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Simulation of the Rayleigh-Benard Convective Flow Problem using a LBEM Method

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Abstract

The natural convection is a traditional phenomenon that is present in different industrial and environmental applications. The lattice Boltzmann equation method (LBEM) is used to simulate a convective flow on laminar regime, Rayleigh numbers Ra < 5E5 and Prandtl number Pr = 0.71, known as Rayleigh-Benard problem (Ra-Be). This method has emerged at the end of last century as one of the most powerful in computational fluid dynamics CFD. The hydrodynamic flow field is calculated from the traditional particles distribution function (FDP) on the lattice model of two-dimensional and nine-speed -D2Q9, and the thermal field is calculated introducing a new internal energy distribution function on the single two-dimensional lattice with five velocities -D2T5. The new thermal model used proved to be stable and the results shown high accuracy compared with the existing theory and other results obtained numerically by other CFD methods.

Keywords: Mesoscopic Method, Natural Convection, Numerical simulation.

1. INTRODUCTION

The Lattice Boltzmann Equation method (LBEM) is a mesoscopic method, as a different approach to conventional computational fluid mechanics (CFD), has achieved considerable recognition in the simulation of science and engineering problems involving fluid flows and transport phenomena. [1] [2] [3] [4]. Some researchers consider that this method has the potential to become a versatile CFD platform, even superior to existing CFD methods based on the continuum theorem [5] [6] [7].

Originally, the method only considered conservation of mass and momentum. However, in many applications that involve transport phenomena it is important, and in most cases critical, to consider the thermal effects of the flow. Therefore, in the present work, the LBEM is used to simulate and analyze the natural convection present in the well-known Rayleigh-Benard (Ra-Be) flow. This flow has been widely used as a benchmark to validate different methods. In this, a horizontal layer of viscous fluid is heated at its lower part while the upper border is kept at a low temperature. There is an analytical solution for this problem, when the velocity is zero in the entire fluid domain (static condition) and the temperature exhibits a linear behavior between the cold and hot layers. However, when the temperature difference between the plates increases to a critical point, the static condition is lost and any disturbance, no matter how small, transforms the flow into a convective system [2].

Generally, the inclusion of thermal effects in LBEMs fit into three models: the multi-speed model, the one-passive scalar model, and the double-function distribution model. An explanation of the first two can be found in [8] [9] [10], respectively. The third model, the double distribution function, used in the present work, is based on the work of He et al. [5]. In this model an internal energy distribution function is introduced to simulate the temperature field, which is analogous to the density distribution function to simulate the velocity field. The stability and precision of the double-function distribution model have been verified in the studies by Kuznik et al. [6], Qiu et al. [10], and Guo et al. [11]. As in much of the bibliography related to natural convection, mainly in cavities, the Boussines approach has been implemented in the present study. This approximation defines the state in which the density changes are small enough to be neglected, except when these differences appear in the term of the gravitational force.

The validation of the computational code developed using the LBEM is done by simulating laminar convective flow, numbers of Ra ≤ 106, which is generated in a square cavity, when it is heated in one of its vertical walls.

2. METHOD

In this section, the two-dimensional LBEM model used for the simulation of the Ra-Be free convection flow is described and the physical domain of the problem is described. Temperature and velocity conditions on vertical walls are assumed to be periodic. Taking into account that the linear stability theory shows that the critical wave number for Ra-Be convection is kc=3.117, which implies that the presence of the vortex that is generated by convection develops faster in cells or domains of aspect ratio equal to 2π / kc = 2.016. Therefore, in the present case a channel with an aspect ratio of 2: 1 has been selected. The Prandt number has been set at Pr = 0.71.

In the Figure 1 a diagram of the simulated problem is shown, there Th and Tc describe the temperature of the hot and cold wall, respectively, the vertical walls, as mentioned above, are adiabatic, u and v describe the horizontal and vertical velocity of the flow at the boundaries, respectively, H the height of the channel and g describe the acceleration of gravity.

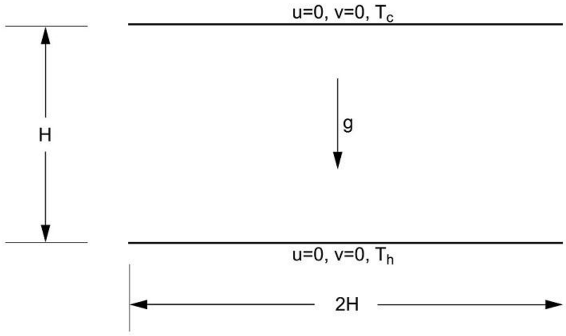

Fig. 1. Setting the Rayleigh-Benard convection flow

Coming up next, a simple description is made of the Boltzmann equation method in networks and the node model used both for the calculation of the velocity field and for the calculation of the Temperature field. The detailed mathematical demonstration of the model can be seen in Li, et al. [12] and in Quian, et al. [13].

The main hypotheses used in the model are:

- The collision term in the modified Boltzmann equation is expressed as a function of a simple relaxation time for the local equilibrium
- Knudsen number is assumed small (continuum theory)
- The flow is incompressible

In the present work, to calculate the energy transport (temperature field) and the amount of movement (velocity field), the LBEM is an iterative scheme where the cavity is represented by a kind of lattice and at each site of this be defines a group of density and temperature distribution functions. These distributions satisfy two transport equations coupled by the flotation term, which, together with appropriate initial and boundary conditions, lead to the solution of different transport phenomena, including natural convection [14].

The density distribution function f_i (x, t) is defined as the probability that a particle located at position x on the lattice, at instant t, has a velocity e_i, with $i = 0$, b-1. The velocities are given by the symmetry of the used grid. In the simulations carried out at the present work, a two-dimensional grid model and 9 velocities (D2Q9) are used, as shown in Figure 2 (a), where $b = 9$, $e_0 = (0,0)$, $e_1 = (1.0)$, $e_2 = (0.1)$, $e_3 = (- 1.0)$, $e_4 = (0, -1)$, $e_5 = (1.1)$, $e_6 = (- 1.1)$, $e_7 = (-1, -1)$, and $e_8 = (1, -1)$.

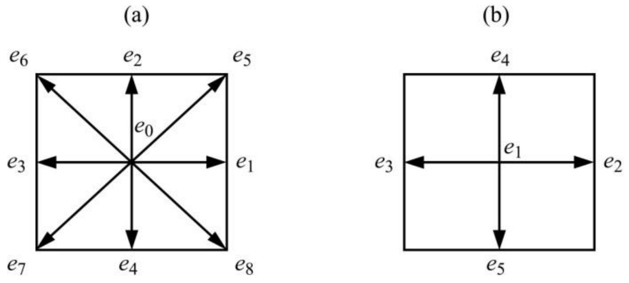

Fig. 2. Discrete directions of a square lattice, for a) model d2q9, and b) model d2q5

That is, the particles that make up the distribution function can be at rest or moving towards their closest neighbor; with a velocity of 1 along the vertical and horizontal directions or a velocity R2 of along the diagonals. The distribution functions of the particle obey the respective Boltzmann transport equation defined by (1).

$$f_i(x + e_i\Delta t, t + \Delta t) - f_i(x,t) = \frac{\Delta t}{\tau_v}\left[f_i^{eq}(x,t) - f_i(x,t)\right] + F_i \qquad (1)$$

Where, $\Delta t = 1$, τ_v is the relaxation time of the distribution function, related to the viscosity of the fluid by $\nu = c^2 (\tau_v - 0.5) / 3$, here c is the lattice constant, iqual to; $c = \Delta x / \Delta t$, which is related to the speed of sound by $c_s^2 = c^2/3$ and F_i is the term that represents the momentum due to the body or buoyant force, and f_i^{eq} is the equilibrium distribution function given by equation (2).

$$f_i^{eq}(x,t) = \omega_i\rho\left[1 + \frac{e_{i\alpha}\cdot V_\alpha}{c_s^2} + \frac{V_\alpha\cdot V_\lambda}{2c_s^2}\left(\frac{e_{i\alpha}e_{i\lambda}}{c_s^2} - \delta_{\alpha\lambda}\right)\right] \qquad (2)$$

where ρ and V are the density and velocity defined by:

$$\rho = \sum_{i=1}^{8} f_i$$
$$\rho V = \sum_{i=1}^{8} e_i f_i \qquad (3)$$

and the subscripts α and λ describe the components of the vector quantities. Furthermore, ω = 4/9, 1/9, and 1/36 for $e_i = 0.1$, respectively, and $\delta_{\alpha\lambda} = 1$ if $\alpha = \lambda$ and $\delta_{\alpha\lambda} = 0$, in any other case [12] [15][16].

Like the particle distribution function, the temperature distribution function T is defined by (4):

$$T(x,t) = \sum_{i=0}^{b-1} T_i(x,t) \qquad (4)$$

Where $b = 5$, and T_i are the respective distribution functions that obey the transport equation

$$T_i(x + \Delta t\, c_i, t + \Delta t) - T_i(x,t) = -\frac{\Delta t}{\tau_T}\left[T_i(x,t) - T_i^{eq}(x,t)\right] \qquad (5)$$

In the equation (5), the term τ_T is the relaxation time for the temperature field and T_{ieq} is the equilibrium temperature distribution function given by (6).

$$T_i^{eq}(x,t) = T\, t_i\left[1 + 3c_i \cdot u\right] \qquad (6)$$

In the equation (6), the temperature T satisfies the diffusion equation with a thermal diffusivity γ, given by $\gamma = c_s^2 (\tau_T - 0.5)$. This considering that $\gamma > 0$, $\tau_T > \tau_0$ [7].

For the simulation of natural convection, the body or buoyant force term at the vertical direction F_i is given by (7).

$$F_i(x,t) = 3\,\omega_i g_0 \beta\left[T(x,t) - T_0\right]c_{iy} \qquad (7)$$

Where c_{iy} is the vertical component of c_i, g_0 is the acceleration of gravity in the lattice, β is the thermal expansion coefficient and $T_0 = (T_h + T_c) / 2$; is the reference temperature. This body force does not contribute to density, but it does change the amount of movement.

The type of lattice used for the temperature distribution function is 2-dimensional with 5 discrete directions (D2T5), as shown in Figure. 2 (b). where $b = 5$, $e_0 = (0,0)$, $e_1 = (1,0)$, $e_2 = (0,1)$, $e_3 = (-1,0)$, $e_4 = (0, -1)$. That is, the particles that make up the distribution function can be at rest or moving towards their closest neighbor; with a velocity of 1, along the vertical and horizontal directions.

2.1 Boundary conditions

The boundary conditions applied to obtain the velocity field have been extensively studied for LBEMs, and for specific details one can refer to [16] [17] [18] [19]. In the present work, two types of them have been used, one for horizontal walls (static with zero speed) and others for vertical ones (periodic). In the former the well-known "Bounce-back" condition was applied. This approach assumes that the particles that collide with the static wall are returned in the same direction but in the opposite way and without loss of energy, which indicates that the collision between the particles and the wall is a totally elastic collision. With reference to Figure 3, if it is assumed that x_l is a fluid node or lattice, it can be determined that $x_l + e_i$ is a solid node or static wall, where ei represents the respective discrete velocity within the lattice. For the condition described, the bounced f's are obtained by (8).

$$f_i\,(x_l, t+1) = f_{i\text{-}2}(x_l, t) \qquad \text{Para } i = 4, 7, 8 \tag{8}$$

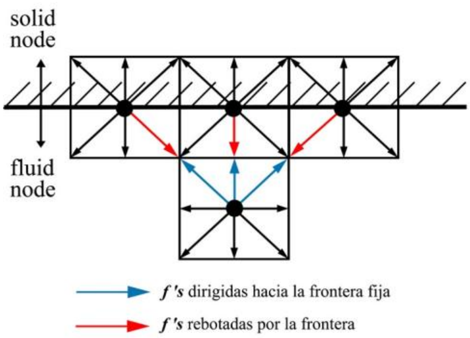

Fig. 3. Bounce-back representation scheme for a static wall

For the implementation of the periodic conditions in the LEBM, as in most methods, it requires, in the propagation step, that the properties at the outlet of the flow return or go directly to the input of the flow, and vice versa. For a problem where the length of the flow is given by $L = n$, and n = the number of nodes in the direction of the flow, in the present case the horizontal distance of the channel, the condition must enforce, for a lattice D2Q9, that at the flow outlet:

$$f\,(i, x{=}1, t) = f\,(i, x{=}n, t) \qquad \text{Para } i = 1, 5, \text{ y } 8 \tag{9}$$

and at the inlet:

$$f\,(i, x{=}n, t) = f\,(i, x{=}1, t) \qquad \text{Para } i = 3, 6, \text{ y } 7 \tag{10}$$

In the above expression, the subscript i indicates the respective direction of the discrete distribution function into the lattice, and the subscript t is time. An example of the application of periodic boundary conditions, using a D2Q9 lattice model, is shown in Figure 4.

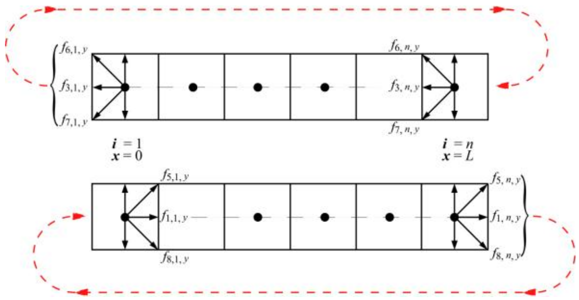

Fig. 4. Scheme of application of the condition of periodic borders, using a D2Q9 lattice

Similarly, to the application of dynamic conditions at the boundaries, to simulate horizontal walls with fixed temperature, Dirichlet conditions (fixed temperature) are used, while for vertical walls periodic conditions are used. For the latter, these conditions do not take into account any disturbance that the flow may present at its boundary, therefore, they are only suitable to apply in physical phenomena where the effects of the surfaces, where the condition is applied, do not play a role. Important at the developed flow and/or where there are not flow disturbances, near the borders themselves, as is the case of the phenomenon analyzed in the present work.

2.2 Solution method

The implementation of a LBEM standard code consists of two main steps, propagation and collision. An example of the flow diagram used to develop the computational code used in the present simulation can be seen in Flórez et al. [16], there it is only necessary to include the temperature calculations in each of the main steps. The grid or domain used in this work has a dimension in x and y of 100 x 50.

To simulate natural convection, it is necessary to define, from the parameters of the problem, an appropriate characteristic velocity ($V_c = \sqrt{\beta\, g_0\, \Delta T\, H}$), where H is the characteristic length of the cavity (number of grids in the vertical direction), to keep the flow within the incompressible regime. In addition, the Prandtl (Pr) and Rayleigh (Ra) numbers are defined, which allow us to have two more equations in the model, one for kinematic viscosity ($\nu = \sqrt{V^2 H^2\, Pr/Ra}$) and another for thermal diffusivity ($\gamma = \nu/Pr$). The number of Ra, the number of Pr, the viscosity ν, and the diffusivity γ are used to calculate the relaxation time of the distribution function of the f's and of the temperature. The viscosity is selected ensuring that the Match number is such that the flow is within the incompressible limit.

The convergence criteria used for both cases; velocity and temperature, respectively, are:

$$max\left|\sqrt{(u^2+v^2)^{n+1}} - \sqrt{(u^2+v^2)^{n}}\right| \leq 10^{-6} \tag{11}$$

$$max|T^{n+1} - T^n| \leq 10^{-6} \quad (12)$$

Where u and v are the horizontal and vertical velocity, respectively at any point on the lattice, T is the temperature and n describe the iteration.

3. RESULTS

For the natural convection at the *Ra-Be* flow, results have been obtained for Rayleigh numbers of; Ra = 103, Ra =104, Ra =105 and Ra =106, and a Prandtl number Pr = 0.71. The simulations are started using a perturbed state in the temperature field, said perturbation is imposed by a Gaussian function along the horizontal line of the domain and centered with respect to it. The initial temperature field is given by:

$$T(x,y) = T_0\, exp\left[-\frac{(x-b)^2}{c}\right] \quad (13)$$

Where b, c and T_0 are real constants, besides $T_0 > 0$ and is taken as the average temperature between the cold and hot walls $T_0 = (T_H + T_C) / 2$, b is the central coordinate in the horizontal direction of the domain, and c defines the width of the function.

When the Rayleigh-Bénard convective flow is established the heat transfer, between the up wall and the down wall, is greater. This increase in heat transfer can be described by the Nusselt number as:

$$Nu = 1 + \frac{\langle u_y \cdot T \rangle}{\chi \Delta T / H} \quad (14)$$

Where u_y is the vertical velocity, ΔT is the difference temperature between the lower and upper wall, H is the height of the channel, and $\langle \cdot \rangle$ represents the average over the entire flow domain. Comparisons with the results of the existing literature are carried out for 1E3 <Ra < 5E5 and Pr = 0.71. The results obtained confirmed the existence of a critical Ra number (R_{ac}), where the disturbances dissipate and the flow velocity gradually decreases to zero, while at $R_a > R_{ac}$ the velocity field stabilizes at a finite value. The calculations carried out revealed a value of R_{ac} = 1710.31, which is remarkably close to the value theoretically established by researchers Reid and Harris [14], R_{ac} = 1707.76. Figure 5 shows the relationship obtained between the Nu number and the Ra number. It also includes the results obtained by He et al. [4] and those obtained from the existing empirical formulation for this type of problem. Then, the results obtained in this work coincide with those obtained by He, up to Ra values less than 1E5. For high Ra numbers, the difference in the results, with respect to the empirical solution, may be because the simulation with the proposed thermal model does not include all the effects of heat transfer.

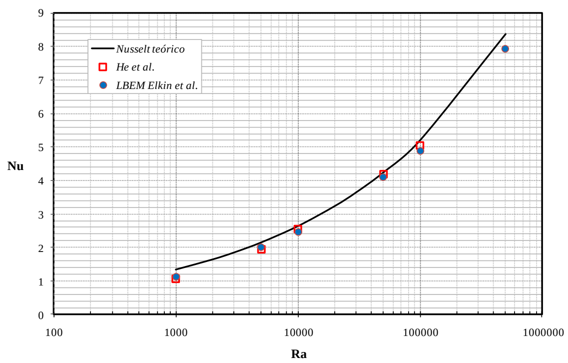

Fig. 5. Behavior of the number of Nu as a function of the number of Ra

In Figures 6 and 7 the results obtained for the temperature distribution and the current flow of the Rayleigh-Bénard convection in steady state can be observed for Ra numbers of 5E4, 11E4, 1E5 and 5E5. As can be seen, the hot fluid near the lower wall flows upward and increases the temperature in the central part of the channel, while the cold fluid, near the upper wall, decreases and decreases the temperature near the lateral limits. When the Rayleigh number increases, two trends are observed for the temperature distribution: the improvement of the mixing of the hot and cold fluids, and an increase in the temperature gradients near the lower and upper limits. Both trends improve heat transfer in the channel.

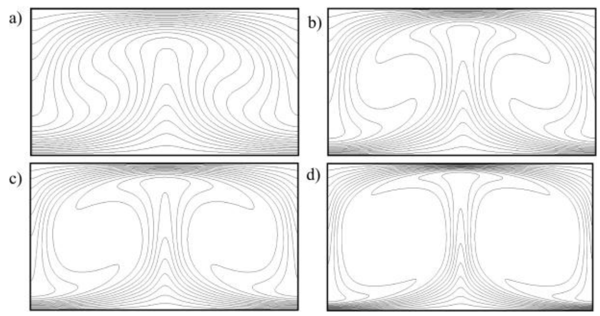

Fig. 6. Isotherms of the flow Ra-Be, for values of Pr = 0.71 and a) Ra = 1E4, b) Ra = 5E4, c) Ra = 1E5, d) Ra = 5E5

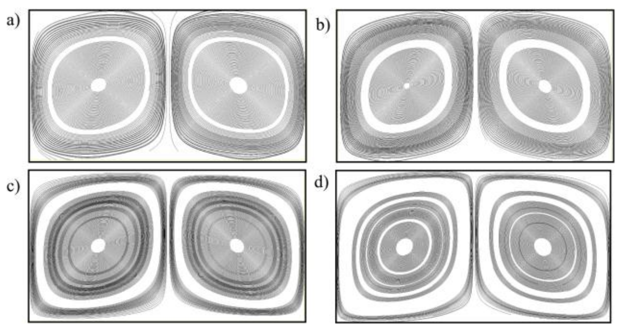

Fig. 7. Streamlines of *Ra-Be* flow, for values of Pr = 0,71 Y a) Ra = 1E4, b) Ra = 5E4, c) Ra = 1E5, d) Ra = 5E5

4. CONCLUSION

A numerical model has been developed from the Boltzmann equation method in networks to simulate the convective flow of *Ra-Be*. This model is derived from kinetic theory and therefore has a firm physical basis. The key point in the model is the use of two types of distributions: the density distribution to simulate the hydrodynamic field and the internal energy distribution to simulate the thermal field.

The *Ra-Be* problem has been selected for the validation of the model considering that this is an application of the convection phenomenon of the most used by researchers due to its analytical and experimental accessibility.

Numerical instability has been a primary concern in thermal LBEM models. In this study, it was found that the main parameters that affect numerical stability are the relaxation times τ_v and τ_T. In the range of parameters used in this study, temperature variation seems to have a minor effect on numerical instability. The lowest value for either τ_v and τ_T was around 0.536 and the highest was 1.129 in the *Ra-Be* convection simulation.

The numerical results have shown that the model of two directions and four directions of temperature distribution presents an acceptable stability, which allows that the computational code developed by the authors can be applied to different types of flow configurations where the thermal effects must be considerate.

The authors consider that, the code developed will allow to provide engineering students interested in the numerical simulation of problems related to fluid flow or fluid mechanics, with thermal affectation, a free tool to validate analytical solutions to problems related to fluids that exist today and in the same way the code can be manipulated to find approximate (numerical) solutions to those problems that, due to their complexity, do not yet present analytical solutions.

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