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Calculation of Instant Centers of Rotation through Arnold Kennedy's Theorem and Screw Theory

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Abstract

In this article the use of Arnold Kennedy's theorem and screw theory for the analytical determination of the center of rotation of four-bar mechanisms is proposed. The natural coordinates are mostly Cartesian coordinates located in the kinematic pairs or in points of interest of the mechanism, where the main advantage of their use is that angular parameters and trigonometric functions can be disregarded, which facilitates the understanding of the modeling, in addition it is not necessary to use trigonometric functions. This tecnique allow to reduce the computational cost in a great way, optimizing problems where the rotation center has to be calculated many times. To achieve this goal, the problem is described and formulated in a robust way using natural coordinates and operating within the body of complex numbers. Then, to deal with the present problem, the mechanism's kinematics position in natural coordinates is modeled and computing and with the obtained equations, a program is implemented in MATLAB® to show the curve described by the rotation centers of links, which is fundamental for the optimal design of polycentric mechanisms.

Keywords Four bar mechanism, rotation center, natural coordinates.

1. INTRODUCTION

The center of rotation of a plane mechanism refers to a common point of two bodies not necessarily connected that share the same velocity [1]. In this case the center of rotation refers to that formed by the coupler and the fixed link, known as the zero-speed center of rotation. For spatial mechanisms, the rotation center's analogue to the center of rotation is the instantaneous axis of rotation, which is the basis of the infinitesimal screw theory [2-4]. The determination of the curve that describes the center of rotation is of vital importance in the optimal design of mechanisms for lower limb prostheses, where the mechanism's center of rotation must approach the curve described by the knee's center of rotation, this curve is known as the poloid [5-7]. Traditionally, the position analysis of a four-bar mechanism is based on the vector-loop closing equations, and therefore the equations that determine the center of rotation are in terms of angular parameters, in addition to trigonometric functions, as is done in Amador [8] and Radcliffe [9].

Natural coordinates were first introduced by De Jalon & Bayo [10], which are mostly Cartesian coordinates located in kinematic pairs or points of interest. The main advantage of using natural coordinates to model mechanisms is that the use of angles and the trigonometric functions can be disregarded, which facilitates the modeling and implementation of the obtained equations [11]. In literature, work on natural coordinates is not very abundant, however there are several important papers on the subject which stand out: Núñez [12], where natural coordinates are used to model vehicle steering mechanisms up to eight bars; in this paper the kinematic equations are solved numerically. In Núñez [13], the problem of position of a four-bar mechanism using natural coordinates was solved analytically. In Rojas & Thomas [14], a general procedure is presented to solve the positioning problem of plane mechanisms, using the concept of bilateration that focuses in two given points to determine the third point of a triangle.

The general objective of this paper is to determine in an analytical way the zero-speed center of rotation in a four-bar mechanism using natural coordinates. The equations obtained are implemented in MATLAB® to show the curve described by the center of rotation, which is of vital importance in many applications where mechanism's stability depends directly on the rotation center.

2. POSITIONING KINEMATICS USING NATURAL COODINATES

Figure 1 display the four-bar mechanism's modeling using natural coordinates, where the natural coordinate vector is given by:

$$q = \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} C_x \\ C_y \\ D_x \\ D_x \end{bmatrix} \quad (1)$$

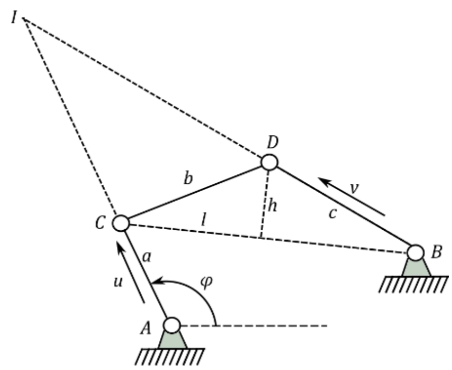

Figure 1. Four-bar mechanism's modeling using natural coordinates. (*Source Author)*

Where C and DD corresponds to the mechanism's coupler kinematic pair coordinates. A and B are the fixed pair coordinates.

To determine the natural coordinates, the procedure developed by Núñez [13] was used; this one is based on triangulation, which consists of determining a point of a triangle by knowing two points of said triangle. From the triangle CBD, the points C and B are known, the point C is determined by:

$$C = A + \begin{bmatrix} a\cos\varphi \\ a\sin\varphi \end{bmatrix} = \begin{bmatrix} A_x + a\cos\varphi \\ A_y + a\sin\varphi \end{bmatrix} \tag{2}$$

Making the distance between C and B be,

$$s = \left\| B - C \right\| \tag{3}$$

The following relations can be written:

$$l = \frac{b^2 + s^2 - c^2}{2s} \tag{4}$$

$$h = t\sqrt{b^2 - l^2} \tag{5}$$

where t takes values of -1 or 1, that represent the four-bar mechanism's two configurations. From Fig. 1 it can be clearly appreciated that D point can be determined by:

$$D = C + \frac{l}{s}\left(B - C\right) + \frac{h}{s}R\left(B - C\right) \tag{6}$$

$$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{7}$$

where R corresponds to a ninety-degree rotation of the vector *(B - C)* The Eq. (6) can be written as follows,

$$D = C + \left[\frac{l}{s}e + \frac{h}{s}R\right]\left(B - C\right) \tag{8}$$

where:

$$\left[\frac{l}{s}e + \frac{h}{s}R\right] = \begin{bmatrix} \frac{l}{s} & -\frac{h}{s} \\ \frac{h}{s} & \frac{l}{s} \end{bmatrix} = N \tag{9}$$

where e is the identity matrix. So, point D is determined by:

$$D = C + N\left(B - C\right) \tag{10}$$

In this way the problem of positioning of four bars is solved, making use of natural coordinates. The obtained equations are simple and easily implemented in a computer. For more information on the natural coordinates, consult Avello [11].

3. ROTATION CENTER DETERMINATION

In this chapter the rotation center is determined by mean Arnold Kennedy theorem and screw theory.

Using Arnold Kennedy theorem

To determine rotation center, I (Fig. 1), the following relation can be written,

$$I = A + \lambda u = B + \beta v \tag{11}$$

where λ*,* β are scalars are u*,* v are unitary vectors determined by:

$$u = \frac{\left(C - A\right)}{\left\| C - A \right\|} \tag{12}$$

$$v = \frac{\left(D - B\right)}{\left\| D - B \right\|} \tag{13}$$

From equations (11) it is understood that,

$$\lambda u - \beta v = \left(B - A\right) \tag{14}$$

written in matrix form as,

$$\begin{bmatrix} u_x & -v_x \\ u_y & -v_y \end{bmatrix}\begin{bmatrix} \lambda \\ \beta \end{bmatrix} = \begin{bmatrix} \left(B_x - A_x\right) \\ \left(B_y - A_y\right) \end{bmatrix} \tag{15}$$

and applying Cramer's rule it can be appreciated that,

$$\lambda = \frac{\begin{vmatrix} \left(B_x - A_x\right) & -v_x \\ \left(B_y - A_y\right) & -v_y \end{vmatrix}}{\begin{vmatrix} u_x & -v_x \\ u_y & -v_y \end{vmatrix}} \tag{16}$$

$$\beta = \frac{\begin{vmatrix} u_x & \left(B_x - A_x\right) \\ u_y & \left(B_y - A_y\right) \end{vmatrix}}{\begin{vmatrix} u_x & -v_x \\ u_y & -v_y \end{vmatrix}} \tag{17}$$

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If, the denominator is equal to zero, the vectors u and v are parallel, and the rotation center will hence tend to infinity. In this way we can determine the center of rotation of a four-bar mechanism for any input angle φ.

Using screw theory

Figure 2 shows the screw system in a four-bar mechanism. It is important to clarify that in this work we do not interest in the movement of link 3 relative to link 1.

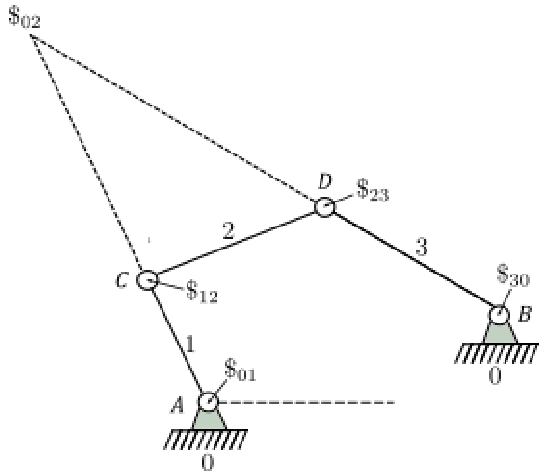

Figure 2. Screw system of four-bar mechanism
(Source Author)

The screw of link 3 relative to link 0 can be represented by,

$$\$_{03} = \$_{01} + \$_{12} + \$_{23} \tag{18}$$

and rearranging it yields

$$\$_{01} + \$_{12} + \$_{23} + \$_{30} = 0 \tag{19}$$

Let be $A = 0$, therefore we have

$$\$_{01} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \omega_{01} \qquad \$_{12} = \begin{bmatrix} 1 \\ C_y \\ -C_x \end{bmatrix} \omega_{12}$$

$$\$_{23} = \begin{bmatrix} 1 \\ D_y \\ -D_x \end{bmatrix} \omega_{23} \qquad \$_{30} = \begin{bmatrix} 1 \\ C_y \\ -C_x \end{bmatrix} \omega_{30}$$

Now taking $\omega_{01} = 1$ as input velocity and using matrix notation in Eq. 19 it becomes

$$\begin{bmatrix} 1 & 1 & 1 \\ C_y & D_y & B_y \\ -C_x & -D_x & -B_x \end{bmatrix} \begin{bmatrix} \omega_{12} \\ \omega_{23} \\ \omega_{30} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

Solving the Eq. 20 we have that

$$\omega_{12} = \frac{\left(B_x D_y - D_x B_y\right)}{\left(B_x - D_x\right)C_y + \left(D_y - B_y\right) - \left(B_x D_y - D_x B_y\right)}$$

Our aim is to determine the screw of link 2 relative to link 0. For this we write

$$\$_{02} = \$_{01} + \$_{12} \tag{21}$$

Hence

$$\$_{02} = \begin{bmatrix} \omega_{01} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_{12} \\ C_y \omega_{12} \\ -C_x \omega_{12} \end{bmatrix} = \begin{bmatrix} \omega_{01} + \omega_{12} \\ C_y \omega_{12} \\ -C_x \omega_{12} \end{bmatrix} = \begin{bmatrix} \omega_{02} \\ v_{02} \end{bmatrix}$$

Where $v_{02} = I \times \omega_{02} + h\omega_{02}$ and this equation can be rewrite as

$$\begin{aligned} \omega_{02} \times v_{02} &= \omega_{02} \times (I \times \omega_{02} + h\omega_{02}) \\ &= \omega_{02} \times (I \times \omega_{02}) + \omega_{02} \times (h\omega_{02}) \\ &= (\omega_{02} \cdot \omega_{02}) I - (\omega_{02} \cdot I)\omega_{02} \\ &= \left\|\omega_{02}\right\|^2 I \end{aligned}$$

Therefore, the rotation center can be computed as

$$I = \frac{\omega_{02} \times v_{02}}{\left\|\omega_{02}\right\|^2} \tag{22}$$

Replacing the correspond values in Eq. 22 result,

$$I = \frac{\omega_{12}}{1 + \omega_{12}} \begin{bmatrix} C_x \\ C_y \end{bmatrix} \tag{23}$$

Where

$\dfrac{\omega_{12}}{1 + \omega_{12}} = \dfrac{\left(B_x D_y - D_x B_y\right)}{\left(B_x - D_x\right)C_y + \left(D_y - B_y\right)C_x}$ and finally, we have the rotation center coordinates

$$I = \begin{bmatrix} \dfrac{\left(B_x D_y - D_x B_y\right)C_x}{\left(B_x - D_x\right)C_y + \left(D_y - B_y\right)C_x} \\ \dfrac{\left(B_x D_y - D_x B_y\right)C_y}{\left(B_x - D_x\right)C_y + \left(D_y - B_y\right)C_x} \end{bmatrix} \tag{24}$$

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The denominator $(B_x - D_x)C_y + (D_y - B_y)C$ can be write as

$$(B_x - D_x)C_y + (D_y - B_y)C_x = [R(B-D)]^T C$$

Where $[R(B-D)]^T C = 0$ when the link 1 and link 3 are parallel therefore the rotation center is located in the infinity as was deduced in the previous method.

Although the screw method is more laborious, it can be used to determine the center of rotation of much more complex planar mechanisms

4. RESULTS

Obtained equations were implemented in the numeric software MATLAB®, for four-bar mechanisms with the dimensions shown in Table 1 and Table 2.

Table 1. Four-Bar Mechanism Dimensions in Centimeters

a	b	c	A_x	A_y	B_x	B_y
1.5	3	2	0	0	4.5	0

Figure 2 shows the path described by the center of rotation, where the entry angle is in the range $-20 \le \varphi \le 100$. The center of rotation for angles outside this range tends to infinity or does not exist because assembly of the mechanism is not possible.

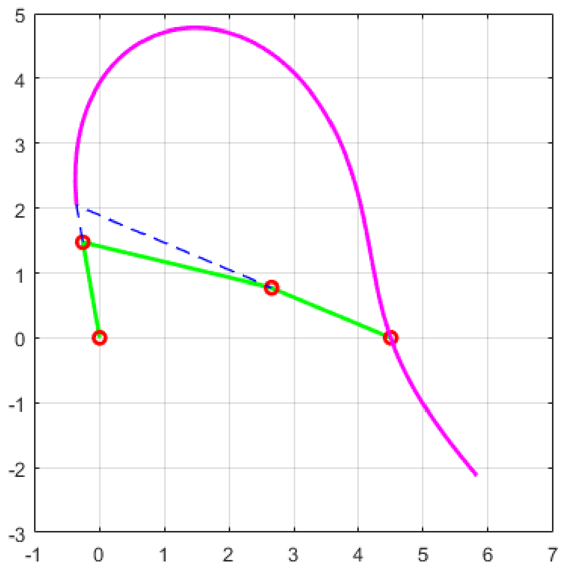

Figure 3. Curve described by the rotation center for t =1
(Source Author)

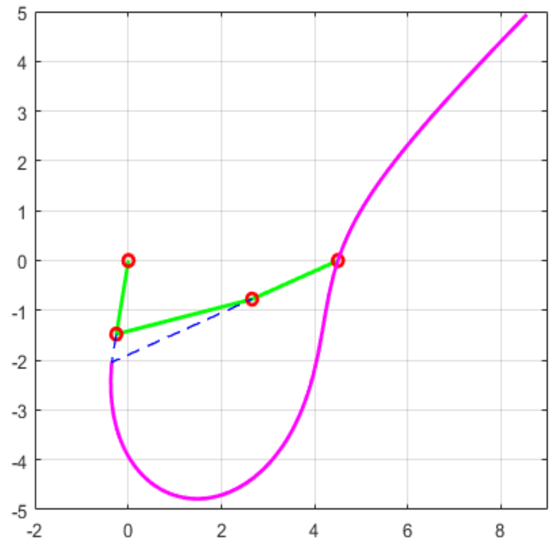

Figure 4. Curve described by the rotation center for t =-1
(Source Author)

The curve described by the center of rotation is shown in Fig. 3, where the angle of entry is in the range $-100 \le \varphi \le 30°$.

Figure 5 shows a double crank mechanism, where it can be seen that the curve formed by the center of rotation is closed. The Table 2 shows the dimension of mechanism of Figure 5.

Table 2. Double-crank Four-Bar Mechanism Dimensions in Centimeters

a	b	c	A_x	A_y	B_x	B_y
7	6	7	0	0	4	0

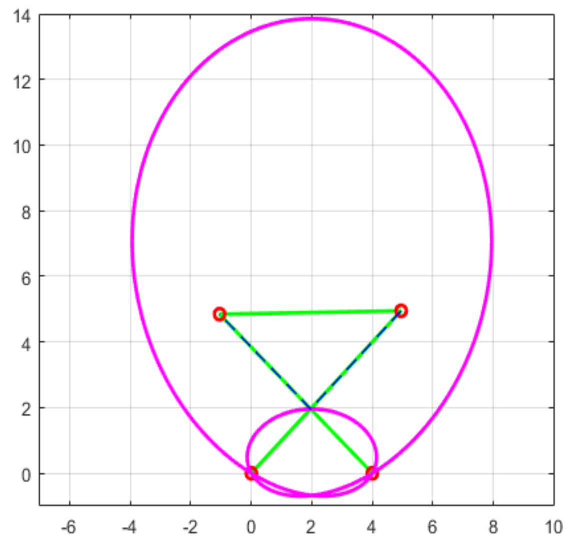

Figure 5. Rotation center double-crank four-bar mechanism for t =-1
(Source Author)

5. DISCUSSION

The equations obtained and implemented in MATLAB® disposes of trigonometric functions, which facilitates the interpretation of the model and decreases the computational cost. The method proposed here for the rotation center's determination can be generalized, and therefore be applied to more complex mechanisms by using screw method. The generalization of this method is intended to be developed in future papers. It should also be mentioned that in the design of lower limb prosthesis, the center rotation is required to describe a given curve, this being one of the rotation center's most important applications [6].

6. CONCLUSIONS

The equations obtained are simple and compact, which makes them ideal to be implemented in a computer.

By the time this paper was written, the use of natural coordinates for the calculation of rotation centers in a mechanism was never described, making it the first paper to make use natural coordinates for that purpose.

The use of angular coordinates is discarded, which facilitates the understanding of the modeling, in addition it is not necessary to use trigonometric functions, which allow to reduce the computational cost in a great way, optimizing problems where the rotation center has to be calculated many times.

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