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Characteristics of Object-oriented Soft Concepts in a Soft Context

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Abstract
We introduced a new type of soft concept called object oriented soft concept(simply, m-concept) based on soft sets, which is independent of the notion of soft concepts in a soft context. The purpose of this work is to study the topological structure in the collection of all the object oriented soft concepts in a soft context. We show that the collection of all the object oriented soft concepts in a soft context is a supratopology. Moreover, we introduce the notions of independent m-concept(object oriented soft concept) and dependent m-concept in a soft context. Using the notions, we show that the set of all independent m-concepts completely determines every m-concept in a given soft context.

Key words and phrases: Formal concepts, soft concepts, object oriented soft concepts, independent m-concepts

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1. INTRODUCTION

FCA (formal concept analysis) was introduced by Wille [11] in 1982, which is an important theory for the research of information structures induced by a binary relation between the set of attributes and objects attributes. The three basic notions of FCA are formal context, formal concept, and concept lattice. A formal context is a kind of information system, which is a tabular form of an object-attribute value relationship [2, 3, 10]. A formal concept is a pair of a set of objects as called the extent and a set of attributes as called the intent.

The concept of soft set was introduced by Molodtsov in 1999 [9], to deal complicated problems and uncertainties. The operations for the soft set theory was introduced by Maji et al. in [4]. Ali et al. [1] proposed new operations modified some concepts introduced by Maji. We have formed a soft context by combining the concepts of the formal context and the soft set defined by the set-valued mapping in [7]. And we introduced and studied the new concepts named soft concepts and soft concepts lattices.

Yao [12] introduced a new concept called *an object oriented formal concept* in a formal context by using the notion of approximation operations.

We recall that: Let (U, A, I) be a formal context in formal concept analysis, where U is a finite nonempty set of objects, A is a finite nonempty set of attributes and I is a binary relation between U and A. For $x \in U$ and $y \in A$, if $(x, y) \in I$, also written as xIy. We will denote $xI = \{y \in A | xIy\}$; and $Iy = \{x \in U | xIy\}$.

And, let us consider two set-theoretic operators,

$^{\Box} : P(U) \to P(A)$: $X^{\Box} = \{y \in A | \forall x \in U(xIy \Rightarrow x \in X)\}$;

$^{\Diamond} : P(A) \to P(U)$: $Y^{\Diamond} = \{x \in U | \exists y \in A(xIy \wedge y \in Y)\}$.

Then a pair (X, Y), $X \subseteq U, Y \subseteq A$, is called *an object oriented formal concept* if $X = Y^{\Diamond}$ and $Y = X^{\Box}$.

Using the facts, we introduced the new notions of object-oriented soft concepts (simply, m-concepts) and studied the notion of m-concepts and basic properties in [8]. The purpose of this work is to study the topological structure in the family of all object-oriented soft concepts. Furthermore, we introduce the notions of independent m-concept and dependent m-concept in a soft context. In particular, we show that the set of all independent m-concepts completely determines every m-concept in a soft context.

2. PRELIMINARIES

A formal context is a triplet (U, A, I), where U is a non-empty finite set of objects, A is a nonempty finite set of attributes, and I is a relation between U and A. Let (U, A, I) be a formal context. For a pair of elements $x \in U$ and $y \in A$, if $(x, y) \in I$, then it means that object x has attribute y and we write xIy. The set of all attributes with a given object $x \in U$ and the set of all objects with a a given attribute $y \in A$ are denoted as the following [10,11]:

$$x^* = \{y \in A | xIy\};\ y^* = \{x \in U | xIy\}.$$

And, the operations for the subsets $X \subseteq U$ and $Y \subseteq A$ are defined as:

$$X^* = \{y \in A | \textit{for all } x \in X, xIy\}; \quad Y^* = \{x \in U | \textit{for all } y \in Y, xIy\}.$$

In a formal context (U, A, I), a pair (X, Y) of two sets

$X \subseteq U$ and $Y \subseteq A$ is called a *formal concept* of (U, A, I) if $X = Y^*$ and $B = Y^*$, where X and Y are called the *extent* and the *intent* of the formal concept, respectively.

Let U be a universe set and A be a collection of properties of objects in U. We will call A *the set of parameters* with respect to U.

A pair (F, A) is called a *soft set* [9] over U if F is a set-valued mapping of A into the set $P(U)$ of all subsets of the set U, i.e.,

$$F : A \to P(U).$$

In other words, for $a \in A$, every set $F(a)$ may be considered as the set of a-elements of the soft set (F, A).

Let $U = \{z_1, z_2, \ldots, z_m\}$ be a non-empty finite set of *objects*, $A = \{a_1, a_2, \ldots, a_n\}$ a non-empty finite set of *attributes*, and $F : A \to P(U)$ a soft set. Then the triple (U, A, F) is called *a soft context* [7].

And, in a soft context (U, A, F), we introduced the following mappings: For each $Z \in P(U)$ and $Y \in P(A)$,
(1) $\mathbf{F}^+ : P(A) \to P(U)$ *is a mapping defined as* $\mathbf{F}^+(Y) = \cap_{y \in Y} F(y)$;
(2) $\mathbf{F}^- : P(U) \to P(A)$ is a mapping defined as $\mathbf{F}^-(Z) = \{a \in A : Z \subseteq F(a)\}$;
(3) $\Psi : P(U) \to P(U)$ is an operation defined as $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z)$.

Then Z is called a *soft concept* [7] in (U, A, F) if $\Psi(Z) = \mathbf{F}^+\mathbf{F}^-(Z) = Z$. The set of all soft concepts is denoted by $sC(U, A, F)$.

In [8], the following operators $\mathbb{F}$ and $\overleftarrow{\mathbb{F}}$ were introduced as follows:

Let (U, A, F) be a soft context. Then for $C \in P(A)$, $X \in P(U)$,

an operator $\mathbb{F} : P(A) \to P(U)$ is defined by $\mathbb{F}(C) = \cup_{c \in C} F(c)$;

an operator $\overleftarrow{\mathbb{F}} : P(U) \to P(A)$ is defined by $\overleftarrow{\mathbb{F}}(X) = \{c \in A : F(c) \subseteq X\}$.

Simply, we denote: For $c \in A$ and $x \in U$ $\mathbb{F}(\{c\}) = \mathbb{F}(c)$ and $\overleftarrow{\mathbb{F}}(\{x\}) = \overleftarrow{\mathbb{F}}(x)$. Obviously, $\mathbb{F}(c) = F(c)$ for $c \in A$.

Theorem 2.1 ([8)] *Let (U, A, F) be a soft context, $S, T \subseteq U$ and $B, C \subseteq A$. Then we have:*

(1) If $S \subseteq T$, then $\overleftarrow{\mathbb{F}}(S) \subseteq \overleftarrow{\mathbb{F}}(T)$; if $B \subseteq C$, then $\mathbb{F}(B) \subseteq \mathbb{F}(C)$;

(2) $\mathbb{F}\overleftarrow{\mathbb{F}}(S) \subseteq S$; $\overleftarrow{\mathbb{F}}\mathbb{F}(B) \subseteq B$;

(3) $\overleftarrow{\mathbb{F}}(S \cap T) = \overleftarrow{\mathbb{F}}(S) \cap \overleftarrow{\mathbb{F}}(T)$, $\mathbb{F}(B \cup C) = \mathbb{F}(B) \cup \mathbb{F}(C)$;

(4) $\overleftarrow{\mathbb{F}}(S) = \overleftarrow{\mathbb{F}}\mathbb{F}\overleftarrow{\mathbb{F}}(S)$, $\mathbb{F}(B) = \mathbb{F}\overleftarrow{\mathbb{F}}\mathbb{F}(B)$.

Let us consider an operator defined as follows: For each $X \in P(U)$ in a soft context (U, A, F),

$\mathfrak{F} : P(U) \to P(U)$ is an operator defined by $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X)$.

Then X is called an *object oriented soft concept* (simply, *m-concept)* [8] in (U, A, F) if $\mathfrak{F}(X) = \mathbb{F}\overleftarrow{\mathbb{F}}(X) = X$. The set of all m-concepts is denoted by $m(U, A, F)$.

Theorem 2.2 ([8)] *Let (U, A, F) be a soft context. Then we have:*

(1) $\mathfrak{F}(X) \subseteq X$ for $X \subseteq U$.

(2) If $X \subseteq Y$, then $\mathfrak{F}(X) \subseteq \mathfrak{F}(Y)$.

(3) $\mathfrak{F}(\mathfrak{F}(X)) = \mathfrak{F}(X)$ for $X \subseteq U$.

(4) $\mathfrak{F}(\emptyset) = \emptyset$.

(5) $\mathfrak{F}(X)$ is an m-concept.

(6) For $B \subseteq A$, $\mathbb{F}(B)$ is an m-concept.

(7) For $a \in A$, $F(a)$ is an m-concept.

(8) X is an m-concept if and only if there is some $B \subseteq A$ such that $X = \mathbb{F}(B)$.

3. MAIN RESULTS

We assume that a soft set (F, A) is *pure* [5], that is, $\cup_{a \in A} F(a) = U$, $\cap_{a \in A} F(a) = \emptyset$, $F(a) \neq \emptyset$ and $F(a) \neq U$ for each $a \in A$.

Theorem 3.1 *Let (U, A, F) be a soft context. Then for $X, Y \in m(U, A, F)$, $\mathfrak{F}(X \cup Y) = \mathfrak{F}(X) \cup \mathfrak{F}(Y)$.*

Proof 3.2 *Let $X, Y \in m(U, A, F)$. Then by (8) of Theorem 2.2, there are $B, C \subseteq A$ satisfying $\mathbb{F}(B) = X$ and $\mathbb{F}(C) = Y$. Then $X \cup Y = \mathbb{F}(B) \cup \mathbb{F}(C) = \mathbb{F}(B \cup C)$, and so again by Theorem 2.2, $X \cup Y$ is also an m-concept. Consequently, $\mathfrak{F}(X \cup Y) = X \cup Y = \mathfrak{F}(X) \cup \mathfrak{F}(Y)$.*

Example 3.3 *Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{a, b, c, d, e, f\}$. Consider a soft context (U, A, F) where a set-valued mapping $F : A \to P(U)$ is defined by*

$$F(a) = F(d) = \{1, 2, 4\}; \quad F(b) = \{2, 4, 5\};$$

$$F(c) = \{2, 4\}; \quad F(e) = F(f) = \{1, 3, 5\}.$$

For $X = \{1, 2, 4\}$ and $Y = \{1, 3, 5\}$, $\mathfrak{F}(X \cap Y) = \mathfrak{F}(\{1\}) = \emptyset$, $\mathfrak{F}(X) \cap \mathfrak{F}(Y) = \{1, 2, 4\} \cap \{1, 3, 5\} = \{1\}$. So, $\mathfrak{F}(X \cap Y) \neq \mathfrak{F}(X) \cap \mathfrak{F}(Y)$.

From Example 3.2, we know that the family $m(U, A, F)$ is not always a topology on U.

A family σ of X is called a *supra topology* [6] on X if σ satisfies the conditions: (1) $X, \emptyset \in \sigma$; (2) the union of any number of sets in σ belongs to σ.

Theorem 3.4 ([8)] *Let (U, A, F) be a soft context and $\mathbf{Im}(\mathbb{F}) = \{\mathbb{F}(C) \mid \mathbb{F} : P(A) \to P(U),\ C \in P(A)\}$. Then*

(1) $\mathbf{Im}(\mathbb{F}) = m(U, A, F)$:

(2) For $C_1, \cdots, C_n \subseteq A$, $\mathbb{F}(C_1) \cup \mathbb{F}(C_2) \cup \cdots, \mathbb{F}(C_n) \in \mathbf{Im}(\mathbb{F})$.

Theorem 3.5 *Let (U, A, F) be a soft context. Then the family $m(U, A, F)$ is a supra topology on U.*

Proof 3.6 *From Theorem 2.2, it is obtained $U, \emptyset \in m(U, A, F)$. For $X_1, \cdots, X_n \in m(U, A, F)$, there are $C_1, \cdots, C_n \subseteq A$ such that $X_i = \mathbb{F}(C_i)$. So. $X_1 \cup \cdots \cup X_n = \mathbb{F}(C_1) \cup \cdots \cup \mathbb{F}(C_n) \in \mathbf{Im}(\mathbb{F}) = m(U, A, F)$. Consequently, $m(U, A, F)$ is a supra topology on U.*

Let (X, σ) be a supratopological space and $\mathcal{B}$ a family of subsets in X. For each supraopen set $G \in \sigma$, G is a union of any subset of $\mathcal{B}$. Then we will call $\mathcal{B}$ *a base* for σ [6].

Theorem 3.7 *For a soft context (U, A, F), the family $\mathcal{F}_A = \{F(a) \mid a \in A\}$ is a base for $m(U, A, F)$.*

Proof 3.8 *Since the soft set (F, A) is pure, $\cup_{a \in A} F(a) = U$. Let $\mathcal{B} = \emptyset \subsetneq \mathcal{F}_A$. Then $\cup_{F(a) \in \mathcal{B}} F(a) = \emptyset$.*

For any $X \in m(U, A, F)$, from (8) of Theorem 2.2, there is some $B \subseteq A$ such that $X = \mathbb{F}(B) = \cup_{b \in B} F(b)$. So, the family $\mathcal{F}_A = \{F(a) \mid a \in A\}$ is a base for $m(U, A, F)$.

Now, to study the property of $\mathcal{F}_A = \{F(a) \mid a \in A\}$, we introduce the following concepts:

Definition 3.9 *Let (U, A, F) be a soft context. Then for $Z \in m(U, A, F)$,*

(1) Z is said to be dependent on $m(U, A, F)$ if there exist $Z_1, \cdots, Z_n \in m(U, A, F)$ satisfying $Z_i \subsetneq Z$ and $Z = \cup Z_i$, $i = 1, \cdots, n$.

(2) Z is said to be independent of $m(U, A, F)$ if Z is not dependent.

We will denote: $mD = \{Z \in m(U, A, F) \mid Z$ is dependent on $m(U, A, F)\}$;

$mI = \{Z \in m(U, A, F) \mid Z$ is independent of $m(U, A, F)\}$.

Example 3.10 *Let $U = \{1, 2, 3, 4, 5\}$ and $A = \{a, b, c, d, e\}$. Consider a soft context (U, A, F), where the set-valued mapping $F : A \to P(U)$ is defined as follows:*

$$F(a) = \{1, 2, 4\};\ F(b) = \{1, 2, 4, 5\};\ F(c) = \{2, 4\};$$

$$F(d) = \{1, 3\};\ F(e) = \{1, 5\}.$$

Then,

$m(U, A, F) = \{\emptyset, \{1, 3\}, \{1, 5\}, \{2, 4\}, \{1, 2, 4\}, \{1, 3, 5\}, \{1, 2, 3, 4\}, \{1, 2, 4, 5\}, U\}$. For $X = \{1, 2, 4, 5\} \in m(U, A, F)$, we can take two m-concepts $Y = F(c) = \{2, 4\}$ and $Z = F(e) = \{1, 5\}$ in $m(U, A, F)$ satisfying $X \supsetneq Y, Z$ and $X = Y \cup Z$. Hence, X is dependent, while the m-concepts Y, Z are independent.

Theorem 3.11 *Let (U, A, F) be a soft context. Then*

(1) $\emptyset$ and U are dependent.

(2) $mD \cap mI = \emptyset$; $mD \cup mI = m(U, A, F)$.

(3) For $Z \in mD$, there is $C \subseteq A$ satisfying for $c \in C$, $F(c) \subsetneq X$ and $\mathbb{F}(C) = Z$.

(4) For $Z \in mI$, there is $c \in A$ satisfying $F(c) = Z$.

Proof 3.12 *(1) For the empty set $\emptyset$, there is $\mathcal{B} = \{Z \in m(U, A, F) | Z \subsetneq \emptyset\} = \emptyset$. So, $\cup_{Z_i \in \emptyset} Z_i = \emptyset$.*

Now, let $\mathcal{B} = \{Z_i \in m(U, A, F) | Z_i \subsetneq U, i = 1, \cdots, n\}$. Then $\mathcal{B} = m(U, A, F) - \{U\}$. Since the soft set (F, A) is pure, for $a \in A$, $F(a) \in \mathcal{B} = m(U, A, F) - \{U\}$ and $\cup_{a \in A} F(a) = U$ and so, U is dependent.

(2) It is obvious.

(3) For $Z \in mD$, there are $Z_1, \cdots, Z_n \in m(U, A, F)$ such that $Z_i \subsetneq Z$ and $Z = \cup Z_i$, $i = 1, \cdots, n$. From Theorem 2.2, it follows that there are $C_1, \cdots, C_n \in P(A)$ such that $\mathbb{F}(C_i) = Z_i$. Therefore, $\mathbb{F}(C_i) \subsetneq Z$ and $Z = \cup \mathbb{F}(C_i) = \mathbb{F}(\cup C_i)$, $i = 1, \cdots, n$. Put $C = \cup_{i=1} C_i$. Then $C \subseteq A$ and $\mathbb{F}(C) = Z \supsetneq F(c)$ for $c \in C$.

(4) Let $Z \in mI$. Then there is $C \subseteq A$ such that $\mathbb{F}(C) = Z$. Suppose that for every $c \in C$, $Z \supsetneq F(c)$, which contradicts to $Z \in mI$. So, there is an element $d \in C$ satisfying $Z = F(d)$.

Theorem 3.13 *Let (U, A, F) be a soft context. Then for each $X \in mD$, there is a family $\mathcal{B} \subseteq mI$ satisfying $X = \cup \mathcal{B}$.*

Proof 3.14 *Let an m-concept X be dependent. Suppose X cannot be represented as a union of only elements of mI.*

Put $\mathcal{S} = \{X \in mD |\ X$ cannot be represented as a union of elements of $mI\}$.
Then, by hypothesis, $\mathcal{S} \neq \emptyset$ and assume that $|\mathcal{S}| = m < |mD|$ where $|mD|$ is the cardinal number of the set mD. First, pick up one element X in $\mathcal{S}$ (say, X_1). Then since $X_1 \in mD$, there is a family $\mathbf{Y_1} = \{Y_{11}, \cdots, Y_{1l}\}$ satisfying $Y_{1i} \in m(U, A, F)$, $Y_{1i} \subsetneq X_1$ and $X_1 = \cup \mathbf{Y_1}$, $i = 1, \cdots, l$. Additionally, since $X_1 \in \mathcal{S}$, $\mathbf{Y_1} \cap \mathcal{S} \neq \emptyset$. Without the loss of generality, we can choose one dependent m-concept in $\mathbf{Y_1} \cap \mathcal{S}$, say X_2. Then $X_1 \supsetneq X_2$, and since $X_2 \in mD$, there is a family $\mathbf{Y_2} = \{Y_{21}, \cdots, Y_{2m}\}$ such that $X_2 \supsetneq Y_{2i} \in m(U, A, F)$ and $X_2 = \cup \mathbf{Y_2}$, $i = 1, \cdots, m$. And since $X_2 \in \mathcal{S}$, $\mathbf{Y_2} \cap \mathcal{S} \neq \emptyset$.

By repeating this process, finally we can pick up the last element X_m in $\mathcal{S}$ that satisfies $X_1 \supsetneq X_2 \supsetneq, \cdots \supsetneq X_{n-1} \supsetneq X_m$.

Since $X_m \in mD$, there is a family $\mathbf{Y_m} = \{Y_{mi} | Y_{mi} \in m(U, A, F),\ \imath = 1, \cdots, r\}$ satisfying $X_m \supsetneq Y_{mi}$ and $X_m = \cup \mathbf{Y_m}$.

But, since $X_1 \supsetneq X_2 \supsetneq, \cdots \supsetneq X_m$ and $|\mathcal{S}| = m$, $\mathcal{S} \cap \mathbf{Y_m} = \emptyset$. So, X_m is not in $\mathcal{S}$.

Since $X_1 \supsetneq X_2 \supsetneq, \cdots \supsetneq X_{n-1} \supsetneq X_m$ and X_m is not in $\mathcal{S}$, X_{m-1} is also not in $\mathcal{S}$.

For the same reason as X_{m-1}, X_{m-2} is also not in $\mathcal{S}$. In the end, it leads to $\mathcal{S} = \emptyset$, which is a contradiction. So, every *dependent m-concept can be represented as a union of only independent m-concepts of mI.*

Theorem 3.15 *In a soft context (U, A, F), mI is the smallest base for $m(U, A, F)$.*

Proof 3.16 *Let $\mathcal{B}$ be a base and $\mathcal{B} \subsetneq mI$. Then for $X \in mI - \mathcal{B}$, there are $S_1, \cdots S_n \in \mathcal{B}$ such that $X = \cup S_i$, which contradicts to $X \in mI$. So, mI is the smallest base.*

Theorem 3.17 *Let (U, A, F) be a soft context. For $B \subseteq A$, if a set-valued mapping $\varphi : B \to mI$ defined by $\varphi(b) = F(b)$ for $b \in B$ is surjective, then $\varphi(B) = \{F(b) \mid b \in B\}$ is a base for $m(U, A, F)$.*

Proof 3.18 *Obvious.*

Remark 3.19 *Let (U, A, F) be a soft context.*

For mI,

$$m(U, A, F) = \{\cup \mathbf{M} | \mathbf{M} \subseteq mI\}.$$

For $\mathcal{F}_A = \{F(a) | a \in A\}$,

$$m(U, A, F) = \{\cup \mathbf{M} | \mathbf{M} \subseteq \mathcal{F}_A\}.$$

For $B \subseteq A$ and a surjective mapping $\varphi : B \to mI$ defined by $\varphi(b) = F(b)$ for $b \in B$,

$$m(U, A, F) = \{\cup \mathbf{M} | \mathbf{M} \subseteq \varphi(B)\}.$$

For $C \subseteq A$ and a bijective mapping $\psi : C \to mI$ defined by $\psi(c) = F(c)$ for $c \in B$,

$$m(U, A, F) = \{\cup \mathbf{M} | \mathbf{M} \subseteq \psi(C)\}.$$

In summary, we have the size relationships for the above bases as follows: For $B, C \subseteq A$,

$$|mI| = |\psi C| \leq |\varphi B| \leq |\mathcal{F}_A| \leq |m(U, A, F)|$$

4. CONCLUSION

We studied the notion of m-dependent and m-independent soft concepts in a given soft context. Additionally, we showed that every m-dependent soft concept is generated by some m-independent soft concepts. In the next study, we will study the various characteristics of such notions and apply these results to object oriented concepts of a formal context.

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