Software-based Simulation of the Injection Molding Process Using Finite Parameters

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Abstract

The input variables and critical stages in the injection process are analyzed, taking into account the numerical methods of finite differences, finite elements and boundary elements commonly used for the solution of the material flow problem, being these methods the basis for the development of software and simulation packages, which can have an impact on the measurements and quality of the parts obtained. Even on the weight of the same.

Keywords: injection molding process, simulation, software development, quality measures.

I. INTRODUCTION

One of the most important achievements in engineering is to be able to explain by means of mathematical models, the physical processes, in our case the modeling of the plastic flow and its route through channels, ducts and the filling of the respective mold, giving rise to the computer software, very common nowadays and that advise the task of the mold maker, giving more reliable and faster results, according to the requirement and accuracy of the model embodied.

One of the critical stages in the injection molding process is the filling stage, and since the molded parts have complex shapes and are three-dimensional and the flow is non-Newtonian and non-isothermal, a simplification is essential for the development of a successful simulation tool at this stage.

Mathematical models are numerical expressions that try to explain approximately a phenomenon of physical, chemical behavior, etc.; in this case the behavior of the polymeric fluid (non-Newtonian), inside a mold cavity, and their study began with simple mathematical models, up to complex models; as well as from models for simple geometric shapes of the mold to others of greater complexity.

These mathematical models were first built based on the approximation of the flow behavior, assuming that it was to fill simple one-dimensional geometries; Researchers such as [1]-[3], developed mathematical models to explain the exponential rheological model of the material used and its dimensional behavior, in a disk geometry centered to determine the injection pressure and the closing force, this disk geometry was constant, in order to simplify the analysis model, other researchers such as [4], [5], developed a mathematical model for the determination of the flow in circular channels, evaluating

physical parameters such as the distribution of temperatures, pressures and velocities that develop in the channel.

They introduce a mathematical model based on the equations of continuity [6], and quantity of motion to evaluate the filling of a circular geometry with inlet in the center; they also use the same principles to determine the behavior of the material in the three phases that are part of the injection process using very simple geometries with great success [7], [8].

According to what was previously stated about the mathematical models, we could suppose that these models could accurately describe the behavior of the plastic flow inside a cavity, but this is not entirely true and this is due in part to the high complexity of the thermal and viscoelastic behavior of polymers, so there are rarely analytical solutions for this complex behavior of non-Newtonian fluids; therefore, for its solution we resort to numerical solutions that describe an approximation of the solution of the conservation equations.

The numerical methods commonly used for the solution of the polymeric material flow problem inside a cavity are typically analyzed by means of three methods: Finite Difference Method (FDM) [9], Finite Element Method (FEM) [10], Boundary Element Method (BEM) [11].

These numerical analysis methods have been the basis for the development of specialized software or simulation packages that today are used in different areas of injection molding, such as the search for process parameters, process control and especially in the quality analysis of the injected parts.

With the increase in the computational potential of computers, these methods have been frequently used to try to explain and analyze the different process conditions, since they can be used to study a great variety of geometric shapes and different processing conditions, thus overcoming some of the restrictions of the mathematical models previously used for the analysis of the flow in the injection process.

These efforts in the use of mathematical models and the development of them through numerical methods allowed the development of computer packages or commercially available simulation software such as C-mold, Moldflow, Cadmould among others; which have allowed to improve the design of parts and molds, and to better understand and optimize the process, although the determination of the processing conditions still requires a high number of simulation to achieve a satisfactory result or to get the right conditions to obtain high quality results and the analysis of people specialized in the subject of plastic injection. Simulation works are based,

assuming the flow as pseudo plastic flow under non-isothermal conditions, called generalized Hele-Shaw flow.

This work was an approximation and determined a very accurate correlation in terms of injection parameters and effects produced in the simulated parts such as prediction of welding lines, cavity filling, flow balance, among others. The FEM allowed the creation of a model that was a direct 3D interpretation of a geometrical part, which allowed enormous progress in this field; since initially 2D elements were simulated but in the space called 2.5D. In 1983 reforms were introduced in the finite element analysis introducing several alternatives such as the analysis based on the advance of the flow front using the control volume method, and it is based on the fact that once a node is filled, the flow front advances towards the other nodes until it fills the last of them on a surface. In this paper, software simulation results for injection molding in parts manufacturing are discussed.

2. METHODOLOGY

The injection molded parts manufacturing industry is growing day by day, so its level of demand increases as the fields of application and competitiveness of this type of industry expand; therefore, it is necessary to use systematic methods for control and optimization so that by means of these methods, the processing windows of the molding process can be established more accurately and quickly.

2.1 Design of Experiments

For the execution of a design of experiments, it is necessary to plan and establish the order in which the experiments will be developed, selecting the appropriate levels, as well as the combination of those factors that intervene in it, to analyze their change and interaction in order to later be able to be treated statistically and in this way generate a response.

In the present thesis the design of experiment will be used to look for the influence of certain factors on the shrinkage and weight of the parts after injection and ejection, since these two characteristics are known to be key in the dimensional stability of the final part. The application of the design of experiments will have as main objective the minimization of shrinkage in the injected parts, according to the best combination of parameters achieved.

The design of experiments is qualified as statistical, since it allows the selection of the most appropriate experimental strategy for each case. In addition, before recording

Some data, it can be verified that the chosen plan has the capabilities to identify

In the manufacturing processes, there are several factors and regulation levels that influence the quality characteristics of the manufactured products; due to the above, the companies have the need to study and investigate simultaneously the effect of these factors and their respective regulation levels to determine the influence that these will exert on the different quality responses evaluated in the final parts. These studies of interaction between factors and the influence they exert on the quality response in the parts produced by the molding process, would require many tests, investment of material, time and money; which would not make them feasible to carry out, since the number of tests required for experimentation tends to grow as the number of factors increases and even more so if their interrelationship was studied at higher order levels.

Statistical objectives of the design of experiments

The primary objective of a design of experiments is the search for the factors that most influence certain responses and the change that will occur in these responses, due to the relationship and dependence of these variables with respect to each other.

As stated above, a design of experiments consists of a series of tests that induce a deliberate change in a system by means of input variables or stimuli, in such a way that it is possible to observe and identify responses or output variables.

A transformation process or system is represented by the combination of machinery, processing methods, personnel and other resources involved, which transform an input or raw material into a specific product, and which are influenced by factors that may be controllable or uncontrollable.

A system can be represented where the input variables of the process are X1, X2, X3...... Xi are the controllable variables; while other Z1, Z2, Z3,......Zj are uncontrollable variables.

Among the statistical objectives of the design of experiments we have the following:

- Determination of the variables with the greatest influence on the response, Yp
- Determination of the best value of the variables Xi that influence Yp, so that Yp, presents a value close to the desired nominal value (almost always).
- Determination of the best value of the variables Xi that influence Yp, so that the variability of Yp is as small as possible.
- Determine the best value of the variables Xi that influence Yp, so that the effects of the uncontrollable variables Z1, Z2, Z3,......Zj are minimized.

The use of the design of experiments has been widely used in modern industry since it allows valid and efficient conclusions to be drawn from a process, analyzing the variables involved in the process and their influence on it.

Classification and types of Experiment Designs. There are several types of experimental plans, and they depend on the type of response to be obtained, variation of study levels of the variables and the number of parameters involved in the respective study. According to the experimental plan we have:

• **Completely Random:** When there is only one factor to study, and it is done by randomly assigning experimental values to the different levels of the factor

involved in the study. Its main objective is to estimate the effects of the treatments.

- **Factorial:** When you want to study the interaction and variation of two or more factors, testing all possible combinations between them, thus achieving a comparison of effects between them, and the responses produced in the system.
- **Block Factorial: There** are two cases; the first is when the number of trials of each factorial is too large to be carried out in a common and satisfactory way and the second is when there are many factors and levels, so it is not possible to test all combinations. This type of experimental plan also aims to study the effects of several factors and their interactions.
- **Response surface:** This type of experimental plan is used when it is necessary to look for an optimal region of experimentation, in which the researcher can have control over them to influence the results of the process. These factors are represented in the space and in which the desired experimental response will remain within it, for its treatment.
- **Mixed designs:** Their operation is very similar to the experimental factorial designs.

Factorial design:

A factorial design is an arrangement of two or more factors in which all possible combinations of levels and interactions are investigated in each experimental trial or test. The effect of a factor in a factorial design is defined as the change in response produced by a change in factor level.

2k two-level factorial design

These are the most commonly used factorial designs, and are carried out by combining k input variables, studied at two levels of experimentation. To code this type of design of experiments, they are made by assigning the lowest value (-1), and the highest value (1). By means of this design of experiments we can study: (k) main effects; 2-factor interactions and 3-factor interactions and a k-factor interaction. In summary, the equation that defines the experimental arrangement is 2k-1 effects (Montgomery, 2004).

2.2 Fractional Factorial Designs

When the interaction of a large number of factors needs to be studied, a 2k factorial design increases exponentially, so the number of experiments to be performed would be too many and could exceed the resources allocated to perform them. For this reason a good design to use are the fractional designs, since for example, if we need to analyze 6 factors at two levels each one we would have a 26 factorial design, and we would have to perform 64 experiments; to find out that of the 63 degrees of freedom (2k-1), 6 correspond to the main effects, 15 to the interactions of two factors and the remaining 42 would

correspond to the interaction of three or more factors; in which case they would be discarded, since according to (Prat 2004) the interactions of order greater than three are negligible; therefore taking into account that only the main interactions and those of order two are important. Having stated the above, we would only need to obtain the same answers with a fraction of the experiment.

These experimental designs are widely used when the number of factors to be analyzed is high, and its main purpose is to find those factors that have more important effects, or those that influence more significantly in the process. In this case, a ²⁸⁻⁴ fractional factorial design was used in the first stage of the suggested methodology, to find which factors influence in an important way, aspects related to the dimensionality of the parts obtained through the injection molding process.

This fractional factorial design is 1/16 of 8 factors in 16 runs, so it was very appropriate to find which factors most influenced the dimensional characteristics of the final part.

In summary, the above tools are useful to know, control and optimize an industrial process, either in continuous or discontinuous form, expressed in the form of results:

- Reduction of process variation.
- Ease of process adjustment.
- Improved product quality and reduction of nonconformities.
- Improved productivity.

The introduction of finite element methods, finite difference and boundary element methods, gave a great impulse to the analysis through simulation of the injection molding process, and already in 1997 the analysis started with a type of mesh that allowed the analysis from 2D to 3D of solid models called Dual Domain, because although the CAD model was 3D, the analysis through CAE simulation required a simplification of the model, through the analysis of the middle plane of the model, this meant creating at least two models; one model to represent the actual details of the modeled part and another for analysis through simulation. All these studies allowed the analysis of more complex models but were impractical due to the expense in calculation time and complexity of the analysis, therefore they had to be simplified to reduce the calculation time which sacrificed the reliability and the accurate calculation of the results when compared with those obtained experimentally.

2.3 Process variables

In the present work, 8 variables of the process are initially considered, in order to later, according to the analysis performed, study only those that have a significant influence on the dimensional aspects and related to the final weight in the parts injected by this injection molding process.

• **Injection time** (tiny): is the time programmed for mold filling, while the machine maintains control of the screw travel speed. This time is the lower limit of the effective cavity filling time, which also includes the

final filling phase under injection pressure control. The difference between the two depends on the remaining set of process conditions. The value of the time, for the simulation stage will be entered directly as an input parameter in the process, and should be expressed at the machine in the form of the value of the screw injection speed. With respect to the above, the speed profile programmed in the machine for mold filling must be taken into account; in our case a constant speed profile will be assumed, so the relationship between the injection speed and the filling time can be obtained by means of the following equation.

$$V_{inyeccion} = \frac{V_{inyeccion}}{V_{cilindro}} * T_{inyeccion} \tag{1}$$

Where:

- Injection [cm3]: the volume of material injected during the filling phase. This volume differs from the total volume injected depending on the compaction of the material and the geometry and cross-section of the cavity.
- **Cylinder** [cm2], the inner section of the injection cylinder.
- **vinjection** [cm/s], the injection velocity.
- injection [s], the injection time.
- **Injection temperature (Tiny.):** is the programmed temperature of the material at the cavity inlet. This temperature corresponds to the temperature programmed on the machine of the last resistance of the heating cylinder, with a percentage in the nozzle temperature of 50%, programmed. The temperatures of the remaining resistances of the cylinder in the different sections of the cylinder will not be taken into account, whose programmed values are established according to the recommendations of the material suppliers.
- Mold Temperature (Tm): is the programmed temperature of the coolant flow at the mold cooling circuit inlet. This temperature corresponds to the temperature programmed in the cooling or heating device (depending on the desired temperature) attached to the injection molding machine. For our study, water was used as the cooling liquid.
- Maintenance Pressure (Pm): it is the value of the maintenance or remanence pressure, and corresponds to the value of the pressure reached at the cavity inlet at the end of the filling phase under spindle speed control to pass to the pressure control. This maintenance pressure can be given as the speed by means of profiles; but, for the present study it is going to be considered constant for the whole process. From the point of view of the injection machine, this parameter must be expressed as the value of the

hydraulic pressure that must be maintained during the compaction of the material, so it must be transformed taking into account the relationship between hydraulic pressure and nozzle pressure.

- **Spindle rotation speed (Vg):** This is the rotation speed of the spindle inside the plasticizing cylinder and should be chosen according to the material used, depending on the viscosity and diameter of the spindle used. This speed is given in terms of percentage and will be entered directly from the machine parameter control.
- Closed mold time (tmc): It is the time programmed directly in the machine, during which the part will be in the mold cavity before its ejection, when it has already reached the necessary temperature for its demolding. This time begins to count from the end of the maintenance or remanence phase when the injection molding carriage is fixed, which is the case taken for our study, but if you want to take it with the mobile injection molding carriage, it will begin to count after reaching the retraction position of the same (Mateu and Solé manual).
- **Holding time:** The holding time corresponds to the time during which the second or remanence pressure is exerted to achieve compaction of the molded part before it is ejected from the respective mold.
- **Back pressure (Cp):** The back pressure is the pressure in the nozzle before the injection pressure acts, and is programmed directly on the injection molding machine control.

2.4 Process Variables in Simulation

There are two types of variables when using CAE packages to analyze the injection molding process:

- Input Variables
- Variables Result.

Input variables: They are introduced directly to the software for analysis and do not need any type of treatment, as if it has to be done when programming in the real process, as for example when we want to program a variable such as the injection time, because it is directly related to the speed at which the polymer is injected into the cavity.

In this simulation analysis, only the variables that are found to be most influential on the dimensional conditions of the analyzed part will be analyzed.

Result Variables: the result variables are the answers offered by the program, in this case the program used is the Moldflow Plastic Insight 4.1 software and they are the following:

Weighted average temperature (Bulk Temperature). [Tpp]

As it is difficult to assess the temperature changes that the material undergoes, not only over time but also through the thickness of the geometric model used, we use the weighted

average temperature or bulk temperature, which will indicate the average temperature of the material when flowing with a certain speed through the entire section of the geometric model used.

$$T_{pp} = T_{media} + \frac{\int_{0}^{h} v (T - T_{media}) dz}{\int_{0}^{h} v dz}$$
(2)

When the flow in the section stops, the Tpp expresses the average value of the temperature of that section, which is appreciated during the compaction and cooling phase. By means of this variable we can analyze hot spots in the filling stage and it can indicate if the Tpp exceeds the maximum temperature recommended for the use of that material, since it would indicate if the material used in the process has suffered any degradation due to the high temperature. Another important aspect to take into account when analyzing this variable response is that the specific volume of the material varies as a function of pressure and temperature, since this variation is directly related to the cooling rate of the material and its behavior through the PvT diagram, which will indicate approximately the degree of shrinkage and deformation that our piece of study will have.

Wall shear stress [Ecp].

The shear stress, or wall shear stress, is the shear stress that appears at the liquid-solid interface during the filling stage. The pressure gradient y is the maximum value of the shear stress appearing across the cavity section at that point. It indirectly gives an indication of the degree of orientation of the molecules/fibers occurring in the material at each position in the cavity. A high value of wall shear stress implies a higher degree of material orientation, especially at the surface of the part, which implies a higher anisotropy of the material and a greater differentiation of the direction of flow. On the other hand, a combination of high shear stress and high material shear rate can lead to material degradation. As a general rule of thumb the shear stress should be kept below 20 times the value of the material.

Cavity pressure [Pc] (Cavity pressure)

So that the fluid can advance through the cavities of the mold, it has to overcome forces that oppose the movement of the same, therefore to overcome these forces and therefore the fluid can advance, it is necessary to apply or supply an energy to the fluid so that it can move, this energy is given in the form of pressure so that the fluid can advance through all areas of the mold cavity to fill it. The pressure in the cavity, increases progressively during the filling phase and then tends to rise to a maximum value as a result of the maintenance of the pressure at the inlet during the remanence phase; also decreasing progressively as the piece cools to reach the value of atmospheric pressure. This variable result is very important since it has significant effects on the part in terms of the degree of contraction that it will undergo after ejection and the possible deformations that may occur after its ejection. This result is, in turn, related to other effects on the part such as final shrinkage, deformations and mechanical characteristics, although these are better characterized by other process results.

Minimum peak pressure [Ppm] (Minimum peak pressure).

The pressure inside the cavity is distributed throughout the cavity, but in the last regions that are filled, there will be points or nodes in which the value of is minimum. This minimum value can indicate the degree of compaction, and the minimum specific volume of the material reached, before its cooling below the transition temperature.

Clamping force required [Fc] (Clamp force)

When we inject a material, this makes a pressure so that the material can enter the mold cavity, so there is a force acting on the two plates, favoring the separation of the same, so it is necessary a force that prevents opening or to keep the mold closed during the process and thus achieve that the material does not escape and therefore burrs and deformations occur in the injected piece.

$$F = \int_{A}^{\cdot} p.\,\overline{n}.\,\overline{dA} \tag{3}$$

Where,

 $\vec{\eta}$ = unit vector according to the direction of flow

P = material pressure

 $\partial \vec{A}$ differential area y = \vec{A} inner surface of the quality.

Percent Cold Layer Fraction [%Cf] (Frozen Layer Fraction)

When the material enters the mold cavity, which is at a colder temperature than the molten material, it solidifies on contact with the mold walls, creating a layer with a wall thickness that depends on the speed at which the fluid is injected, called the cold layer; therefore the percentage of cold layer represents the thickness of the layer of material that has solidified at a given instant and at a given point, in relation to the total thickness of the section at that point. This value oscillates over time between zero and one. A high value of %Cf indicates that the thickness of solidified material in the section is large and, therefore, results in a decrease of the useful section through which the material can pass to continue its path along the cavity. During filling, the thickness of this layer reaches a value that tends to remain constant when balancing the heat flux lost through the mold wall with the heat input due to the flow. When the flow of material ceases, heat is no longer supplied and, from this moment on, the layer of solidified material increases rapidly. The cold layer has a great influence on the resistance to flow, since the decrease in the thickness of the fluid vein has a great influence on the fluidity of the material as it is proportional to the cube of the thickness. If the CFR value reaches unity before complete filling of the cavity in any section of the filling system or part, this will lead to incomplete parts, so this process result is a suitable indicator of whether or not the part has filled the total volume of the mold cavity.

3. RESULTS

Analysis of input variables and influence on dimensional quality measured longitudinally.

The dimensional quality response in the measurement of the part analyzed, the longitudinal measurement of the part was taken and only the significant effects were taken into account to perform the respective analysis of variance (table 1, figure 1 and 2).



Fig. 1. Pareto. Length Measurement



Fig 2. Normal Probability. Length Measurement

Table 1. Significant effects Longitudinal Measure

Estimated effects for Ml	The following effects are confused:
average = 27.0942 A:Pm = 0.123375 B:Tm = -0.021875 C:tinj = 0.014375 D:tmc = -0.001125 E:tman = -0.009375 F:Vg = -0.003625 G:Tiny = -0.0046875 H:Cp = -0.007875	• AB+CG+DH+EF = 0.002125 • AC+BG+DF+EH = 0.015375 • AD+BH+CF+EG = -0.001625 • AE+BF+CH+DG = -0.008875 • AF+BE+CD+GH = -0.010125 • AG+BC+DE+FH = 0.012125 However, in this part of the study we will not study the main effects and their influence on the quality characteristic analyzed.

Analysis of input variables and influence on transversal measurement dimensional quality. The response of dimensional quality in the measurement of the part analyzed, the transversal measurement was taken and only the significant effects were taken into account to perform the respective analysis of variance (Figure 3, 4 and 5).



Fig. 3. Pareto chart. Cross-sectional measurement



Fig. 4. Normal Probability Plot. Cross-sectional measurement

Mathematical equation for quality response, Transversal measurement

$$\begin{split} Mt &= 20.0839 + 0.002625 \text{*Cp} + 0.06125 \text{*Pm} + \\ 0.00425 \text{*tinj} - 0.015625 \text{*Tiny} + 0.004875 \text{*Tm} - \\ 0.001875 \text{*tman} - 0.0005 \text{*tmc} + 0.001 \text{*Vg} \end{split}$$



Figure 5. Interaction of Main Effects Cross-sectional Measurement

Analysis of input variables and influence on dimensional quality measured by weight. The response of dimensional quality on the weight of the piece analyzed was taken by weighing the ten samples in each experimental design and also, the significant effects were taken into account to perform the respective analysis of variance (Figure 6).



Fig. 6. Pareto chart. Weight

4. CONCLUSIONS

Mathematical methods that have been used for the development of specialized packages such as finite elements, finite differences, which have allowed the simulation in injection processes, providing a better response to detect how it affects the parameters in the quality of developed parts were analyzed. Allowing the reduction of materials and prices in a manufacturing chain, either in the improvement of the weight, size and geometry of the same.

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