International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 14, Number 8 (2021), pp. 782-787

Unsteady MHD Flow and Heat Transfer Through Porous Medium Between Parallel Plates with Periodic Magnetic Field and Constant Pressure Gradient

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Abstract

A theoretical model is performed to study the flow and thermal behavior of an electrically conducting fluid flow between two infinite parallel plates with porous medium. A perpendicular periodic magnetic field is applied while the fluid is subjected to a constant presssure gradient. A constant heat flux on the upper plate is applied while the lower plate is kept at constant temperature. The dimensionless governing momentum and energy equations are solved numerically using a finite difference technique. The equations are solved also analytically for a constant magnetic field and zero Grashoff number using eigenfunction expansion method and the numerical and analytical results are found to be in full agreement.

The effect of different physical parameters on the transient velocity and temperature profiles , such as Grashof's number, magnetic parameter, magnetic frequency is studied. The results are presented graphically and discussed.

Keywords: Magnetohydrodynamics (MHD), parallel plates, periodic magnetic field.

I. INTRODUCTION

The unsteady MHD flow between parallel plates has wide applications such as MHD pumps and power generators, cooling systems, petroleum industries accelerators and many other applications.

The effect of magnetic field on flow between two parallel plates with porous medium and different conditions had been studied by many researchers. Attia [1] studied the transient flow of dusty conducting fluid between parallel porous plates with effect of temperature dependent viscosity and constant pressure gradient. Chauhan et al [2] studied MHD viscous fluid flow and heat transfer in a parallel plate channel when it is partially filled with a porous medium with an inclined magnetic field and a rotating system. Guchhait et al [3] presented the effects of Hall currents on the flow of an electrically conducting fluid between two horizontal parallel porous plates channel in a rotating system where the transverse magnetic field is uniform. Manyonge et al [4] examined the steady flow between two infinite parallel porous plates under the effect of uniform magnetic field and constant pressure gradient. Raju et al [5] studied the steady MHD forced convective flow of a fluid in a porous medium over a horizontal channel where the bottom wall was insulated and impermeable with effect of viscous dissipation and joule heating. Yadav and Sharma [6] investigated steady convection flow of an electrically conducting fluid along a semi infinite vertical plate in the presence of heat generation and a convective surface boundary condition. The incompressible two-dimensional magneto hydrodynamic flow and heat transfer of a micropolar fluid between parallel porous plates was studied by Ojjela and Naresh [7]. The MHD transient free convection flow through infinite vertical parallel plates in a porous medium with heat source and chemical reaction had been investigated by Sasikumar and Govindarajan [8]. Kuiry and Surya [9] studied the steady MHD flow and thermal behavior of a fluid between two infinite horizontal porous plates. The effect of inclined magnetic field on the flow through a horizontal channel through porous medium is studied by Dwivedi et al [10]. A theoretical model is performed by Abdullah [11] to study the effect of magnetic field and periodic wall temperature on transient free convection flow of a fluid past an accelerated vertical plate. The effect of thermal radiation and chemical reaction on unsteady MHD Couette flow through a porous medium of a fluid between two parallel porous plates was investigated by Anyanwu et al [12].

Other studies had been presented to study the effect of no uniform magnetic field on the behavior of MHD flows. Shliomis and Shinichi [13] studied the oscillatory pipe flow in a no uniform magnetic field subject to the quasielastic magnetic force. Moreau et al [14] investigated the problem of magneto hydrodynamic flow in a rectangular duct with a non-uniform magnetic field. Asghar et al [15] presented an exact solution to the transient Couette flow with no uniform magnetic field. Effect of a no uniform magnetic field on ferrofluid flow with heat transfer in a channel was studied by Goharkhah et al [16]. Ghaffarpasand [17] studied numerically the MHD mixed convection of ferrofluid in a cavity in the presence of alternating magnetic field.

In the present paper a study of the transient convection MHD flow between two infinite parallel plates with porous medium is presented. The fluid is subjected to a perpendicular periodic magnetic field and a constant presssure gradient. The dimensionless equations are solved numerically and verified by analytical solutions.

II. MATHEMATICAL FORMULATION

Consider an MHD incompressible electrically conducting viscous flow through a porous medium between two infinite horizontal parallel plates separated by a distance h. Using the rectangular cartesian coordinates, the x* axis is assumed to be in the flow direction and y* axis perpendicular to it as shown in Fig. 1.

The fluid is assumed to has a density ρ , a dynamic viscosity μ, and an electrical conductivity σ. The flow is subjected to a constant pressure gradient and a non-uniform transverse magnetic field of the form $B(t^*) = B_0(1 + A\sin(\omega^* t^*))$ with a flux density B_o directed along the y* axis.

Initially, at time t* ≤ 0, the plates and the fluid are assumed to be stationary and have the same temperature T_∞. At time t* > 0, the upper plate starts moving in its own plane with a constant velocity and a constant heat flux, while the lower plate is heated with a constant temperature and maintained stationary.

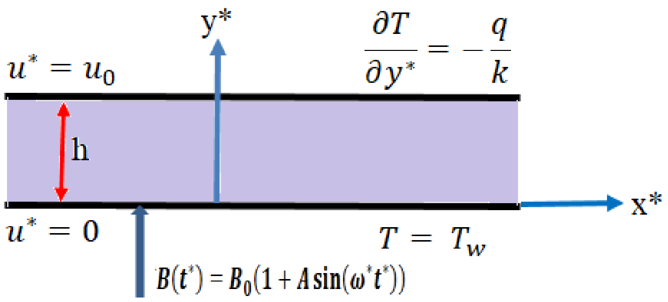

Fig. 1. Geometry of the Physical System

By applying the periodic magnetic field with Boussinesq approximation, the transient governing equations representing the fluid velocity and temperature are:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p}{\partial x^*} + g\beta(T - T_\infty) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho}[1 + A\,sin(\omega^* t^*)]u^* - \frac{\nu}{k^*}u^* \tag{1}$$

$$\rho c_p \frac{\partial T}{\partial t^*} = k\frac{\partial^2 T}{\partial y^{*2}} \tag{2}$$

With corresponding initial and boundary conditions as follows:

$t^* \le 0$: $u^* = 0$ $T = T_\infty$ *for all* y^*

$t^* > 0$: $u^* = 0$ $T = T_w$ $at\ y^* = 0$

$u^* = u_0$ $\frac{\partial T}{\partial y^*} = -\frac{q}{k}$ $at\ y^* = h$ (3)

Now to produce a dimensionless form of the governing equations, the following dimensionless quantities are employed :

$$x = \frac{x^*}{h},\quad y = \frac{y^*}{h},\quad t = \frac{\nu t^*}{h^2},\quad \omega = \frac{h^2\omega^*}{\nu}, u = \frac{u^*}{u_0},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\quad P = \frac{ph}{u_0\mu} \tag{4}$$

$$M = \frac{\sigma h^2 B_0^2}{\mu},\quad Gr = \frac{g\beta h^2 (T_w - T_\infty)}{\nu u_0},\quad Pr = \frac{\mu c_p}{k},\quad K^2 = \frac{h^2}{k^*}$$

Then the dimensionless equations become:

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + G_r\theta + \frac{\partial^2 u}{\partial y^2} - [K^2 + M(1 + A\,sin(\omega t))]u \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

The corresponding boundary conditions can be written as :

Initially, $t \le 0$: $u = 0$ $\theta = 0$ for all y

$t > 0$: $u = 0$ $\theta = 1$ $at\ y = 0$

$u = 1$ $\frac{\partial \theta}{\partial y} = -1$ $at\ y = 1$ (7)

III. NUMERICAL ANALYSIS

The dimensionless momentum and energy equations 5 and 6 are solved numerically using the Crank Nicolson technique which is a fully implicit finite difference method. The central difference approximations are used to produce the finite difference equations in implicit form.

The method with a large amount of time steps is employed. At each time the tridiagonal matrix is solved directly using Thomas algorithm.

IV. ANALYTICAL SOLUTIOM

The numerical solution can be verified by comparing it with analytical solution. The analytical eigenfunction expansion method is used to solve the momentum and energy equation.

IV.I TEMPERATURE SOLUTION

The dimensionless energy equation is:

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

The boundary conditions are:

Initially,$t \le 0$: $\theta = 0$ for all y

$t > 0$: $\theta = 1$ $at\ y = 0$

$\frac{\partial \theta}{\partial y} = -1$ $at\ y = 1$ (9)

The boundary conditions are non homogeneous and they are converted to homogeneous ones by introducing:

$$H(y,t) = \theta(y,t) + y - 1 \tag{10}$$

Then the energy equation becomes:

$$\frac{\partial H}{\partial t} = \frac{1}{Pr}\frac{\partial^2 H}{\partial y^2} \quad (11)$$

With the following corresponding boundary conditions

Initially, $t \le 0$: $H = y - 1$ for all y

$t > 0$: $H = 0$ $at\ y = 0$

$\frac{\partial H}{\partial y} = 0$ $at\ y = 1$ (12)

Let $H(y,t) = \emptyset(y)\delta(t)$

Taking the derivatives and substituting into (11) yields the eigenvalue problem as:

$$\frac{d^2\emptyset}{dy^2} + \lambda\emptyset = 0 \qquad \emptyset(0) = \frac{\partial\phi}{\partial y}(1) = 0 \quad (13)$$

Where λ is the separation constant.

By applying the boundary conditions the solution of the above equation will be

$$\emptyset_n(y) = sin(\sqrt{\lambda}y) \quad (14)$$

With eigenvalues

$$\lambda_n = \left(\frac{2n-1}{2}\pi\right)^2 \quad (15)$$

For each n, the solution for δ(t) is $\delta_n(t) = e^{-\frac{\lambda_n t}{pr}}$

Hence the series solution for H(y,t) is:

$$H(y,t) = \sum_{n=1}^{\infty} B_n\, sin(\sqrt{\lambda}y)e^{\frac{-\lambda}{pr}t} \quad (16)$$

Which will satisfy the non-homogeneous initial condition $H(y,0) = (y-1)$.

Hence

$$B_n = \frac{2}{\lambda}\left(\sin\sqrt{\lambda} - \sqrt{\lambda}\right)$$

Where $n = 1,2,\dots\dots,\infty$

Then the final form of solution is

$$\theta(y,t) = \left(\sum_{n=1}^{\infty}\frac{2}{\lambda}\left(sin\sqrt{\lambda} - \sqrt{\lambda}\right) sin(\sqrt{\lambda}y)\, e^{\frac{-\lambda}{pr}t}\right) + (1-y) \quad (17)$$

The coefficient of heat transfer (Nusselt number), is given by:

$$Nu = \frac{-h\left(\frac{\partial T}{\partial y^*}\right)}{(T - T_\infty)} \quad (18)$$

Using the temperature solution given in (17), The Nusselt number at the lower and upper plates can be written as:

$$Nu_0 = \frac{-1}{\theta(0,t)}\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$

$$= \left(\sum_{n=1}^{\infty}\frac{2}{\sqrt{\lambda}}\left(sin\sqrt{\lambda} - \sqrt{\lambda}\right)\, e^{\frac{-\lambda}{pr}t}\right) - 1 \quad (19)$$

$$Nu_1 = \frac{-1}{\theta(1,t)}\left(\frac{\partial\theta}{\partial y}\right)_{y=1} = \frac{1}{\theta(1,t)} \quad (20)$$

IV.II VELOCITY SOLUTION

The momentum equation is also solved analytically for the case of uniform magnetic field (ω=0) and negligible Grashof number (Gr=0).

By introducing p*=-∂P/∂x the dimensionless time dependent velocity equation can be written as:

$$\frac{\partial u}{\partial t} = p^* + \frac{\partial^2 u}{\partial y^2} - [K^2 + M]u \quad (21)$$

The corresponding boundary conditions are :

Initially, $t \le 0$: $u = 0$ for all y

$t > 0$: $u = 0$ $at\ y = 0$

$u = 1$ $at\ y = 1$ (22)

To convert the non homogeneous boundary conditions to homogeneous ones, we introduce

$$F(y,t) = \theta(y,t) - y \quad (23)$$

Then the energy equation becomes:

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - [K^2 + M]F + p^* - [K^2 + M]y \quad (24)$$

With the following boundary conditions

Initially, $t \le 0$: $F = -y$ for all y

$t > 0$: $F = 0$ $at\ y = 0$

$F = 0$ $at\ y = 1$ (25)

Taking the homogeneous part of equation 24, the equation becomes:

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - [K^2 + M]F \quad (26)$$

Assuming $F(y,t) = \alpha(y)a(t)$, taking the derivatives and substituting into (26), the eigenvalue problem will be:

$$\frac{d^2\alpha}{dy^2} + \eta\alpha = 0 \qquad \alpha(0) = \alpha(1) = 0 \quad (27)$$

Where η is the separation constant.

The solution of the above equation will be

$$\alpha_n(y) = \sin(\sqrt{\eta}y) \quad (28)$$

With eigenvalues

$$\eta_n = (n\pi)^2 \quad (29)$$

And for each n, the solution for a(t) is

$$a_n(t) = \frac{2(-1)^n}{n\pi} e^{-(K^2+M+\eta_n)t} + \frac{1}{(K^2+M+\eta_n)} \left(\frac{2(-1)^n}{n\pi} (K^2 + M - p^*) + \frac{2p^*}{n\pi} \right) \left(1 - e^{-(K^2+M+\eta_n)t}\right)$$

Hence the series solution for F(y,t) is:

$$F(y,t) = \sum_{n=1}^{\infty} a_n(t) \sin(\sqrt{\lambda} y) \quad (30)$$

Substituting into equation 23, the final form of solution will be

$$u(y,t) = \sum_{n=1}^{\infty} a_n(t) \sin(\sqrt{\lambda} y) + y \quad (31)$$

V. RESULTS AND DISCUSSION

The effect of periodic magnetic field with constant pressure gradient on a viscous incompressible fluid flow through a porous medium between infinite parallel plates is studied. The governing momentum and energy equations are solved numerically by using the Crank-Nicolson technique and the results are verified by solving the problem analytically in case of a constant magnetic field and zero Grashoff number.

The effects of different parameters on the velocity and temperature profiles are shown through graphs. The parameter values that used to get the results are: Gr = 5, Pr = 7, M = 5, P* = 5, A = 0.5, K=2, t = 10.

Fig. 2 shows the effect of Grashof number on the dimensionless velocity profile. It is seen that the velocity of the fluid increases with the increase of Grashoff number.

The effect of Magnetic field strength is shown in Fig. 3, where the increase in the applied magnetic field strength causes retardation the flow.

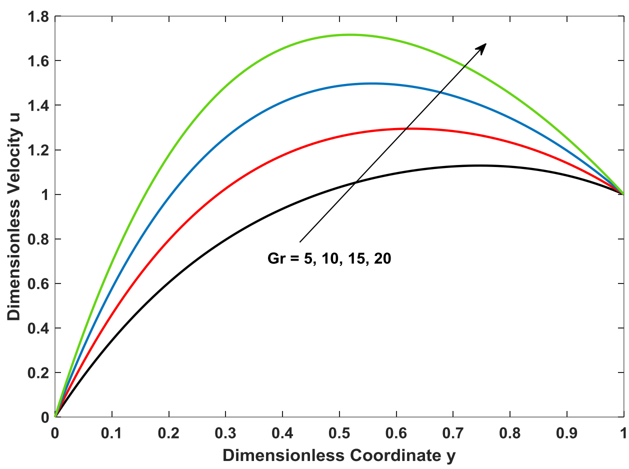

Fig. 2. Velocity profile for different Grashof number

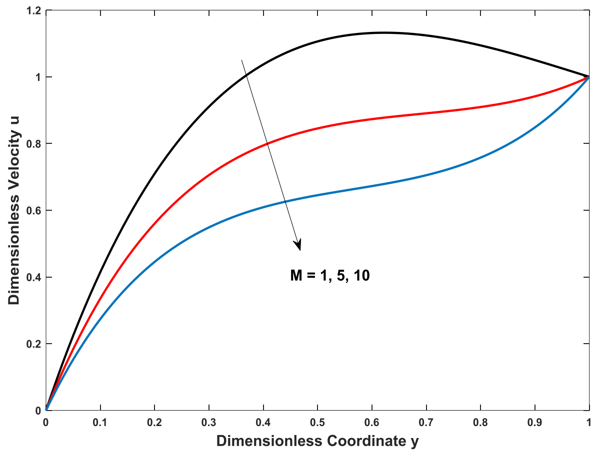

Fig. 3. Velocity profile for different magnetic parameter

Fig. 4 illustrate the effect of permeability parameter on the velocity profile. It is noticed that the effect of increasing K is to decrease the velocity.

The effect of pressure gradient is shown in Fig. 5. It is observed that the velocity of the fluid increases due to an increase in the pressure gradient.

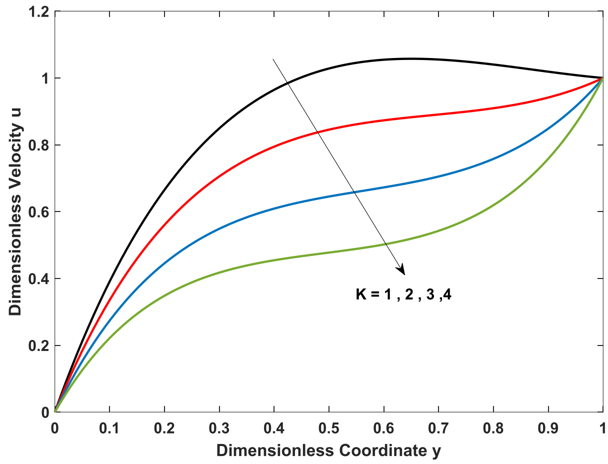

Fig. 4. Velocity profile for different permeability parameter

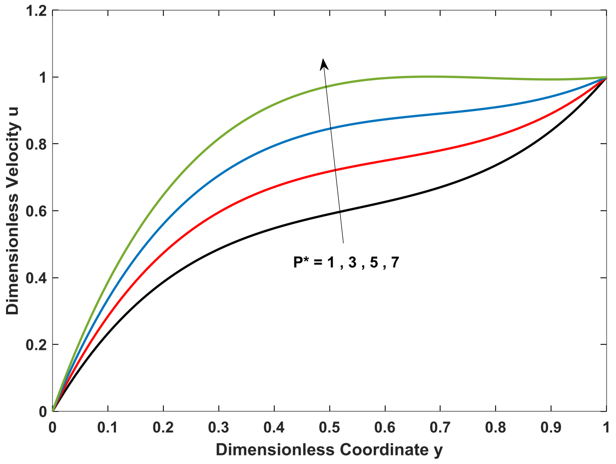

Fig. 5. Velocity profile for different pressure gradient

The transient velocity profile hase been shown in Fig. 6 and Fig. 7 for different locations on the fluid and different magnetic field frequencies. It is seen from figure 6 that the velocity increases until it reach the steady state, and that the same periodic behavior of the velocity is shown for different locations of the flow unless that the amplitude increases as moving toward the mid point between the plates. Fig. 7 studies the effect of magnetic frequency on the transient velocity. It is seen that the velocity has the oscillatory behavior with same amplitude and different time period for different frequencies.

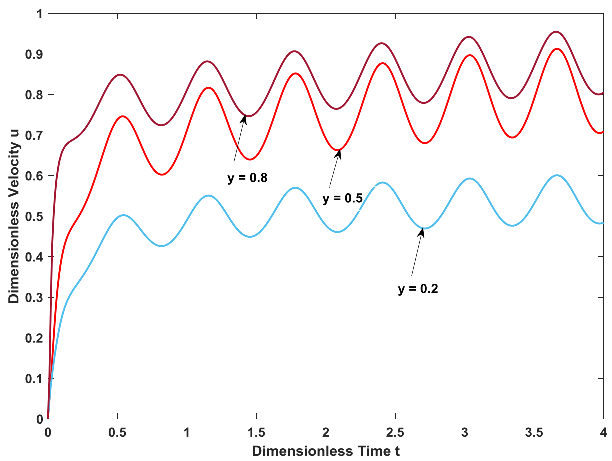

Fig. 6. Transient velocity at different points on the y coordinate

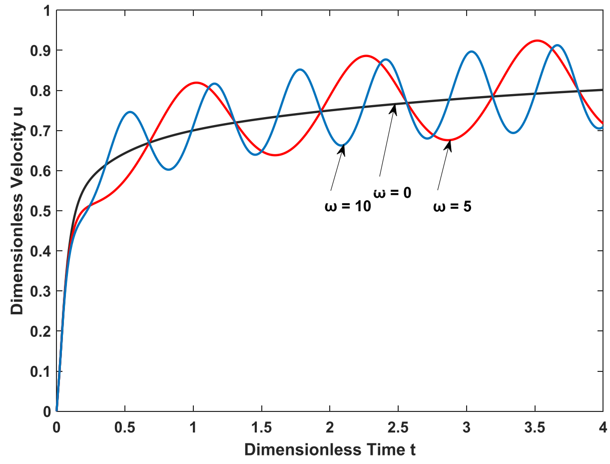

Fig. 7. Transient velocity for different values of magnetic frequency

It is observed from Fig. 8 that the temperature decreases as the prandtl number increases.

A comparison between numerical solution and analytical solution for the transient velocity and temperature is given in table 1. It is noticed from the table that the results are in a very good agreement with each other.

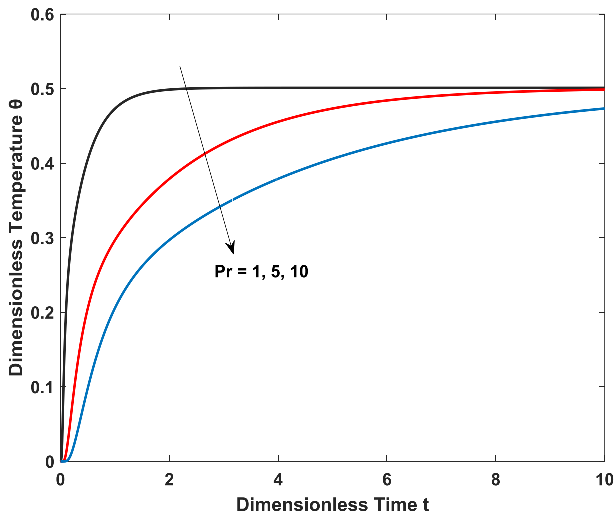

Fig. 8. Temperature profile for different Prandtl number

Table 1. Comparison of numerical and analytical results

	Time t	0.1	0.2	0.3	0.4	0.5	0.6	0.7
u	Numerical	0.3934	0.5087	0.5275	0.5306	0.5311	0.5312	0.5312
	Analytical	0.4086	0.5126	0.5284	0.5308	0.5312	0.5312	0.5312

	Time t	1	2	3	4	5	6	7	8	9	10
θ	Numerical	0.2540	0.3380	0.3869	0.4208	0.4447	0.4614	0.4732	0.4815	0.4873	0.4913
	Analytical	0.2548	0.3384	0.3871	0.4210	0.4447	0.4615	0.4732	0.4815	0.4873	0.4913

VI. CONCLUSION

The effect of periodic magnetic field on a fluid flow through a porous medium between two infinite parallel plates with constant pressure gradient is considered. The dimensionless governing partial differential equations are solved using a fully implicit numerical technique and verified by an eigen function expansion method. The velocity and temperature profiles are shown through graphs with the effect of various physical parameters. The conclusions of the study are:

1. The velocity of the fluid increases as the Grashoff number or the pressure gradient inreases.

2. The velocity of the fluid decreases due to an increase in Prandtl number, magnetic field strength or permeability parameter.

3. It is found that the transient velocity profile has a periodic behavior when using a periodic magnetic field.

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