \in V | \textit{ for all } x \in X, xIy\}; \quad Y^* = \{x \in U | \textit{ for all } y \in Y, xIy\}.$$

In a formal context (U, V, I), a pair (X, Y) of two sets $X \subseteq U$ and $Y \subseteq V$ is called a *formal concept* of (U, V, I) if $X = Y^*$ and $B = Y^*$, where X and Y are called the *extent* and the *intent* of the formal concept, respectively.

Let U be a universe set and E be a collection of properties of objects in U. We will call E *the set of parameters* with respect to U.
A pair (F, E) is called a *soft set* [8] over U if F is a set-valued mapping of E into the set $P(U)$ of all subsets of the set U, i.e.,

$$F : E \to P(U).$$

In other words, for $a \in E$, every set $F(a)$ may be considered as the set of a-elements of the soft set (F, E).
Let $U = \{z_1, z_2, \ldots, z_m\}$ be a non-empty finite set of *objects*, $E = \{e_1, e_2, \ldots, e_n\}$ a non-empty finite set of *attributes*, and $F : E \to P(U)$ a soft set. Then the triple (U, E, F) is called *a soft context* [6].
And, in a soft context (U, E, F), we introduced the following mappings:
For each $Z \in P(U)$ and $Y \in P(E)$,
(1) $\mathbf{F}^+ : P(E) \to P(U)$ is a mapping defined as $\mathbf{F}^+(Y) = \cap_{y \in Y} F(y)$;
(2) $\mathbf{F}^- : P(U) \to P(E)$ is a mapping defined as $\mathbf{F}^-(Z) = \{a \in E : Z \subseteq F(a)\}$;
(3) $\Psi : P(U) \to P(U)$ is an operation defined as $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z)$.
Then Z is called a *soft concept* [6] in (U, E, F) if $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z) = Z$. The set of all soft concepts is denoted by $sC(U, E, F)$.

In [7], we introduced the notion of m-concepts which is independent of the notion of soft concepts to each other as the following: For each $X \in P(U)$,
For each $Z \in P(U)$ and $Y \in P(E)$,
(1) $\mathbb{F} : P(A) \to P(U)$ is a mapping defined as $\mathbb{F}(C) = \cup_{c \in C} F(c)$;

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(2) $\overleftarrow{\mathbb{F}} : P(U) \to P(A)$ is a mapping defined as $\overleftarrow{\mathbb{F}}(X) = \{c \in A : F(c) \subseteq X\}$;

Theorem 2.1 ([7])**.** Let (U, A, F) be a soft context, $S, T \subseteq U$ and $B, C \subseteq A$. Then we have:
(1) If $S \subseteq T$, then $\overleftarrow{\mathbb{F}}(S) \subseteq \overleftarrow{\mathbb{F}}(T)$; if $B \subseteq C$, then $\mathbb{F}(B) \subseteq \mathbb{F}(C)$;
(2) $\mathbb{F}\overleftarrow{\mathbb{F}}(S) \subseteq S$; $\overleftarrow{\mathbb{F}}\mathbb{F}(B) \subseteq B$;
(3) $\overleftarrow{\mathbb{F}}(S \cap T) = \overleftarrow{\mathbb{F}}(S) \cap \overleftarrow{\mathbb{F}}(T)$, $\mathbb{F}(B \cup C) = \mathbb{F}(B) \cup \mathbb{F}(C)$;
(4) $\overleftarrow{\mathbb{F}}(S) = \overleftarrow{\mathbb{F}}\mathbb{F}\overleftarrow{\mathbb{F}}(S)$, $\mathbb{F}(B) = \mathbb{F}\overleftarrow{\mathbb{F}}\mathbb{F}(B)$.

Let $\Phi : P(U) \to P(U)$ be an operation defined by $\Phi(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X)$ for $X \in P(U)$.

Then for $X \in P(U)$, X is called an *m-concept* (or *object oriented soft concept*) [7] in (U, A, F) if $\Phi(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = X$. The set of all m-concepts is denoted by $m(U, A, F)$.

3. MAIN RESULTS

First, for a soft context (U, A, F) and $C \subseteq A$, we consider a set-valued mapping $F_C : C \to P(U)$ defined by $F_C(c) = F(c)$ for all $c \in C$. Then the set-valued mapping F_C induces a soft set (F_C, C) and a soft context (U, C, F_C). Then we consider the operations $\mathbb{F}_C, \overleftarrow{\mathbb{F}_C}, \Phi_C$ as the following:
$\mathbb{F}_C : P(C) \to P(U)$ is a mapping defined by $\mathbb{F}_C(B) = \cup_{b \in B} F_C(b)$ for each $B \in P(C)$.
$\overleftarrow{\mathbb{F}_C} : P(U) \to P(C)$ is a mapping defined by $\overleftarrow{\mathbb{F}_C}(X) = \{c \in C : F_C(c) \subseteq X\}$ for each $X \in P(U)$.
An associated operation $\Phi_C : P(U) \to P(U)$ is also well defined by for each $X \in P(U)$, $\Phi_C(X) = \mathbb{F}_C\overleftarrow{\mathbb{F}_C}(X)$.

Lemma 3.1. Let (U, A, F) be a soft context, $C \subseteq A$ and $X \subseteq U$. Then
(1) $\overleftarrow{\mathbb{F}_C}(X) \subseteq \overleftarrow{\mathbb{F}}(X)$.
(2) $\overleftarrow{\mathbb{F}_C}(X) = \overleftarrow{\mathbb{F}}(X) \cap C$.

Proof. Obvious. □

Theorem 3.2. Let (U, A, F) be a soft context, $X, Y \subseteq U$ and $B, C, E \subseteq A$. Then we have the following things:
(1) If $X \subseteq Y$, then $\overleftarrow{\mathbb{F}_C}(X) \subseteq \overleftarrow{\mathbb{F}_C}(Y)$; if $B \subseteq E$, then $\mathbb{F}_C(B) \subseteq \mathbb{F}_C(E)$;
(2) $\mathbb{F}_C\overleftarrow{\mathbb{F}_C}(X) \subseteq X$; $\overleftarrow{\mathbb{F}_C}\mathbb{F}_C(B) \subseteq B$;
(3) $\overleftarrow{\mathbb{F}_C}(X \cap Y) = \overleftarrow{\mathbb{F}_C}(X) \cap \overleftarrow{\mathbb{F}_C}(Y)$, $\mathbb{F}_C(B \cup E) = \mathbb{F}_C(B) \cup \mathbb{F}_C(E)$;
(4) $\overleftarrow{\mathbb{F}_C}(X) = \overleftarrow{\mathbb{F}_C}\mathbb{F}_C\overleftarrow{\mathbb{F}_C}(X)$, $\mathbb{F}_C(B) = \mathbb{F}_C\overleftarrow{\mathbb{F}_C}\mathbb{F}_C(B)$.

Proof. It is obvious from the notions of $\mathbb{F}_C, \overleftarrow{\mathbb{F}_C}$ and Φ_C. □

Let (U, A, F) be a soft context, $X \in P(U)$ and $C \subseteq A$. Then X is called *m-concept* in (U, C, F_C) if $\Phi_C(X) = \mathbb{F}_C\overleftarrow{\mathbb{F}}_C(X) = X$. The set of all m-concepts will be denoted by $m(U, C, F_C)$.

Theorem 3.3 ([7])**.** Let (U, A, F) be a soft context. Then we have:
(1) $\Phi(\emptyset) = \emptyset$.
(2) $\Phi(X)$ is an m-concept.
(3) For $B \subseteq A$, $\mathbb{F}(B)$ is an m-concept.
(4) For $a \in A$, $F(a)$ is an m-concept.
(5) X is an m-concept if and only if there is some $B \subseteq A$ such that $X = \mathbb{F}(B)$.

By Theorem 3.3, the next theorem is obviously obtained:

Theorem 3.4. Let (U, A, F) be a soft context, $X \subseteq U$ and $B, C \subseteq A$. Then
(1) $\Phi_C(\emptyset) = \emptyset$.
(2) $\Phi_C(X)$ is an m-concept in (U, C, F_C).
(3) For each $B \subseteq C$, $\mathbb{F}_C(B)$ is an m-concept in (U, C, F_C).
(4) For each $c \in C$, $F(c)$ is an m-concept in (U, C, F_C).
(5) X is an m-concept in (U, C, F_C) if and only if $X = \mathbb{F}_C(B)$ for some $B \in P(C)$.

Theorem 3.5. Let (U, A, F) be a soft context and $C \subseteq A$. Then
(1) $m(U, C, F_C) = \mathbf{Im}(\mathbb{F}_C)$.
(2) If $\mathbb{F}_C(B_1), \cdots, \mathbb{F}_C(B_n) \in \mathbf{Im}(\mathbb{F}_C)$, then $\mathbb{F}_C(B_1) \cup \cdots \cup \mathbb{F}_C(B_n) \in \mathbf{Im}(\mathbb{F}_C)$.

Proof. (1) By (3) of Theorem 3.4, it is easily obtained.
(2) For $B_1 \cdots B_n \in P(C)$, by (3) of Theorem 3.2, $\mathbb{F}_C(B_1) \cup \cdots \cup \mathbb{F}_C(B_n) = \mathbb{F}_C(B_1 \cup \cdots \cup B_n)$. Since $B_1 \cup \cdots \cup B_n \in P(C)$, by (3) of Theorem 3.4, the statement (2) is obtained. □

Theorem 3.6. Let (U, A, F) be a soft context and $\mathcal{S}_C = \{F_C(c) \mid c \in C \subseteq A\}$ for the soft set (F_C, C). Then
(1) $\mathcal{S}_C \subseteq m(U, C, F_C)$:
(2) For each $X \in m(U, C, F_C)$, there is $S_1, S_2, \cdots, S_n \in \mathcal{S}_C$ satisfying $X = \cup S_i$, $i = 1, 2, \cdots, n$.

Proof. (1) By (4) of Theorem 3.4, it is obvious.
(2) By (4) of Theorem 3.4, there is a $B \in P(C)$ satisfying $X = \mathbb{F}_C(B)$. So, $X = \mathbb{F}_C(B) = \cup_{b \in B} F_C(b)$ and $F_C(b) \in \mathcal{S}_C$. □

Theorem 3.7. Let (U, A, F) be a soft context. Then for $C \subseteq A$, $m(U, C, F_C) \subseteq m(U, A, F)$.

Proof. For $X \in m(U, C, F_C)$, by Theorem 3.4, there is $B \in P(C)$ satisfying $X = \mathbb{F}_C(B)$. From Lemma 3.1, $X = \mathbb{F}_C(B) = \mathbb{F}(B)$ for $B \in P(C)$. From $m(U, A, F) = \mathbf{Im}(\mathbb{F})$ in [7], it implies $X \in m(U, A, F)$. So, $m(U, C, F_C) \subseteq m(U, A, F)$. □

Now, we define an order between two m-soft concepts in $m(U, A, F)$ as the following:

Definition 3.8. Let (U, A, F) be a soft context and $X, Y \in m(U, A, F)$.

$$X \preceq Y \text{ if and only if } X \subseteq Y.$$

X is called a *sub-m-concept* of Y, and Y is called a *super-m-concept* of X. For the ordered set $(m(U, A, F), \preceq)$, the infimum $\wedge$ and supremum $\vee$ are defined by:

$$X \wedge Y = \Phi(X \cap Y); \quad X \vee Y = X \cup Y.$$

Example 3.9. For $U = \{1, 2, 3, 4, 5\}$, $A = \{a, b, c, d, e\}$, Let us consider a soft context (U, A, F) as shown in Table 1.

Table 1:A formal context

-	a	b	c	d	e
1	1	1	0	1	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	0	0	0	0
5	0	0	1	0	1

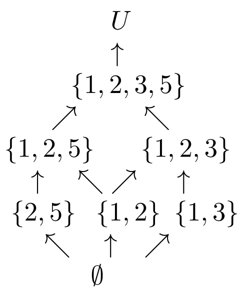
Then, (F, A) is a soft set as follows:

$$F(a) = \{1, 2\};\ \ F(b) = \{1, 3\};\ \ F(c) = \{2, 5\};$$

$$F(d) = \{1, 2, 3\};\ \ F(e) = \{1, 2, 5\}.$$

For the soft context (U, A, F),
$m(U, A, F) = \mathbf{Im}(\mathbb{F})$
$= \{\mathbb{F}(C)|\mathbb{F}(C) = \cup_{c\in C} F(c) \text{ for } C \in P(A)\}$
$= \{\emptyset, \{1, 2\}, \{1, 3\}, \{2, 5\}, \{1, 2, 3\}, \{1, 2, 5\}, \{1, 2, 3, 5\}, U\}.$

Hence, $mL(U, A, F)$ is obtained as shown in the below diagram:



$mL(U, A, F)$, where $A = \{a, b, c, d, e\}$

Theorem 3.10. Let (U, A, F) be a soft context. Then $(m(U, A, F), \preceq, \wedge, \vee)$ is complete lattice.

Proof. (1) Let $X, Y \in m(U, A, F)$. Then from Theorem 3.4, there exist $B, C \in P(A)$ such that $\mathbb{F}(B) = X$ and $\mathbb{F}(C) = Y$. By Theorem 3.2, $\mathbb{F}(B) \cup \mathbb{F}(C) = \mathbb{F}(B \cup C)$ and $X \cup Y = \mathbb{F}(B \cup C)$. It implies $X \cup Y \in m(U, A, F)$, and so $X \vee Y = X \cup Y \in m(U, A, F)$.

(2) For $X, Y \in m(U, A, F)$, let $Z \in m(U, A, F)$ satisfying $Z \subseteq X \cap Y$ and $X \wedge Y \preceq Z$. Then from $X \wedge Y \preceq Z$, $\Phi(X \cap Y) \subseteq Z$. Since $Z \subseteq X \cap Y$, from Theorem 3.4, $\Phi(Z) \subseteq \Phi(X \cap Y)$. It implies $Z = \Phi(Z) = \Phi(X \cap Y) = X \wedge Y$, and so $X \wedge Y = Z \in m(U, A, F)$. □

The complete lattice $(m(U, A, F), \preceq, \wedge, \vee)$ is called *m-concept lattice* (or *object oriented soft concept lattice*) and simply will be denoted by $mL(U, A, F)$.

Definition 3.11. Let $mL(U, B, F)$ and $mL(U, C, G)$ be two m-concept lattices. $mL(U, B, F)$ is said to be finer than $mL(U, C, G)$, which is denoted by

$$mL(U, B, F) \leq mL(U, C, G) \Leftrightarrow m(U, C, G) \subseteq m(U, B, F).$$

If $mL(U, B, F) \leq mL(U, C, G)$ and $mL(U, C, G) \leq mL(U, B, F)$, then two m-concept lattices are said to be *isomorphic* to each other, and denoted by

$$mL(U, B, F) \cong mL(U, C, G).$$

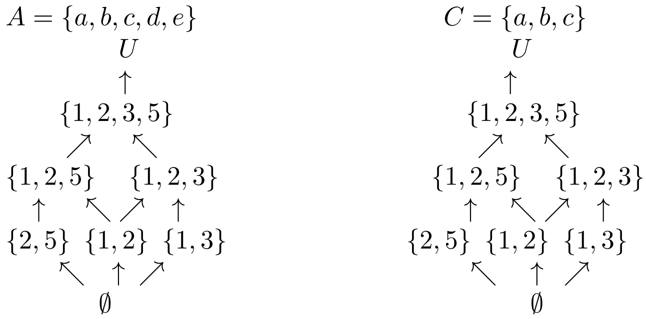
Theorem 3.12. Let $mL(U, A, F)$ be an m-concept lattice. Then for $C \subseteq A$, $mL(U, A, F) \leq mL(U, C, F_C)$.

Proof. From Theorem 3.7, we know that $m(U, C, F_C) \subseteq m(U, A, F)$. So, we have $mL(U, A, F) \leq mL(U, C, F_C)$. □

Theorem 3.13. Let (U, A, F) be a soft context and $C \subseteq A$. Then $mL(U, A, F) \cong mL(U, C, F_C)$ if and only if $\mathbf{Im}(\mathbb{F}) = \mathbf{Im}(\mathbb{F}_C)$.

Proof. By Theorem 3.5, $\mathbf{Im}(\mathbb{F}) = \mathbf{Im}(\mathbb{F}_C)$ if and only if $m(U, A, F) = m(U, C, F_C)$ if and only if $mL(U, A, F) \cong mL(U, C, F_C)$. So, the theorem is obtained. □

Example 3.14. As in Example 3.9, let us consider a soft context (U, A, F). For a subset $C = \{a, b, c\}$ of A, (U, C, F_C) is a soft context. Then we easily find that $m(U, C, F_C) = \{\emptyset, \{1, 2\}, \{1, 3\}, \{2, 5\}, \{1, 2, 3\}, \{1, 2, 5\}, \{1, 2, 3, 5\}, U\}$. So, $m(U, A, F) = m(U, C, F_C)$ and $\mathbf{Im}(\mathbb{F}) = \mathbf{Im}(\mathbb{F}_C)$. Consequently, $mL(U, A, F) \cong mL(U, C, F_C)$. The following diagrams are induced by A and $C \subseteq A$, respectively.



$mL(U, A, F) \cong mL(U, C, F_C)$

4. CONCLUSION

We showed that the set of all m-concepts of a given m-context together with the order relation between two m-concepts is a complete lattice, and found what is the condition for the isomorphic relation between two m-concept lattices. In the next research, we will study the relationships between m-concept lattices and formal concept lattices.

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