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Necessary Optimality Conditions for Determining of the Position of the Boundary of Oil Deposit

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Abstract:

The essential criteria was discovered in the thesis to define free boundary over the nonlinear partial differential equations. The task is to find co-ordinates of each point of the border so that to minimize functional. Such task can be treated as an optimum control problem. It is a problem with restriction. In accordance with a method of the interfaced functions we will pass to a problem of optimization without restrictions, having entered new functional. The purpose of the given work is calculation of the first variation of functional. It also defines a necessary condition of optimality for definition of position of border of an oil layer. The considered method for determining free boundaries is based on solving problems of determining the boundary of a system that describes a nonlinear partial differential equation.

Keywords - Differential Equation, Functional, Dirac Function, Ecessary Optimality, Initial and Boundary Conditions, Gradient, Laplacian of the Functions, Variation, Integration.

I. INTRODUCTION

The task of the current study is to find co-ordinates of each point of the border of the oil deposit so that to minimize functional. Such task can be treated as an optimum control problem. It is a problem with restriction.

The purpose of the given work is calculation of the first variation of functional.

This subsection considers the necessary optimality condition for determining of the position of the boundary of the oil deposit.

Let us consider the problem of determining of the boundary for a system described by nonlinear differential equations in partial derivatives.

$$u_t = f(x,y,t,u,u_x,u_y,u_{xx},u_{yy}) \qquad (1)$$

where $u(x,y,t) - m$-dimensional a vector function of the state of the system, defined in a two-dimensional region $\sigma \in E^2$; u_t, u_x, u_{xx} and so on - denoting the partial derivatives with respect to time and spatial coordinates, respectively.

The initial and boundary conditions are given in general form

$$u(x,y,0) = u_0(x,y) \quad (x,y) \in \sigma \qquad (2)$$

$$g(t,x,y,u,u_n) = 0 \quad (x,y) \in \partial\sigma \qquad (3)$$

where u_n denotes the normal derivative of the vector u to the boundary of the region $\partial\sigma$.

II. MATERIAL AND METHODS

In accordance with a method of the interfaced functions it is supposed to pass to a problem of optimization without restrictions, having entered new functional.

The considered method for determining free boundaries is based on solving problems of determining the boundary of a system that describes a nonlinear partial differential equation. Principles for solving ill-defined problems are considered in the works of Zakirov and Lapuk [1], Lions [2], Bubnov [3], Bulygin [4-5], Gutnikov et al. [6-8].

Applied to the inverse problems of the filtration theory in the works of Zhirov [9], Shazhdekeeva and Mukhambetzhanov [10], Shazhdekeeva et al. [11], Frolov [12].

III. RESULTS AND DISCUSSION

The system evolves over a period of time $t \in [0,T]$, during which the measurements are made.

Observations are represented by the q-dimensional (q <m) vector z (x, y, t), which for convenience can be considered a continuous function $z \in E^q(\sigma \times [0,T])$. Suppose that the vector z is related to the state of the system by the relation

$$z(x,y,t) = h(u) + \xi(x,y,t), \qquad (4)$$

where $\xi(x,y,t) -$ measurement error. The problem is to determine the region σ (i.e., finding the coordinates of each boundary point $\partial\sigma$) in such a way as to minimize the functional

$$J = \int_0^T \int_\sigma \int_\sigma [z(x,y,t) - h(u(x,y,t))]' G(x,y,\xi,\eta,t) \times [z(\xi,\eta,t) - h(u(\xi,\eta,t))] dxdyd\xi d\eta dt \qquad (5)$$

The weight matrix $G(x,y,\xi,\eta,t)$ is continuous in its arguments, positive definite and symmetric ($G = G'$).In this case, the problem under consideration can be treated as an optimal control problem, in which the position of the boundary is a control variable.

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In practice, the case of measuring the state vector of a system in M-discrete points (x_j, y_j) is often encountered, i.e. a case

$$z(x_j, y_j, t) = A_j u(x_j, y_j, t) + \zeta_j(t),\ j = 1,2,...,M \qquad (6)$$

where $A_j (j = 1,2,...,M) -$ matrices of dimension ($q \times m$),whose elements are equal to zero or one. In this case, the original functional J can be represented as follows:

$$J = \int_0^T \sum_{j=1}^M [z(x_j, y_j, t) - A_j u(x_j, y_j, t)]' G_j(t) [z(x_j, y_j, t) - A_j u(x_j, y_j, t)] dt \qquad (7)$$

The notation (1.5.7) can be reduced to a more general form (5) by introducing into the weight function σ - a Dirac function. Therefore, in the sequel, for convenience, a more general formula (5) is used.

The optimal control problem formulated above is a problem with the constraint imposed by equation (1.5.1). In accordance with the method of conjugate functions, pass to the optimization problem without restrictions, introducing a new functional

$$J_\psi = J + \int_0^T \int_\sigma \psi'(x, y, t)[f - u_t] dxdydt, \qquad (8)$$

where $\psi(x, y, t) - m$-dimensional conjugate vector-valued function. If $\partial\sigma$ -boundary, minimizing J , then it also minimizes and J_ψ .

Suppose that the boundary $\partial\sigma$ is subjected to a perturbance and transforms $d\sigma^* = d\sigma + \delta(d\sigma)$ with the corresponding transformation of the region σ in $\sigma^* = \sigma + \delta\sigma$. Perturbance of the border $\partial\sigma$ leads to perturbance J_ψ . Our goal is to calculate the first variation J_ψ .

Suppose that a new area σ^* (its coordinates are indicated x^*, y^*), which depends on the parameter ε , can be converted to the original region σ by transformations

$$x^* = \Phi_1(x, y, u, \nabla u, \nabla^2 u; \varepsilon); \qquad (9)$$

$$y^* = \Phi_2(x, y, u, \nabla u, \nabla^2 u; \varepsilon), \qquad (10)$$

where ∇u and $\nabla^2 u$ gradient, and Laplacian of the functions u . The new value of the function $u^* = u^*(x^*, y^*)$ is reduced to the original function $u(x, y)$ by the transformations (9), (10) and the transformation

$$u^* = \Phi_3(x, y, u, \nabla u, \nabla^2 u; \varepsilon) \qquad (11)$$

It is assumed that these transformations are continuous, invertible, differentiable, and that to the values $\varepsilon = 0$ correspond identical transformations

$$x = \Phi_1(x, y, u, \nabla u, \nabla^2 u; 0) \qquad (12)$$

$$y = \Phi_1(x, y, u, \nabla u, \nabla^2 u; 0) \qquad (13)$$

$$u = \Phi_3(x, y, u, \nabla u, \nabla^2 u; 0) \qquad (14)$$

If ε - a is small value, then equation (9) - (11) can be represented in the form

$$x^* = \Phi_1\big|_{\varepsilon=0} + \varepsilon \frac{\partial \Phi_1}{\partial \varepsilon}\Big|_{\varepsilon=0} + O(\varepsilon) = x + \varepsilon\varphi_1 + 0(\varepsilon); \qquad (15)$$

$$y^* = \Phi_2\big|_{\varepsilon=0} + \varepsilon \frac{\partial \Phi_2}{\partial \varepsilon}\Big|_{\varepsilon=0} + 0(\varepsilon) = y + \varepsilon\varphi_2 + 0(\varepsilon); \qquad (16)$$

$$u^* = \Phi_3\big|_{\varepsilon=0} + \varepsilon \frac{\partial \Phi_3}{\partial \varepsilon}\Big|_{\varepsilon=0} + 0(\varepsilon) = y + \varepsilon\varphi_3 + 0(\varepsilon); \qquad (17)$$

The first variations $x,\ y,\ u$ are defined as follows:

$$\delta x = x^* - x = \varepsilon\varphi_1; \qquad (18)$$

$$\delta y = y^* - y = \varepsilon\varphi_2; \qquad (19)$$

$$\delta u = u^*(x^*, y^*) - u(x, y) = \varepsilon\varphi_3 \qquad (20)$$

The first variation J_ψ due to the perturbance is the principal linear part of (relative to ε) of the difference:

$$J_\psi[u^*(x^*, y^*)] - J_\psi[u(x, y)] = \int_0^T \int_{\sigma^*} \int_{\sigma^*} [z - h(u^*)]' \times G(x^*, y^*, \xi^*, \eta^*, t)[z - h(u^*)] dx^* dy^* d\xi^* d\eta^* dt -$$

$$- \int_0^T \int_\sigma \int_\sigma [z - h(u)]' G(x, y, \xi, \eta, t)[z - h(u)] \times dxdyd\xi d\eta dt + \int_0^T \int_{\sigma^*} \psi'[f - u_t^*] dx^* dy^* dt -$$

$$- \int_0^T \int_\sigma \psi'[f - u_t] dxdydt. \qquad (21)$$

Using the Jacobian of the transformation, we reduce equation (21) to the form

$$J_\psi[u^*] - J_\psi[u] = \int_0^T \int_\sigma \int_\sigma [z - h(u^*)]' G [z - h(u^*)] \times \left| \frac{\partial(x^*, y^*, \xi^*, \eta^*)}{\partial(x, y, \xi, \eta)} \right| dxdyd\xi d\eta dt -$$

$$- \int_0^T \int_\sigma \int_\sigma [z - h(u)]' G \times [z - h(u)] dxdyd\xi d\eta dt + \int_0^T \int_\sigma \psi'[f - u_t^*] \times \left| \frac{\partial(x^*, y^*)}{\partial(x, y)} \right| dxdydt -$$

$$- \int_0^T \int_\sigma \psi'[f - u_t] dxdydt \qquad (22)$$

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where

$$\left|\frac{\partial(x^*,y^*,\xi^*,\eta^*)}{\partial(x,y,\xi,\eta)}\right| \approx 1+\varepsilon\frac{\partial\varphi_1}{\partial x}+\varepsilon\frac{\partial\varphi_2}{\partial y}+\varepsilon\frac{\partial\varphi_1}{\partial\xi}+\varepsilon\frac{\partial\varphi_2}{\partial\eta},$$

$$\left|\frac{\partial(x^*,y^*)}{\partial(x,z)}\right| \approx 1+\varepsilon\frac{\partial\varphi_1}{\partial x}+\varepsilon\frac{\partial\varphi_2}{\partial y}.$$

Expanding the integrals in the expression (22) in a Taylor series and preserving terms of the first order with respect to ε , obtain the first variation J_ψ in the form

$$\delta J_\psi = -\int_0^T\int_\sigma\int_\sigma 2[z(\xi,\eta,t)-h(u(\xi,\eta,t))]'G'(x,y,\xi,\eta,t)\times h_u(u(x,y,t))[\overline{\delta u}+u_x\delta x+$$

$$+u_y\delta y]dxdyd\xi d\eta dt + +\int_0^T\int_\sigma\int_\sigma 2[z(x,y,t)-h(u(x,y,t))]'G(x,y,\xi,\eta,t)\times$$

$$\times[z(\xi,\eta,t)-h(u(\xi,\eta,t))][\frac{\partial\delta x}{\partial x}+\frac{\partial\delta y}{\partial y}]dxdyd\xi d\eta dt + +\int_0^T\int_\sigma[\psi'(f-u_t)(\frac{\partial\delta x}{\partial x}+\frac{\partial\delta y}{\partial y})+$$

$$+\delta\psi'(f-u_t)+\psi' f_x\delta x+\psi' f_y\delta y+\psi' f_u\overline{\delta u}+\psi' f_u u_x\delta x+\psi' f_u u_y\delta y+\psi' f_{u_x}\overline{\delta u_x}+$$

$$+\psi' f_{u_x}u_{xy}\delta y+\psi' f_{u_y}\overline{\delta u_y}+\psi' f_{u_y}u_{yx}\delta x+\psi' f_{u_y}u_{yy}\delta y+\psi' f_{u_{xx}}\overline{\delta u_{xx}}+\psi' f_{u_{xx}}u_{xxx}\delta x+\psi' f_{u_x}u_{xx}\delta x+$$

$$+\psi' f_{u_{xx}}u_{xxy}\delta y+\psi' f_{u_{yy}}\overline{\delta u_{yy}}+\psi' f_{u_{yy}}u_{yyx}\delta x+\psi' f_{u_{yy}}u_{yyy}\delta y-\psi'\overline{\delta u_t}-\psi' u_{tx}\delta x-\psi' u_{ty}\delta y]dxdydt, \quad (23)$$

there

$$\delta\psi = \psi(x^*,y^*,t)-\psi(x,y,t) = \psi_x\delta x+\psi_y\delta y, \overline{\delta u} = u^*(x,y,t)-u(x,y,t),$$

The following relations are also used:

$$\delta u = \overline{\delta u}+u_x\delta x+u_y\delta y;$$

$$\delta u_x = \frac{\partial u^*(x^*,y^*,t)}{\partial x^*}-\frac{\partial u(x,y,t)}{\partial x} = \frac{\partial u^*(x,y,t)}{\partial x}-\frac{\partial u(x,y,t)}{\partial x}+u_{xx}\delta x+u_{xy}\delta y = \overline{\delta u_x}+u_{xx}\delta x+u_{xy}\delta y;$$

$$\delta u_y = \overline{\delta u_y}+u_{yx}\delta x+u_{yy}\delta y;$$

$$\delta u_{xx} = \overline{\delta u_{xx}}+u_{xxx}\delta x+u_{xxy}\delta y;$$

$$\delta u_{yy} = \overline{\delta u_{yy}}+u_{yyx}\delta x+u_{yyy}\delta y.$$

In addition, it is assumed that if the point (x,y) does not fall in the region σ^* , then $u^*(x,y)$ it can still be represented in the form $u^*(x^*,y^*)$, where (x^*,y^*) - is the point in σ^* , which corresponds to the point (x,y) be reason of the transformations (9), (10).

Using equations

$$\frac{\partial}{\partial x}(\psi' f\delta x)+\frac{\partial}{\partial y}(\psi' f\delta y) = \psi' f\delta x_x+\psi' f\delta y_y+\frac{\partial\psi'}{\partial x}f\delta x+\frac{\partial\psi'}{\partial y}f\delta y+\psi' f_x\delta x+\psi' f_u u_x\delta x+\psi' f_{u_x}u_{xx}\delta x+$$

$$+\psi' f_{u_y}u_{yx}\delta x+\psi' f_{u_{xx}}u_{xxx}\delta x+\psi' f_{u_{yy}}u_{yyx}\delta x+\psi' f_y\delta y+\psi' f_u u_y\delta y+\psi' f_{u_x}u_{xy}\delta y+\psi' f_{u_y}u_{yy}\delta x+$$

$$+\psi' f_{u_{xx}}u_{xxy}\delta y+\psi' f_{u_{yy}}u_{yyy}\delta y; \quad (24)$$

$$\frac{\partial}{\partial x}(\psi' u_t\delta x)+\frac{\partial}{\partial y}(\psi' u_t\delta y) = \frac{\partial\psi'}{\partial x}u_t\delta x+\psi' u_{tx}\delta x+\psi' u_t\delta x_x+\frac{\partial\psi'}{\partial y}u_t\delta y+\psi' u_{ty}\delta y+\psi' u_t\delta y_y; \quad (25)$$

$$\psi' f_{u_x}\overline{\delta u_x}+\psi' f_{u_y}\overline{\delta u_y} = \frac{\partial}{\partial x}(\psi' f_{u_x}\overline{\delta u})-\frac{\partial}{\partial x}(\psi' f_{u_x})\overline{\delta u}+\frac{\partial}{\partial y}(\psi' f_{u_y}\overline{\delta u})-\frac{\partial}{\partial y}(\psi' f_{u_y})\overline{\delta u}; \quad (26)$$

$$\psi'\overline{\delta u_t} = \frac{\partial}{\partial t}(\psi'\overline{\delta u})-\frac{\partial\psi'}{\partial t}\overline{\delta u}; \quad (27)$$

$$\psi' f_{u_{xx}}\overline{\delta u_{xx}}+\psi' f_{u_{yy}}\overline{\delta u_{yy}} = -\frac{\partial}{\partial x}[(\psi' f_{u_{xx}})_x\overline{\delta u}]+\frac{\partial^2}{\partial x^2}(\psi' f_{u_{xx}})\overline{\delta u}+\frac{\partial}{\partial x}(\psi' f_{u_{xx}}\overline{\delta u_x})-\frac{\partial}{\partial y}[(\psi' f_{u_{yy}})_y\overline{\delta u}]+$$

$$+\frac{\partial^2}{\partial y^2}(\psi' f_{u_{yy}})\overline{\delta u}+\frac{\partial}{\partial y}(\psi' f_{u_{yy}}\overline{\delta u_y}) \quad (28)$$

and the relation $\overline{\delta u}(x,y,0)=0$, obtain the first variation of the functional δJ_ψ in the form

$$\delta J_\psi = \int_0^T\int_\sigma\{\frac{\partial\psi'}{\partial t}+\psi' f_u-\frac{\partial}{\partial x}(\psi' f_{u_x})-\frac{\partial}{\partial y}(\psi' f_{u_y})+\frac{\partial^2}{\partial x^2}(\psi' f_{u_{xx}})+\frac{\partial^2}{\partial y^2}(\psi' f_{u_{yy}})-2\int_\sigma[z(\xi,\eta,t)-$$

$$-h(u(\xi,\eta,t))]'G'(x,y,\xi,\eta,t)h_u(u(x,y,t))d\xi d\eta\}\times\overline{\delta u}dxdydt+\int_0^T\int_\sigma\{\frac{\partial}{\partial x}[\psi' f_{u_x}\overline{\delta u}-(\psi' f_{u_{xx}})_x\overline{\delta u}+$$

$$+\psi' f_{u_{xx}}\overline{\delta u_x}+(\int_\sigma[z(x,y,t)-h(u(x,y,t))]'\times G(x,y,\xi,\eta,t)[z(\xi,\eta,t)-h(u(\xi,\eta,t))]d\xi d\eta)\delta x]+$$

$$+\frac{\partial}{\partial y}[\psi' f_{u_y}\overline{\delta u}-(\psi' f_{u_{yy}})_y\overline{\delta u}+\psi' f_{u_{yy}}\delta u_y+2(\int_\sigma[z(x,y,t)-h(u(x,y,t))]'G(x,y,\xi,\eta,t)\times[z(\xi,\eta,t)-$$

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$$-h(u(\xi,\eta,t))]d\xi d\eta)\delta y]\}dxdydt-\int_{\sigma}\psi'\overline{\delta u}\,\Big|_{t=T}dxdy\,. \qquad (29)$$

Let the conjugate vector $\psi(x,y,t)$ satisfy the system of equations

$$\psi_t=-f'_u\psi+(f'_{u_x}\psi)_x+(f'_{u_y}\psi)_y-(f'_{uxx}\psi)_{xx}-(f'_{u_{yy}}\psi)_{yy}+2\int_{\sigma}h'_u(x,y,t)G(x,y,\xi,\eta,t)\times$$

$$\times[z(\xi,\eta,t)-h(u(\xi,\eta,t))]d\xi d\eta \qquad (30)$$

with the condition at the end of the time interval

$$\psi(x,y,T)=0\,. \qquad (31)$$

The substitution of the equations (30) and (31) into the expression (29) and the application of the Gauss-Ostrogradskii theorem gives the following representation for the variation of the functional:

$$\delta J_{\psi}=\int_0^T\int_{\partial\sigma}\{[\psi'f_{u_x}-(\psi'f_{u_{xx}})_x]\cos\nu+[\psi'f_{u_y}-(\psi'f_{u_{yy}})_y]\sin\nu\}\overline{\delta u}dsdt+$$

$$+\int_0^T\int_{\partial\sigma}(\psi'f_{u_{xx}}\overline{\delta u_x}\cos\nu+\psi'f_{u_{yy}}\overline{\delta u_y}\sin\nu)dsdt+\int_0^T\int_{\partial\sigma}\{2\int_{\sigma}[z(x,y,t)-h(u(x,y,t))]'G(x,y,\xi,\eta,t)\times$$

$$\times[z(\xi,\eta,t)-h(u(\xi,\eta,t))]d\xi d\eta\}[\delta x\cos\nu+\delta y\sin\nu]dsdt, \qquad (32)$$

where ν – the angle between the positive direction of the x axis and the outer normal to the boundary of the region $\partial\sigma$, s - the arc length of the contour $\partial\sigma$.

To determine the boundary conditions for a function ψ it is more convenient to use a δu. Expressing $\overline{\delta u}$ through δu and substituting this expression in (32), obtain

$$\delta J_{\psi}=\int_0^T\int_{\partial\sigma}\{[\psi'f_{u_x}-(\psi'f_{u_{xx}})_x]\cos\nu+[\psi'f_{u_y}-(\psi'f_{u_{yy}})_y]\sin\nu\}\delta udsdt+$$

$$+\int_0^T\int_{\partial\sigma}(\psi f_{u_{xx}}-\overline{\delta u_x}\cos\nu+\psi f_{u_{yy}}\overline{\delta u_y}\sin\nu)dsdt+$$

$$+\int_0^T\int_{\partial\sigma}\{S(x,y,t,)(\delta x\cos\nu+\delta y\sin\nu)-[(\psi'f_{u_x}-(\psi'f_{u_{xx}})_x)\cos\nu+$$

$$+(\psi'f_{u_y}-(\psi'f_{u_{yy}})_y)\sin\nu][u_x\delta x+u_y\delta y)\}dsdt, \qquad (33)$$

where is denoted

$$S(x,y,t,)=2\int_{\sigma}[z(x,y,t)-h(u(x,y,t))]'G(x,y,\xi,\eta,t)[z(\xi,\eta,t)-h(u(\xi,\eta,t))]d\xi d\eta$$

If we express both $\overline{\delta u_x}$ and $\overline{\delta u_y}$ in terms of the normal $\overline{\delta u_n}$ and tangential $\overline{\delta u_s}$ derivatives of the function u on $\partial\sigma$, and also assume that the contour $\partial\sigma$ is closed, then equation (1.5.33) is transformed to the form

$$\delta J_{\psi}=\int_0^T\int_{\partial\sigma}\{[\psi'f_{u_x}-(\psi'f_{u_{xx}})_x]\cos\nu+[\psi'f_{u_y}-(\psi'f_{u_{yy}})_y]\sin\nu+$$

$$+\frac{\partial}{\partial x}\left[\psi'f_{u_{xx}}\sin\nu\cos\nu-\psi'f_{u_{yy}}\cos\nu\sin\nu\right]\}\delta udsdt+$$

$$+\psi'f_{u_{yy}}\sin^2\nu)\overline{\delta u_n}dsdt+\int_0^T\int_{\partial\sigma}\{S(x,y,t)(\delta x\cos\nu+\delta y\sin\nu)-[(\psi'f_{u_x}-(\psi'f_{u_{xx}})_x)\cos\nu+$$

$$+$$

$$(\psi'f_{u_y}-(\psi'f_{u_{yy}})_y\sin\nu][u_x\delta x+u_y\delta y]-\frac{\partial}{\partial s}(\psi'f_{u_{xx}}\sin\nu\cos\nu-$$

$$-\psi'f_{u_{yy}}\sin\nu\cos\nu)(u_x\delta x+u_y\delta y)\}dsdt \qquad (34)$$

For convenience of calculations, the variation $\overline{\delta}u_n=\partial u^*(x,y)/\partial n-\partial u(x,y)/\partial n$ can be expressed in terms of $\delta u_n=\partial u^*(x^*,y^*)/\partial n^*-\partial u(x,y)/\partial n.$

Indeed, since the angle between n and n^*, obtain

$$\frac{\partial u^*(x,y)}{\partial n}-\frac{\partial u(x,y)}{\partial n}=\frac{\partial u^*(x^*,y^*)}{\partial n^*}-\frac{\partial u(x,y)}{\partial n}-\frac{\partial^2u(x,y)}{\partial n^2}\delta n-\frac{\partial^2u(x,y)}{\partial n\partial s}\delta s, \qquad (35)$$

where n - normal direction to $\partial\delta$ at the point (x,y); s - the tangent direction to $\partial\delta$ at the point (x, y); n^* - the normal direction to the perturbed boundary $\partial\sigma^*$ at the point (x^*,y^*), which after deformation corresponds to the point (x,y) on $\partial\delta$; ∂n and ∂s – are the normal and tangential components of the variations δx and δy.

The substitution of (35) into (34) gives

$$\delta J_{\psi}=\int_0^T\int_{\partial\sigma}\{[\psi'f_{u_x}-(\psi'f_{u_{xx}})_x]\cos\nu+[\psi'f_{u_y}-(\psi'f_{u_{yy}})_y]\sin\nu+\frac{\partial}{\partial x}[\psi'f_{u_{xx}}\sin\nu\cos\nu-$$

$$+\frac{\partial}{\partial x}\psi'f_{u_{xx}}\sin\nu\cos\nu-$$

$$\psi'f_{u_{yy}}\sin\nu\cos\nu]\}\delta udsdt+$$

$$+\int_0^T\int_{\partial\sigma}(\psi'f_{u_{xx}}\cos^2\nu+\psi'f_{u_{yy}}\sin^2\nu)\delta u_ndsdt-\int_0^T\int_{\partial\sigma}(\psi'f_{u_{xx}}\cos^2\nu+$$

$$+\psi'f_{u_{yy}}\sin^2\nu)(\frac{\partial^2u}{\partial n^2}\delta n+\frac{\partial^2u}{\partial n\partial s}\delta s)dsdt+\int_0^T\int_{\partial\sigma}\{S(x,y,t)(\delta x\cos\nu+\delta y\sin\nu)-$$

$$-[(\psi'f_{u_x}-(\psi'f_{u_{xx}})_x\cos\nu+(\psi'f_{u_y}-(\psi'f_{u_{yy}})_y)\sin\nu)][u_x\delta x+u_y\delta y]-$$

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$$-\frac{\partial}{\partial s}(\psi^{/} f_{u_{xx}} \sin\nu\cos\nu - \psi^{/} f_{u_{yy}} \sin\nu\cos\nu)(u_x\delta x + u_y\delta y)\}dsdt \tag{36}$$

It follows from (3) that

$$g_x\delta x + g_y\delta y + g_u\delta u + g_{u_n}\delta u_n = 0,$$
$$x, y \in \partial\sigma,$$
$$x^*, y^* \in \partial\sigma^* \tag{37}$$

Solving (37) with respect to δu_n and substituting the result in (36), obtain

$$\delta J_\psi = \int_0^T\int_{\partial\sigma}\{[\psi' f_{u_x} - (\psi' f_{u_{xx}})_x]\cos\nu + [\psi' f_{u_y} - (\psi' f_{u_{yy}})_y]\sin\nu + \frac{\partial}{\partial s}[\psi' f_{u_{xx}}\sin\nu\cos\nu -$$

$$-\psi' f_{u_{yy}}\sin\nu\cos\nu - (\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)g^{-1}{}_{u_n} g_u\}\delta u dsdt +$$

$$\int_0^T\int_{\partial\sigma} \cos\nu -$$

$$-[(\psi' f_{u_x} - (\psi' f_{u_{xx}})_x)\cos\nu + (\psi' f_{u_y} - (\psi' f_{u_{yy}})_y)\sin\nu]u_x - \frac{\partial}{\partial s}(\psi' f_{u_{xx}}\sin\nu\cos\nu -$$

$$-\psi' f_{u_{yy}}\sin\nu\cos\nu)u_x - (\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu g^{-1}{}_{u_n} g_x - (\psi' f_{u_{xx}}\cos^2\nu +$$

$$\psi' f_{u_{yy}}\sin^2\nu(\frac{\partial^2 u}{\partial n^2}\cos\nu - \frac{\partial^2 u}{\partial n\partial s}\sin\nu)\}\delta x dsdt + \int_0^T\int_{\partial\sigma}\{S(x,y,t)\sin\nu -$$

$$[(\psi' f_{u_x} - (\psi' f_{u_{xx}})_x)\cos\nu +$$

$$(\psi' f_{u_y} - (\psi' f_{u_{yy}})_y)\sin\nu]u_y - \frac{\partial}{\partial s}(\psi' f_{u_{xx}}\sin\nu\cos\nu - \psi' f_{u_{yy}}\sin\nu\cos\nu)u_y - (\psi' f_{u_{xx}}\cos^2\nu +$$

$$+$$

$$\psi' f_{u_{yy}}\sin^2\nu)g^{-1}{}_{u_n} g_y - (\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)(\frac{\partial^2 u}{\partial n^2}$$

$$\sin\nu + \frac{\partial^2 u}{\partial n\partial s}\cos\nu)\}\delta y dsdt. \tag{38}$$

We now choose boundary conditions for equation (30), so that the first term on the right-hand side of expression (38) vanishes. In view of the arbitrariness of the variation δu on the contour, obtain

$$[\psi' f_{u_x} - (\psi' f_{u_{xx}})_x]\cos\nu + [\psi' f_{u_y} - (\psi' f_{u_{yy}})_y]\sin\nu + \frac{\partial}{\partial s}[\psi' f_{u_{xx}}\sin\nu\cos\nu - \psi' f_{u_{yy}}\sin\nu\cos\nu] -$$

$$-(\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)g^{-1}{}_{u_n} g_u = 0. \tag{39}$$

Substituting (30) into (29) and changing the order of integration, obtain the final expression

$$\delta J_\psi = \int_{\partial\sigma} L_1(x,y)\delta x(x,y)ds + \int_{\partial\sigma} L_2(x,y)\delta y(x,y)ds, \tag{40}$$

where

$$L_1(x,y) = \int_0^T\{S(x,y,t)\cos\nu - [(\psi' f_{u_x} - (\psi' f_{u_{xx}})_x)\cos\nu + (\psi' f_{u_y} - (\psi' f_{u_{yy}})_y)\sin\nu]u_x -$$

$$-\frac{\partial}{\partial s}(\psi' f_{u_{xx}}\sin\nu\cos\nu - \psi' f_{u_{yy}}\sin\nu\cos\nu)u_x - (\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)g^{-1}{}_{u_n} g_x -$$

$$-(\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)(\frac{\partial^2 u}{\partial n^2}\cos\nu - \frac{\partial^2 u}{\partial n\partial s}\sin\nu)\}dt, \tag{41}$$

$$L_2(x,y) = \int_0^T\{S(x,y,t)\sin\nu - [(\psi' f_{u_x} - (\psi' f_{u_{xx}})_x)\cos\nu + (\psi' f_{u_y} - (\psi' f_{u_{yy}})_y)\sin\nu]u_y -$$

$$-\frac{\partial}{\partial s}(\psi' f_{u_{xx}}\sin\nu\cos\nu - \psi' f_{u_{yy}}\sin\nu\cos\nu)u_y - (\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)g^{-1}{}_{u_n} g_y -$$

$$-(\psi' f_{u_{xx}}\cos^2\nu + \psi' f_{u_{yy}}\sin^2\nu)(\frac{\partial^2 u}{\partial n^2}\sin\nu + \frac{\partial^2 u}{\partial n\partial s}\cos\nu)\}dt, (x,y)\in\partial\sigma \tag{42}$$

Note that the perturbance of the contour $\delta(\partial\sigma)$ is represented by perturbance of the corresponding coordinates δx and δy contour at each point of the originally defined boundary. The necessary conditions for optimality (of the first order) will be the conditions:

$$L_1(x,y) = 0, \tag{43}$$

$$L_2(x,y) = 0. \tag{44}$$

IV. CONCLUSIONS

The article discovered necessary optimality conditions for determining of the position of the boundary of oil deposit. The essential criteria was discovered in the thesis to define free boundary over the nonlinear partial differential equations.

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