Optimization of Transmission Ratios for Two-stage Bevel Helical Gearboxes Based on Mass Function

Tran Thi Hong[1], Vu Trung Tuyen[2], Bui Thanh Danh[3], Tran Thi Phuong Thao[4], Nguyen Thi Thanh Nga[4], Tran Ngoc Giang[4], Nguyen Thanh Tu[4] and Vu Ngoc Pi[4,*]

[1]*Center of Excellence for Automation and Precision Mechanical Engineering, Nguyen Tat Thanh University, Vietnam.*

[2]*National Research Institute of Mechanical Engineering, Vietnam.*

[3]*Mechanical Engineering Faculty, University of Transport and Communications, Vietnam.*

[4]*Mechanical Engineering Faculty, Thai Nguyen University of Technology, Vietnam.*

Abstract:

This study presents a methodology to compute the mass of a two-stage bevel helical gearbox. The objective function depending on the transmission ratios of two stages was examined with the constraint conditions of the important input parameters. These input parameters were given for designing the screening experiment with the help of Minitab 19. The results indicate that not only input parameters but also their interactions have important influences on the optimal transmission ratios. Based on the regression and variance analysis, a mathematical model to compute the optimum transmission ratios was resulted as an explicit equation. The reliability of the mathematical model is verified by the results of the screening experiment. Therefore, the transmission ratios are determined simply. This can increase the economic and technical effectiveness in gearbox design process.

Keywords: Bevel Gearboxes, Gear ratio, Gearbox design, Optimization gearbox design.

1. INTRODUCTION

In a process of calculating gearboxes, the selection of the partial transmission ratios plays an important role because of its effects on the size, mass, and cost of gearboxes. In regard to methods for determining the partial transmission ratios, the graphical method was investigated in [1–6], which normally uses graphs to calculating gear ratios in a design process. With the practical method, partial transmissions ratios are commonly given in a table by the professional experiences from expert designers [7]. For the model method as found in [8–11], gear ratios are regularly led to mathematical models. For concerning the study on types of gearboxes, the partial transmission ratios have been investigated for two-step gearboxes [5,6,12–16], for three-step gearboxes [17–22], and for four-step gearboxes [4,23].

In recent years, there have been several researches regarding bevel gears. The calculation of gear ratios for three-stage bevel helical gearboxes has been investigated in [19,24]. Vu and Nguyen [25] proposed the determination partial ratios of a chain drive and gear ratios of a two-step bevel helical gearbox. In this work, both the partial ratio of a chain drive and gear ratios were obtained by minimizing the cross-section function. Besides, examining a two-step bevel helical reducer was also found in [8]. The gear ratios were determined based on the minimum reducer length. However, for two-step bevel helical gearboxes, the optimization problem related to the gearbox mass has not been investigated. Moreover, various factors affecting gear ratios also have not been mentioned in these previous works. Therefore, in this study, we present a methodology to determine transmission ratios based on minimizing the gearbox mass function. Additionally, several input parameters were considered to discuss the effects of them on the optimal transmission ratios. To do this, the screening experiment was conducted with the input parameters. The regression and variance were used to build the explicit equation of the optimum gear ratio.

2. OPTIMIZATION PROBLEM

This section presents the procedure to compute the objective function based on the mass of a two-stage bevel helical gearbox. The diagram of a two-stage bevel helical gearbox is described in Figure 1. The main components of this gearbox include the gears, the gearbox housing, and the shafts, which mainly affect the mass of the gearbox. Therefore, the gearbox mass, denoted by m_{gb}, can be calculated by

$$m_{gb} = m_g + m_{gh} + m_s \qquad (1)$$

Where, m_g, m_{gh} and m_s are the gear mass, the gearbox housing mass, and the shaft mass, respectively.
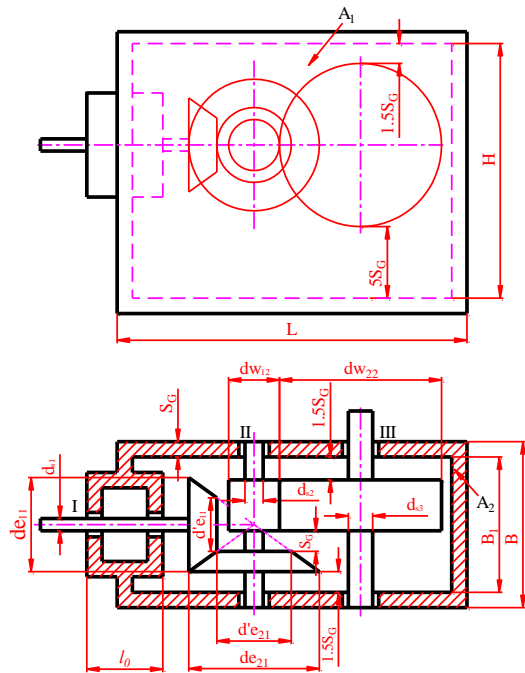

Figure 1. Diagram of calculating gearbox mass for a two-stage bevel helical gearbox

2.1. Gear mass calculation

As shown in Figure 1, there are two bevel gears of the first stage and two helical gears of the second stage. The total mass of gears is therefore written as

$$m_g = m_{bg1} + m_{bg2} + m_{hg1} + m_{hg2} \qquad (2)$$

In which, m_{bg1} and m_{bg2} are representative for the mass of the pinion and bevel gears (kg), and m_{hg1} and m_{hg2} are the mass of the pinion and helical gears (kg), respectively. In general, the mass of each gear can be computed by multiplying the weight density and the volume of gear. Thus, m_{bg1}, m_{bg2}, m_{hg1}, and m_{hg2} can be calculated by:

$$m_{bg1} = \rho_g \cdot \{(\pi/3) \cdot [d_{e11}^2 \cdot R_e \cdot cos\delta_1/4 - d_{e11}'^2 \cdot (R_e - b) \cdot cos\delta_1/4]\} \qquad (3)$$

$$m_{bg2} = \rho_g \cdot \{(\pi/3) \cdot [d_{e21}^2 \cdot R_e \cdot cos\delta_2/4 - d_{e21}'^2 \cdot (R_e - b) \cdot cos\delta_2/4]\} \qquad (4)$$

$$m_{hg1} = \rho_g \cdot \pi \cdot e_1.d_{w12}^2 \cdot b_w/4 \qquad (5)$$

$$m_{hg2} = \rho_g \cdot \pi \cdot e_2.d_{w22}^2 \cdot b_w/4 \qquad (6)$$

In Eqs (3) and (4), ρ_g is the weight density of gear material (kg/m³); with gear made by steel material, $\rho_g = 7.82$ [26]; b is the gear width of the bevel gear set (mm); $b = k_{be} \cdot R_e$

(mm) [27]; R_e is the cone distance (mm) which can be calculated by [27]:

$$R_e = k_R \cdot \sqrt{u_1^2 + 1} \cdot \sqrt[3]{T_{11} \cdot k_{h\beta1}/[(1 - k_{be}) \cdot k_{be} \cdot u_1 \cdot [\sigma_H]^2]} \quad (7)$$

In here, k_R is face width coefficient; for a steel straight bevel gear, $k_R = 50$ (MPa) [27]; $k_{H\beta}$ is the contacting load ratio for pitting resistance; for a straight bevel gear set $k_{H\beta} = 1.04 \div 1.18$ [27] and choosing $k_{H\beta} = 1.11$; k_{be} is the face width coefficient, $k_{be} = 0.25 \div 0.3$ and choosing $k_{be} = 0.27$; T_{11} is the torque on the pinion; d_{e11} and d_{e21} are respectively the outer pitch diameters of the pinion and the gear. With straight bevel gear set, T_{11}, d_{e11} and d_{e21} can be computed as [27]:

$$T_{11} = T_{out}/(u_g \cdot \eta_{bg} \cdot \eta_{hg} \cdot \eta_{be}^3) \qquad (8)$$

$$d_{e11} = 2 \cdot R_e/(1 + u_1^2)^{1/2} \qquad (9)$$

$$d_{e21} = 2 \cdot u_1 \cdot R_e/(1 + u_1^2)^{1/2} \qquad (10)$$

In which, u_g is the total gearbox ratio; η_{bg} is the efficiency of the bevel gear set, $\eta_{hg} = 0.95 \div 0.97$ [27]; η_{hg} is the efficiency of helical gear set, $\eta_{hg} = 0.96 \div 0.98$ [27]; η_{be} is the efficiency of a rolling bearing pair $\eta_{be} = 0.99 \div 9.995$ [27]. Choosing $\eta_{bg} = 0.96$; $\eta_{hg} = 0.97$, $\eta_{be} = 0.992$, T_{11} can be rewritten as:

$$T_{11} = 1.1001 \cdot T_{out}/u_g \qquad (11)$$

In Eqs. (5) and (6), e_1 and e_2 are volume coefficients of the pinion and the gear of the helical gear set; $e_1 = 1$ and $e_2 = 0.6$ [27]; b_w is the gear width of the helical gear set (mm); $b_w = X_{ba} \cdot a_w$ (mm) [27]; d_{w12} and d_{w22} are the pitch diameters of the second step which can be expressed as [27]:

$$d_{w12} = 2 \cdot a_w/(u_2 + 1) \qquad (12)$$

$$d_{w22} = 2 \cdot a_w \cdot u_2 \cdot/(u_2 + 1) \qquad (13)$$

In Eq. (12) and (13), a_w is the center distance of the helical gear set which is found by [27]:

$$a_w = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta}/([\sigma_{H2}]^2 \cdot u_2 \cdot X_{ba})} \quad (14)$$

With

$$T_{12} = T_{out}/(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2) \qquad (15)$$

and $\eta_{hg} = 0.97$, $\eta_{be} = 0.992$ as mentioned above, Eq. (15) can be expressed as:

$$T_{12} = 1.0476 \cdot T_{out}/u_2 \qquad (16)$$

In Eq. (14), $k_{H\beta}$ is the contacting load ratio for pitting resistance; for the helical gear set $k_{H\beta} = 1.01 \div 1.16$ [27]. It

can be chosen $k_{H\beta} = 1.08$; $[\sigma_{H2}]$ is the allowable contact stress of the first stage (MPa); k_a is the material coefficient; as the gear material is steel, $k_a= 43$ [27]; X_{ba} is the coefficient of wheel face width of the helical gear set.

2.2. Determination of gearbox housing mass

Using a similar procedure as shown in Sec. 2.1, the gearbox housing mass can be computed as

$$m_{gh} = \rho_{gh} \cdot V_{gh} \tag{17}$$

In Eq. (17), ρ_{gh} is the weight density of gearbox housing material, $\rho_{gh} = 7,2$ (kg/m3) with the cast iron material [26]; V_{gh} is the volume of the gearbox housing (m³). For determination of V_{gh}, the volume gearbox housing can be separated into three parts i.e., V_{A1} is the volume of font and back sides of the gearbox; V_{A2} is the volume of the left and right sides, and V_b is the volume of the top and bottom sides. Thus, V_{gh} can be written as

$$V_{gh} = 2 \cdot V_b + 2 \cdot V_{A1} + 2 \cdot V_{A2} \tag{18}$$

In which V_b, V_{A1} and V_{A2} can be expressed as

$$V_b = L \cdot B \cdot S_G \tag{19}$$

$$V_{A1} = L \cdot H \cdot S_G \tag{20}$$

$$V_{A2} = B_1 \cdot H \cdot S_G = (B - 2 \cdot S_G) \cdot H \cdot S_G \tag{21}$$

In Eqs. (19), (20), and (21), L, H, B_1 and S_G can be determined by:

$$L = (2 \cdot l_0/3 + d_{e21}/2 + d_{w12}/2 + d_{w22} + 2.5)/0.975 \text{ [28]} \tag{22}$$

$$H = \max(d_{e21}; d_{w22}) + 6.5 \cdot S_G \tag{23}$$

$$B = b \cdot cos\delta_2 + b_w + 4 \cdot S_G \tag{24}$$

$$S_G = 0.005 \cdot L + 4.5 \text{ [28]} \tag{25}$$

In here, $l_0 = 3 \cdot d_{s1}$ [27] with d_{s1} is the diameter of the first shaft which will be determined in the next section; δ_2 is the gear pitch cone angle and $tan\delta_2 = u_1$.

2.3. Shaft mass calculation

As shown in Figure 1, the mass of all shafts can be expressed as

$$m_s = m_{s1} + m_{s2} + m_{s3} \tag{26}$$

In which, m_{s1}, m_{s2} and m_{s3} are respectively the mass of shaft 1, 2 and 3 of the gearbox (kg). They can be computed as

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1}/4 \tag{27}$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2}/4 \tag{28}$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3}/4 \tag{29}$$

In Eqs. (27), (28), and (29) ρ_s is the weight density of shaft material (kg/m³); l_{s1}, l_{s2} and l_{s3} are the length of shaft 1, 2 and 3 of the gearbox. They can be written as (see Figure 1)

$$l_{s1} = 1.3 \cdot l_0 + 1.2 \cdot d_{s1} \tag{30}$$

$$l_{s2} = B \tag{31}$$

$$l_{s3} = B + 1.2 \cdot d_{s3} \tag{32}$$

In Eqs. (30), (31), and (32), preliminary diameters of the first, second and third shafts i.e., d_{s1}, d_{s2}, and d_{s3} are described as [27]:

$$d_{s1} = [T_{11}/(0.2 \cdot [\tau])]^{1/3} \tag{33}$$

$$d_{s2} = [T_{12}/(0.2 \cdot [\tau])]^{1/3} \tag{34}$$

$$d_{s3} = [T_{13}/(0.2 \cdot [\tau])]^{1/3} \tag{35}$$

In which, $[\tau]$ is the allowable shear stress (MPa); $[\tau] = 15 \div 20$ (MPa) [27] and we can choose $[\tau] = 17$ (Mpa).

2.4. Objective function

The objective function as shown in Eq. (1) must be minimized

$$Minimize \ m_{gb} \tag{36}$$

From the above procedure, all calculation parameters of the gearbox mass depend on the transmission ratios of the two stages in the gearbox, i.e., $u_1 \ and \ u_2$. Thus, the objective function in Eq. (1) can be expressed as

$$m_{gb} = m_{gb}(u_1, u_2) \tag{37}$$

Subject to the following constraints:

$$1 \leq u_1 \leq 9 \tag{38}$$

$$1 \leq u_2 \leq 9$$

As known that the total ratio of the gearbox, u_g as shown in Eqs. (8) and (11)**Error! Reference source not found.** has the relation with the transmission ratio of the two stages, i.e., $u_g = u_1.u_2$. In general, u_g should be used in an interval. Therefore, the objective function in Eq. (37) can be rewritten as

$$Minimize \ m_{gb} = m_{gb}(u_1) \tag{39}$$

3. EXPERIMENTAL DESIGN

The optimum transmission ratio of the first stage u_1 is obtained by solving Eq. (39) with a computation program. From the relation between the total ratio of the gearbox and gear ratios, the transmission ratio of the second stage is simply determined, i.e., $u_2 = u_g / u_1$. Furthermore, a screening

experiment using Minitab 19 was established to depict the effects of input parameters on the optimum transmission ratio u_1. As presented in Sec. 2, the input parameters such as the total gearbox ratio u_g, the coefficient of the face width of the bevel gear set K_{be}, the coefficient of wheel face width of the second stage X_{ba}, the allowable contact stress of the first and second stages AS_1 and AS_2, and the output torque T_{out} were

taken to design the screening experiment. The value ranges of the input parameters are shown in Table 1. Using 2-level factorial in Minitab with the six parameters, the number of experiments is 2^6 i.e., 64 test runs. The experimental design of the input parameters and the output response values are presented in Table 2.

Table 1. Input parameters and their level

Factor	Code	Unital	Low	High
Total gearbox ratio	u_g	-	7	25
Coefficient of the face width of bevel gear set	K_{be}	-	0.25	0.3
Coefficient of wheel face width of the second stage	X_{ba}	-	0.3	0.4
Allowable contact stress of the first stage	AS_1	MPa	360	420
Allowable contact stress of the second stage	AS_2	MPa	360	420
Output torque	T_{out}	Nm	1000	10000

Table 2. Experimental design of input parameters and output response

StdOrder	RunOrder	CenterPt	Blocks	u_g	K_{be}	X_{ba}	AS_1	AS_2	T_{out}	u_1
64	1	1	1	25	0.3	0.4	420	420	10000	5.27
16	2	1	1	25	0.3	0.4	420	360	1000	5.74
52	3	1	1	25	0.3	0.3	360	420	10000	5.03
34	4	1	1	25	0.25	0.3	360	360	10000	5.61
4	5	1	1	25	0.3	0.3	360	360	1000	5.51
44	6	1	1	25	0.3	0.3	420	360	10000	5.7
…										
62	63	1	1	25	0.25	0.4	420	420	10000	5.46
35	64	1	1	7	0.3	0.3	360	360	10000	1.83

4. RESULTS AND DISCUSSIONS

This section presents the results obtained from the screening experiment above. The results are exhibited as the following.

4.1. The influence of input parameters on the optimum transmission ratio

Figure 2 shows the effect of each factor on the optimum transmission ratio of the first stage u_1. It is clearly seen that the relation between the optimal transmission ratio and each factor is described by the straight lines. As observed in Figure 2, the straight line of u_g has the largest slope angle among the

input parameters; thus, u_g brings the strongest influence on the optimum transmission. The other input parameters have less effects on the output respond compared to the total gearbox ratio.
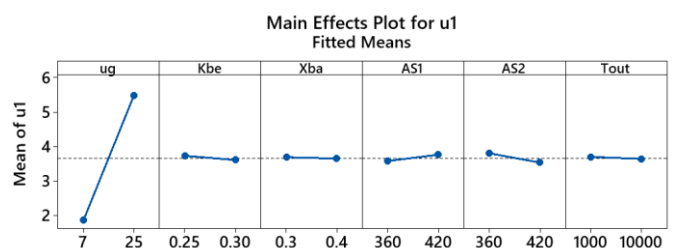

Figure 2. Effects of the factors on the output respond u_1

In order to evaluate the trend of the effects of the input parameters on the output response, the normal plot of the standardized effects presents the distribution of the input parameters and their interactions as shown in Figure 3. It is observed from Figure 3 that positive effects of the factors and interactions (see the squares in the right side of the reference line) increase the optimum transmission ratio u_1 when the settings change from the low value of the factors and interactions to the high value. Otherwise, negative effects located on the left side of the reference line decrease the optimal transmission ratio u_1 when they change from the low value to the high value. The effects further from the reference line are more statistically significant e.g., the total gearbox ratio u_g (factor A).
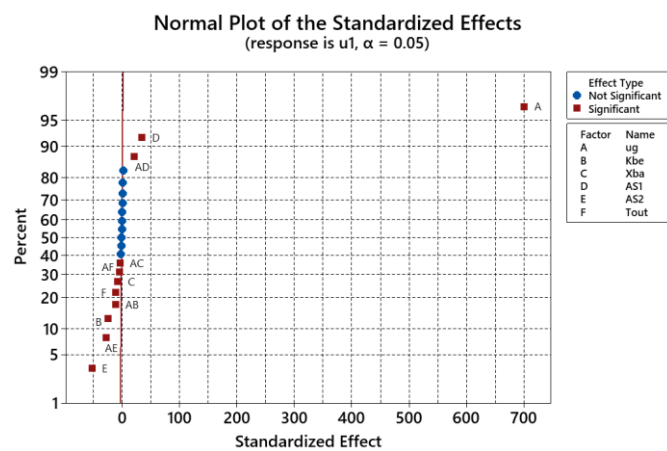

Figure 3. The trend of the effects of factors on the output respond u_1
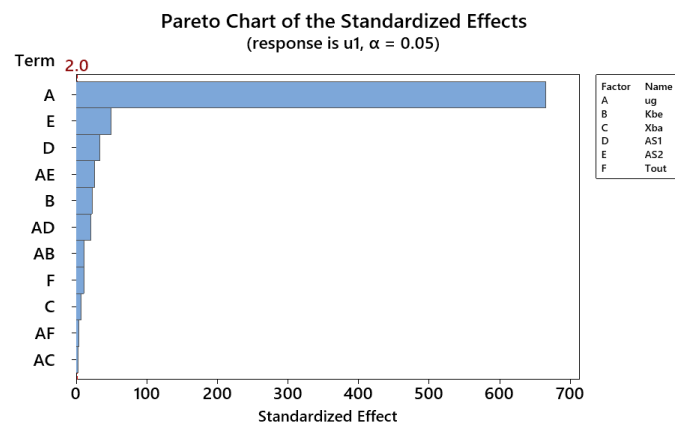

Figure 4. The magnitude of the effects of the factors on the output response u_1

Figure 4 presents the Pareto chart of the standardized effects with statistical significance at the 0.5 level. In this figure, the influences of the input parameters on the output response are more clearly evaluated by the magnitude of these parameters. As seen in Figure 4, the absolute values of the factors and the interactions are distributed from the lowest to the highest

values; therefore, influences of these parameters and interactions are arranged from smallest to largest effects on the optimum transmission ratio u_1. From the observation in this figure, the total gearbox ratio u_g (factor A) has the most important effect on the output response because of its largest magnitude among the factors and interactions.

Besides the influences of the single input factors on the output response, Figure 5 depicts the influences of the interactions of the factors on the optimal transmission ratio. As seen that the interactions including AB (u_g*K_{be}), AC (u_g*X_{ba}), AD (u_g*AS_1), AE (u_g*AS_2), AF (u_g*T_{out}) have significant effects on u_1.
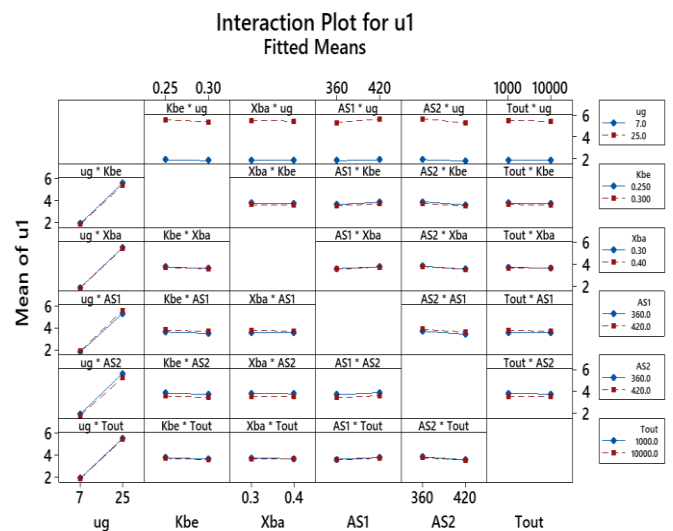

Figure 5. Effects of interactions on the output response u_1

4.2. Regression and variance analysis

After neglecting the insignificant effects on u_1, the estimated effects and coefficients for the optimum transmission ratio are presented in Table 3. As seen that the input parameters and the interactions have the p-values that are smaller than the significant level ($\alpha = 0.05$). Thus, they absolutely have the significant effects on the output response. In addition, the influences of factors and interactions are measured on the second column. The coefficients of those parameters are also shown in the third column. Therefore, the mathematical model can be built by using the analysis regression technique in order to compute the transmission ratio of the gearbox. This model depending on the six input parameters and their interactions can be expressed as

$$
\begin{aligned}
u_1 = {} & 0.780 + 0.26884\, u_g - 0.361\, K_{be} - 0.086\, X_{ba} - \qquad (40) \\
& 0.000247\, AS_1 - 0.000270\, AS_2 - 0.000002\, T_{out} \\
& - 0.1306\, u_g*K_{be} - 0.01806\, u_g*X_{ba} \\
& + 0.000205\, u_g*AS_1 - 0.000265\, u_g*AS_2 - \\
& 0.00000001\, u_g*T_{out}
\end{aligned}
$$

To evaluate the reasonability of the mathematic modal as shown in Eq. (40), the correlation coefficient reaches 99,99% (see Table 3). Due to this the proposed model in Eq. (40) is a good fit to the experimental data. From the model, this can be simple to compute the optimal transmission ratios.

Table 3. Estimated effects and coefficients for u_1

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		3.66406	0.00273	1341.90	0.000	
u_g	3.63062	1.81531	0.00273	664.83	0.000	1.00
K_{be}	-0.12250	-0.06125	0.00273	-22.43	0.000	1.00
X_{ba}	-0.03750	-0.01875	0.00273	-6.87	0.000	1.00
AS_1	0.18188	0.09094	0.00273	33.30	0.000	1.00
AS_2	-0.27063	-0.13531	0.00273	-49.56	0.000	1.00
T_{out}	-0.05875	-0.02938	0.00273	-10.76	0.000	1.00
u_q*K_{be}	-0.05875	-0.02938	0.00273	-10.76	0.000	1.00
u_g*X_{ba}	-0.01625	-0.00812	0.00273	-2.98	0.004	1.00
u_q*AS_1	0.11062	0.05531	0.00273	20.26	0.000	1.00
u_g*AS_2	-0.14312	-0.07156	0.00273	-26.21	0.000	1.00
u_q*T_{out}	-0.02125	-0.01062	0.00273	-3.89	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0218441	99.99%	99.99%	99.98%

Variance analysis was used for more clearly evaluating the significant influences of the input parameters and their interactions on the response. It is observed from the table that all factors and their interactions have important influences on the optimum transmission ratio because their p-values are smaller than the statistically significant value (i.e., $\alpha = 0.05$). Furthermore, the correlation coefficients are approximate 100% (see Table 4). Therefore, the variance analysis results are consistent with the regression analysis as shown in Table 3.

Table 4. Variance analysis

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	11	213.512	19.410	40678.29	0.000
Linear	6	212.922	35.487	74370.70	0.000
u_q	1	210.903	210.903	441993.20	0.000
K_{be}	1	0.240	0.240	503.18	0.000
X_{ba}	1	0.022	0.022	47.15	0.000
AS_1	1	0.529	0.529	1109.17	0.000
AS_2	1	1.172	1.172	2455.78	0.000
T_{out}	1	0.055	0.055	115.74	0.000
2-Way Interactions	5	0.590	0.118	247.39	0.000
u_q*K_{be}	1	0.055	0.055	115.74	0.000
u_g*X_{ba}	1	0.004	0.004	8.85	0.004
u_q*AS_1	1	0.196	0.196	410.35	0.000
u_q*AS_2	1	0.328	0.328	686.88	0.000
u_q*T_{out}	1	0.007	0.007	15.14	0.000
Error	52	0.025	0.000		
Total	63	213.537			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0218441	99.99%	99.99%	99.98%

The regression model in Eq. (40) was also examined by the normal probability and versus order plots as shown in Figure 6. As seen the normal probability plot (Figure 6.a), the residuals are closely distributed to the standard line (straight line). Additionally, the residuals in the versus order randomly arrange both above and under the standard line (the horizontal axis) (Figure 6.b). Due to these results, the mathematic model is highly reliable to calculate the optimal transmission ratio of the gearbox.
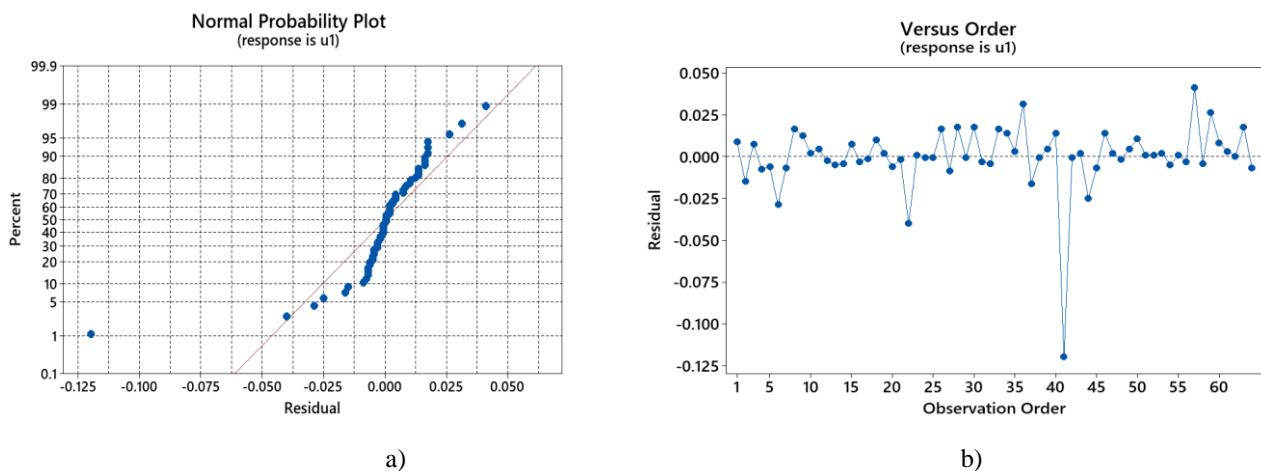

a)

b)

Figure 6. Normal Probability Plot and Versus order plot for the response of u_1

5. CONCLUSIONS

In this paper, the mass analysis of a two-stage bevel helical gearbox was proposed in order to determine the optimum transmission ratios based on minimizing the mass function. From the mass analysis, the six input parameters including the total gearbox ratio, the coefficient of the face width of the bevel gear set, the coefficient of wheel face width of the second stage, the allowable contact stress of the first and second stages, and the output torque were investigated. By using Minitab 19 for the experimental design, the influences of the input parameters on the optimum transmission ratio were examined. The results of this study are drawn as the following

- The total gearbox ratio brings the strongest influence on the optimal transmission ratio of the first stage compared to the other input parameters.

- The positive effects of the total gearbox ratio u_g and the allowable contact stress of the first stage AS_1 increase the optimum transmission ratio. Whereas the other parameters i.e., the coefficient of the face width of bevel gear set K_{be}, the coefficient of wheel face width of the second stage X_{ba}, the allowable contact stress of the second stage AS_2, and the output torque T_{out} have the negative effects to make the decrease of the output response.

- The regression model was carefully evaluated by the correlation coefficients, the p-values, and the normal probability and versus order plots. Therefore, the model can be used to simply compute the optimal transmission ration of the first stage.

References

[1] V.N. Kudreavtev, I.A. Gierzaves, Design and calculus of gearboxes (in Russian), Mashinostroenie Publishing, Sankt Petersburg, 1971.

[2] A.N. Petrovski, B.A. Sapiro, N.K. Saphonova, About optimal problem for multi-step gearboxes (in Russian), Vestn. Mashinostr. (1987) 13–14.

[3] N. T. Hiep, N. V. Lam, Design of machine elements (in Vietnamese), Educational Republishing House, Hanoi, 1979.

[4] V.N. Pi, Optimal determination of partial transmission ratios for four-step helical gearboxes with first and third step double gear-sets for minimal mass of gears, in: Appl. Comput. Eng., 2008: pp. 53–57.

[5] V.N. Pi, A Study on Optimal Determination of Partial Transmission Ratios of Helical Gearboxes with Second-Step Double Gear-Sets, Int. J. Mech. Mechatronics Eng. 2 (2008) 26–29.

[6] N.P. Vu, A New Study on Optimal Calculation of Partial Transmission Ratios of Helical Gearboxes with First-step Double Gear-sets, in: Lect. Notes Eng. Comput. Sci., 2008: pp. 162–165.

[7] G. Milou, G. Dobre, F. Visa, H. Vitila, Optimal Design of Two Step Gear Units, regarding the Main Parameters, VDI Berichte. 1230 (1996) 227–244.

[8] T.H. Tran, N. V. Cuong, L.H. Ky, N.K. Tuan, N.P. Vu, Calculating Optimum Gear Ratios of Two Step Bevel Helical Reducer, 14 (2019) 3494–3499.

[9] N.T.H. Cam, N.P. Vu, T.T. Hong, L.. H. Ky, L.A. Tung, A study on calculation of optimum gear ratios of a two-stage helical gearbox with second stage double gear sets, Int. J. Mech. Prod. Eng. Res. Dev. 9 (2018) 613–620.

[10] N.K. Tuan, N.P. Vu, N.T.H. Cam, T.T.P. Thao, H.K. Thanh, L.X. Hung, H.T. Tham, Determining Optimal Gear Ratios of a Two-stage Helical Reducer for Getting Minimal Acreage of Cross Section, in: Asia Conf. Mech. Mater. Eng., 2018: pp. 1–5.

[11] N.K. Tuan, T.T. Hong, N. V. Cuong, L.H. Ky, N.T. Tu, L.A. Tung, L.X. Hung, N.P. Vu, A Study on Determining Optimum Gear Ratios of Mechanical Driven Systems Using Two-Step Helical Gearbox with First Step Double Gear Sets and Chain Drive, in: Adv. Eng. Res. Appl., 2020: pp. 85–93.

[12] N.P. Vu, N.K. Tuan, L.X. Hung, L.A. Tung, A New Study on Determination of Optimum Gear Ratios of a Two-Stage Helical Gearbox, in: Adv. Mater. Sci. Eng., 2020: pp. 107–115.

[13] N.T.H. Cam, N.P. Vu, N.K. Tuan, L.X. Hung, T.T.P. Thao, Determining Optimal Partial Transmission Ratios of Mechanical Driven Systems Using a V-Belt Drive and a Helical Reducer with Second-Step Double Gear-Sets, in: Int. Conf. Eng. Res. Appl., 2019: pp. 261–269.

[14] N.K. Tuan, V.P. Pi, Optimum Calculation of Partial Transmission Ratios of Mechanically Driven Systems Using a V-Belt and a Helical Gearbox with First-Step Double Gear Sets, Int. J. Mech. Eng. Robot. Res. 8 (2019) 323–326.

[15]   A.T. Luu, T.T. Hong, N. V. Cuong, L.H. Ky, N.T. Tu, L.X. Hung, N.P. Vu, A Study on Determination of Optimum Gear Ratios of a Two-Stage Worm Gearbox, in: Adv. Eng. Res. Appl., 2020: pp. 76–84.

[16]   T.H. Tran, N. V. Cuong, L.H. Ky, N.T. Tu, V.N. Pi, Calculating Optimum Gear Ratios of Mechanical Driven Systems Using Worm-Helical Gearbox and Chain Drive, Int. J. Appl. Eng. Res. 14 (2019) 3211–3218.

[17]   N.T.H. Cam, N.P. Vu, N.K. Tuan, L.X. Hung, T.P.T. Tran, Determining Optimum Partial Transmission Ratios of Mechanical Driven Systems Using a V-Belt Drive and a Three-Stage Helical Reducer, in: Adv. Eng. Res. Appl., 2019: pp. 81–88.

[18]   N.K. Tuan, T.P.T. Tran, N.T.H. Cam, L.X. Hung, N.P. Vu, Optimum Calculation of Partial Transmission Ratios of Mechanical Driven Systems Using a V-Belt and a Three-Step Bevel Helical Gearbox, in: Adv. Eng. Res. Appl., 2019: pp. 469–476.

[19]   N.P. Vu, T.T.P. Thao, T.T. Hong, N.K. Tuan, L.X. Hung, L.A. Tung, Determination of optimum gear ratios of a three stage bevel helical gearbox, in: Mater. Sci. Eng., 2019: pp. 1–8.

[20]   N.P. Vu, K.T. Nguyen, X.H. Le, T.H.C. Nguyen, T.P.T. Ttan, Determining Optimum Partial Transmission Ratios of Mechanical Driven Systems Using a V-Belt Drive and a Three-Stage Helical Reducer, in: Adv. Mater. Sci. Eng., 2020: pp. 1–6.

[21]   N.P. Vu, A new study on the optimal prediction of partial transmission ratios of three-step helical gearboxes with second-step double gear-sets, Appl. Theory Mech. 2 (2007) 229–238.

[22]   O. Buiga, 3 Stage helical speed reducer partial gear ratios optimal determination using genetic algorithms, Proc. Rom. Acad. Ser. A - Math. Phys. Tech. Sci. Inf. Sci. 20 (2019) 156–163.

[23]   N.P. Vu, Optimal calculation of partial transmission ratios for four-step helical gearboxes with first and third step double gear-sets for minimal gearbox length, in: Proc. Am. Conf. Appl. Math., 2008: pp. 29–32.

[24]   T.T.P. Thao, T.T. Hong, N. V. Cuong, L.H. Ky, N.T. Tu, L.X. Hung, P. V. Ngoc, Determining Optimum Gear Ratios of Mechanical Driven Systems Using Three Stage Bevel Helical Gearbox and Chain Drive, in: Adv. Eng. Res. Appl., 2020: pp. 249–261.

[25]   V.N. Pi, N.K. Tuan, Determining optimum partial transmission ratios of mechanical driven systems using a chain drive and a two-step bevel helical gearbox, Int. J. Mech. Eng. Robot. Res. 8 (2019) 708–712.

[26]   T.E. Toolbox, Density, Specific Weight and Specific Gravity, (n.d.).

[27]   T. Chat, L. V. Uyen, Calculation of mechanical driven systems (in Vietnamese), Education Publisher, 1996.

[28]   I. RÖMHILD, H. Linke, Gezielte Auslegung Von Zahnradgetrieben mit minimaler Masse auf der Basis neuer Berechnungsverfahren. Konstruktion, Konstruktion. 44 (1992) 229–