

Assessment of Free Electron Laser and a Laser Plasma Wakefield Accelerator and Using Puffin simulation code

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Abstract

In review, the free electron laser radiation produces by laser plasma accelerator that accelerates the electron beam on a few centimeter distances for high peak current to produce high peak brightness radiation for the usefulness and application. The goal of this work is studying free electron laser (FEL) radiation and evaluation to investigate a chirped electron pulse from the laser plasma acceleration (LPA) and resolve energy chirp reduction. It is a potential assessment of the interaction of electron beams in 3D radiation fields, momentum space and 3D spatial dimensions in an unaverage 3 dimensional model of FEL implementing numerical techniques with polarization and the electron expands during propagation that off-axis magnetic fields in a 1D dimension using simulation with utilization of Puffin code. The FEL electron beam is applied to generate an ultra-low emittance electron bunch with high brightness that utilizes numerical simulation for model approximations of radiation generation process parameters of the 3D PIC code VSim for the FEL electron beam. The plasma acceleration (LPA) is studying to analysis the values of current, emittance, dgamma and gamma from the output file of accelerator and evaluates the essential steps to resolve a chirped electron pulse from the laser. The Puffin code uses to an assessment of the trojan horse parameters of stronger chirp electron pulse radiate coherent radiation, which leads to coherent effect. The simulation implements a Laser plasma accelerator parameters to demonstrate the advantage of brightness radiation to enhance the short-wavelength of the FEL. The chirp is requested to remove before the Puffin code energy to improve the electron beams quality for the FEL application. It is prospective of the electron beam to assess the impact of the energy chirp of FEL power by using the script to resolve the focusing channel for plasma acceleration.

Keywords: Free electron laser, laser Plasma accelerator, Trojan horse, simulation, Puffin code.

1.1 Introduction:

The Free electron laser (FEL) is a device including different sources that offer coherent radiation through a beam of free electrons from the unbound electron in the atom or molecular

states system that varies from microwaves to X-rays laser. The FEL device is tools used to transfer electron kinetic energy into electromagnetic radiation, which is the fundamental of a procedure on a beam of relativistic electrons and an undulator magnet.

The electron beam passes through the undulator magnet affected by Lorentz force and provides necessary kinematical and dynamical conditions, which transverse component of motion oscillations and emit radiation at a fixed frequency.

In 1979 laser wakefield electrons acceleration the proposal by Tajima and Dawson are the techniques of generating 100 GV/m electric field, which are interesting techniques to develop electron beams of the unique characteristics of bunch length of few fs peak currents of few KA and emittance 1π mm.rad [1]. The sources of radiation of ultra-short pulsed development using compact high brightness radiation, including short wavelength free electron laser has received recently more interest to improve the equality and stability of the electron beam that produced in gas jet and capillary discharge targets.

The Lorentz force of the single free single electron governs the dynamic motion of single particles on the E- and B-fields, which produce two fields from outside (E, B) and internal fields (E, B) of the mass of a free electron [2].

$$\frac{d(mv)}{dt} = -e(E + v \times B) \quad (1)$$

Where m is electron rest mass, v speed and E electrical field and B magnetic field.

The electron beam movement and energy exchange forms in the presence of the optical and undulator are leading to electrons that emit very bright radiation. Fig.1 shows a typical high gain FEL configuration with a highly relativistic electron beam propagating through undulator or (wiggler) characterised by the dimensions undulator parameter

$$a_u = \frac{eB_u \lambda_u}{2\pi mc}, \text{ where, } B_u \text{ is the RMS undulator magnetic field}$$

strength, λ_u is the undulator period, e and m are the electron charge and rest-mass respectively. The resonant FEL wavelength is then given by $\lambda_r = \frac{\lambda_u}{2\gamma_0^2} (1 + a_u^2)$, where γ_0 is

the mean Lorentz factor. The wavelength produce exchanged in electromagnetic radiation regions by changing electron beam energy and the undulator parameters, which emits a spectrum of the short time and line shape characteristic of the discharge process [3]. The electrons radiation move side to side on the undulator magnetic field of the stored transverse radiation, which is working on oscillating electrons. The electrons accelerate in the FEL oscillator forced to move in the forward direction that radiates spontaneous emission in the resonator, while the radiation power grows due to the many passes in the undulators. The electrons pass in the operating FEL forced to transport the beam of the relativistic electrons nearly the speed of light, which randomly spreads radiation wavelengths and energy exchange takes place on the undulator and resonator [3].

$$B = \frac{2\mu_0 NI}{g} \quad (2)$$

$$K = 2.35 \times 10^{-4} NI \frac{\lambda_u}{g}$$

IN ampere turn, K is constant relative to IN value

The peak current range from several amperes to hundreds of amperes and the electron beam energy varies from a few MeV to GeV. The field strength B of undulator in the range of usual kG, while the wavelength varies in a few centimeters and the undulator length in the range of few meters length, while the period number N extends from 20 to several hundred [3].

$$\lambda_0 \frac{\lambda_u}{2\gamma^2} (1 + k^2) \quad (3)$$

Where λ_u is the undulator period, γ the relativistic factor and K is the undulator parameter proportional to field inside the undulator.

$$k = \frac{eB_0 \lambda_u}{2\pi m_e c^2} \quad (4)$$

The electrons beam of a few GeV generates by laser plasma accelerator that accelerates on the distance a few cm of high peak current with a relatively low emittance of the beam. The transverse undulator magnetic field variation effects on beam large energy spread the potential application to produce free electron laser radiation. However, laser plasma accelerators electron beam are the sources of free electron laser used to generate high peak brightness radiation of the short beam duration with the high peak current beam in the range of (1 to 10 KA) [4].

The electrons accelerate in the density wake in plasma that drives by laser beam, while the energy from laser transferred to electron bunch. The electron bunches assessment to study due to their latest energy, energy spread, charge and

emittance, which were resulting in a higher plasma density lead in accelerating charge and a higher energy spread, emittance and lower latest energy [5].

The plasma density perturbs by laser beam propagating through the plasma and the plasma density wake governs by plasma oscillation of frequency dependent on plasma density, which plasma density wakes drive getting behind the laser pulse. The laser plasma accelerator generates experimentally source of the energetic electron beam up to a few GeV of ultra-low transverse emittance of 0.1 mm rad, as well as large accelerating gradients that intrinsically produce femto second (fs) electron bunches.

Generation of FEL coherent radiation of high energy electron beam with low peak emittance produces by compact laser plasma. The energy spread hinders the potential application compensated by the method of introducing a transverse filed variation into the FEL undulator that to decrease the effect of electron beam spread [6]. The large energy spread of Laser plasma accelerators for the short wavelength produced by using a transverse gradient undulator together with a properly dispersed beam for FEL amplification. In review, we have demonstrated the significant advantage of using TGUs to enhance the short-wavelength FEL performance of laser-plasma accelerators with 1D analysis and 3D simulations [7]. There are several practical effects that were not included in these simple considerations. They include electron energy correlation with bunch longitudinal coordinates and the method of generating the required beam dispersion. The plasma electron density is lower due to dephasing and diffraction in laser plasma accelerator and that limited the energy gain and relative to relativistic self-focusing in plasma [8].

The large electric plasma fields that pre-ionized by low ionization potential of 13.6 eV of hydrogen or self-ionization by electron bunch driver accelerated the electrons and excite plasma wave. The electron bunch driver of higher ionization threshold such as helium (24.6 eV) and the mixture of helium and hydrogen that fully ionized the electron displacement, which excites the plasma wave based on hydrogen electron and helium remains present in a mixture according to gas laws. The laser pulse focuses to liberate high ionization threshold electrons via ionization with short Rayleigh length into the inside of the lower ionization threshold plasma blowout to peak intensity. The laser energies < 100 μ J focused up to sub-relativists intensities for helium is enough to excite by Ti:sapphire laser pulse [9].

The nitrogen at a higher gas pressure to produce plasma density due to ionization injection in the experiment investigate by B. S. Rao et al, which shows electrons of higher charge, broad spectrum and lowers energy on the enhanced beam loading effect. Thus, the experimental work of pure nitrogen as a gas jet target is using a titanium sapphire laser of 3 TW useful to developed laser Wakefield accelerators for various applications [1].

The idea of Trojan horse (TH) is to used laser pulses for locally confined release of the electrons directly from electron bunches and drivers to set up the plasma blowout cavity a long acceleration length. The experimental setup of the Trojan

horse acceleration in principle is multi-component, plasma medium that consists of hydrogen and higher ionization thresholds such as helium and under-dense photo-cathode electron bunches of a high current electron bunch with a release synchronized laser pulse [9]. The high electric field used to accelerate the electrons in laser wake field acceleration to reach high energy required duration of a few femtoseconds that to an achieved generation of short electron bunches. The plasma electrons from the ions pushed away by laser pulse passing through the plasma that accompanying electric field, which leads electrons oscillating around their original position and plasma wave are moving behind laser pulse [10].

Laser plasma accelerator of high peak current (10 KA), high energy (2 GeV) and low emittance (0.1m) beam used to produce soft x-ray. In 2005 at DESY in Germany the first soft x-ray FEL facility has operated for users. The Linac coherent light source at SLAC has produced the first hard x-ray FEL facility in 2009 [6].

Theoretical analysis and numerical simulations that basis on laser plasma accelerators demonstrated by Zhirong Huang et al on seeded ultraviolet and soft x-ray FEL. The review of the concept of a transverse gradient of undulator and the modification theory of the FEL simulation code Genesis and the particle in cell simulations of laser-plasma accelerator of TGU was assessed by Wang and T.Zhang [4] (4).

Currently, the simulations code basis for numerical integration were using two different FEL codes such as Genesis code ID Simulation and Puffin paralleled 3D FEL code. Genesis code used in 3D FEL simulation code for both the radiation field and electron macro-particles as a time-dependent. However, Genesis code computed harmonic emission as a function of undulator magnetic field, electron pulse and the radiation field, which reduces the computation time of the resonant period. The numerical model of the FEL used for the field envelope approximation that averages the radiation envelope and radiation wavelength over it's the motion of an individual undulator period. The computational time improved by period average, while the Genesis applies the paraxial and limited in the range of frequencies.

Puffin code the simulation model is an unaverage 3 dimensional model of FEL and interaction of electron beams than in 3 dimensions in momentum space, 3D spatial dimensions and 3D radiation fields, which implements in Fortran90 can drastically reduce computation time. The undulator magnetic field simulates in a 1D dimension in applying Puffin code and the electron expands during propagation as electron pulses that off-axis parabolic magnetic fields. Puffin is a parallel approximation code using new numerical techniques with polarization along with electron beam optics elements and flexible array of undulator tunings to produce improved temporal coherence [11]. Dijke et al was investigating the effect of laser pulse intensity on the experimental parameters basis of plasma density and length and first energy of the injected electrons on the acceleration. The electromagnetic field effects on the particle motion studies with the framework using simulation the basis on experimental parameters, which is introducing simple plasma,

wave a model consisting of a harmonic wave with the period and amplitude [10].

Mourou and Strickland in the mid-1980s developed the chirped pulse amplification (CPA) of laser pulses are known transformed the pulse by stretching the laser pulse in time before amplification and then compressed back into original length. The CPA-based laser systems were the elimination of developed the damage a threshold of the gain medium effect of limiting on intensity into the generation of peak powers of hundreds of terawatts with durations below (50fs). LUSIA collaboration 9 research groups from 6 countries studying the intense attosecond light pulses used undulator concepts offers light sources for producing intense single-cycle light pulses with a duration in the attosecond (10^{-18} s) and ultra-pulse energy in accelerator research, which is providing the basis high-brilliance attosecond light sources [12].

1.2 Results and discussions:

The FEL codes utilize numerically simulation for model approximations of FEL radiation generation process, which an important tool in evaluating the performance of averaging procedures or unaverage code over different length scales. Fig.1 shows the FEL with planer undulator and the electron is isolated through the undulator that emitting resonant radiation. The simulation parameters of the 3D PIC code VSim for the FEL electron beam used in the Trojan horse technique [13]. The plasma acceleration mechanism was used to generate an ultra-low emittance electron bunch with high brightness that of an ultra-low emittance electron bunch with high brightness that supported the efficacy and application [14]. The electrons are injected by a focused laser that produces a witness bunch speed up electron beam into GV/m fields [15].

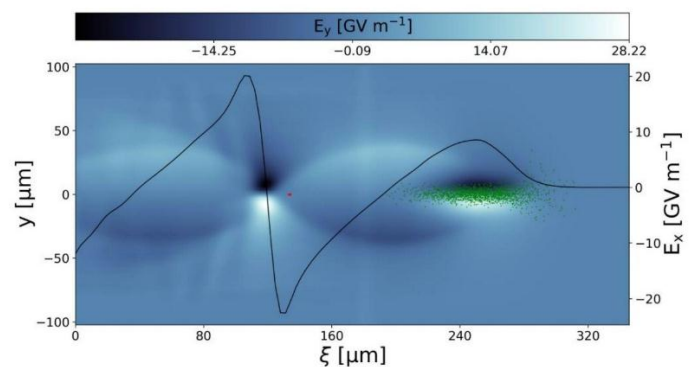


Fig.1 The Trojan Horse (TH) technique in three dimensional model

The driver beam parameters are used for the Trojan horse technique shown in table.1. The FEL operation procedure is designed and simulation using multiple codes, in addition to Puffin code implemented as a simulation tool for an unaveraged three-dimensional model of FEL [16,17]. The aim of the work to is studying and evaluates the essential steps to resolve a chirped electron pulse from the laser, plasma

acceleration (LPA), which the electrons pulse a product of the plasma accelerator code [18]. The data in table 1 presents the results obtained from the preliminary of data of the parameters of the driver beam. The table 2 below illustrates some of the main parameters of the PWFA photocathode laser. Table 3 provides the output parameters of the electron beam from LPA using VSim code.

Table 1. The parameters of the Driver beam

Driver parameters	FACET(II)
Energy	10 GeV
RMS Energy Spread	2%
RMS Emittance	50E-6 m-rad
RMS Width	3.5 um
Charge	0.8 nC

Table 2. The parameters of the PWFA photocathode laser

Laser parameters	FACET
Wavelength	800 nm
a_0	0.018
τ_0	30 fs
w_0	7 um
Distance Behind Driver	115 um
Focus position	2 mm

Table 3. The output parameters of the electron beam from LPA using VSim code

Parameters	Values
The charge of driver beam (nC)	0.8
Witness charge (pC)	3.59
Witness mean energy (Mev)	248.01
Witness emittance (m rad)	y=1.68 E-08 z=1.70 E-08
Witness energy spread rms	0.01
Witness max current (kA)	1.6
Witness 5D brightness A/(m ² .rad ²)	1.09E19

The simulation calculation used to find the values of the peak current I_{peak} , emittance ϵ_n , energy spread σ_γ , and energy γ an analysis that uses the proper script from the Puffin code to convert the VSim file to units [19]. Fig.2 shows the values of the plasma accelerator simulation output file. In addition, to the analysis to find the values of current, emittance, dgamma and gamma from the output file of accelerator are displayed in Fig.3.

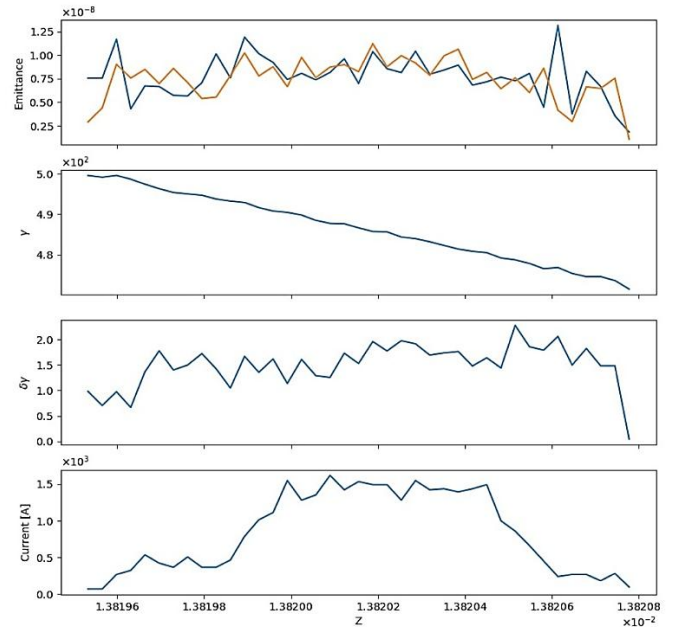


Fig. 2 shows the analysis step to determinate the values of (current, emittance, dgamma, and gamma) from the output file of plasma accelerator.

The study of FEL implied to resolve two problems in the coming work that one to match the focusing channel and electron beam using the script [20]. The second problem is to assess the impact of the energy chirp while processing in the output power of FEL[3].

Currently, two types of indicators are the used basis for generating of FEL radiation that usually depending on electron beam through undulators either helical or planer. Electrons beam paths helically or a sinusoidal that will be linear of circularly polarized radiation. In this work, planer undulator considered to using Puffin code [21].

In z- axis direction the electrons beam propagation in a long undulator axis, while in planner undulator the magnetic field in the center of undulator described as.

$$B_u = -B_u \sin(k_u z) \hat{y} \quad (5)$$

Where B_u the peak is magnetic field of the undulator, and $k_u = \frac{2\pi}{\lambda_u}$ is undulator wave-number.

The electrons pass through planer undulator are oscillating in

the system and the relativistic electron beam produced spontaneously synchrotron radiation, which enables the radiation amplified in undulator of FEL [22].

The electrons propagate through undulator emits radiation that the energy changes gave due to the following expression:

Where, the amplification in the FEL is exponential in $e^{\frac{z}{L_g}}$ Where L_g is the gain length and z the longitudinal coordinate along the undulator, where the gain length L_g is the parameters related to electron beam according to the following expression:

$$L_g = \left[\frac{\lambda_u}{4\pi\sqrt{3}} \right] \gamma \left(\frac{\sigma_x \sigma_y}{I_{peak}} \right)^{\frac{1}{3}} \quad (6)$$

Where λ_u is the undulator period, γ the Lorentz factor, I_{peak} the peak current and $\sigma_x \sigma_y$ the transverse beam sizes.

The electrons propagate through undulator emits radiation that the energy changes gave due to the following expression [14]:

$$\frac{dW}{dt} = \frac{ec\hat{K}E_x(z)}{2\gamma} (\cos\psi + \cos\chi) \quad (7)$$

Where, $E_x(Z)$ is the position dependent radiation field amplitude, Ψ and χ are phases, Ψ is the first phase, called pondermotive phase:

$$\Psi = (k_u + k_L)z - \omega_L t + \phi_0$$

$$\chi = (k_u + k_L)z - \omega_L t + \phi_0$$

The particle position relatives to the light wave that gained or lose energy. The light amplification of the wave is taking place during light particle interaction and pondermotive phase remain constant, while energy transfer constant in FEL.

$$\frac{d\Psi}{dt} = (k_u + k_L)\bar{v}_z - \omega_L = 0 \quad (8)$$

Where we used $= \bar{v}_z t$. The condition is $\Psi = const \leftrightarrow \dot{\Psi} = 0$

The resonance condition of FEL is resolved wavelength with respect to λ_r .

$$\lambda_r = \frac{\lambda_u}{2\gamma_0^2} \left(1 + \frac{K_0^2}{2} \right) \quad (9)$$

The radiation amplification of electron energy for certain resonant provides the resonance wavelength on axis, which defines specifics undulator set up, and the undulator period λ_u and the dimensionless undulator parameters K [23]. The constant amplification achieves by the electron that slips back on one wavelength λ_r per undulator period [24]. The energy modulation of the electron on the scale of the wavelength provides when the energy exchange between the electron and

the radiation field occur. Therefore, current density modulation (micro-bunching) is exchanged with energy modulation due to the electron motion. The electron coherent radiation is depending on the modulation is periodic on the scale of the wavelength, which the field amplitude growths exponentially [25].

1.3 Conclusion:

The study of laser plasma accelerator has demonstrated the advantage of brightness radiation to enhance the short-wavelength of the FEL[26]. The driver parameters, laser parameters and the simulation calculation show the peak current I_{peak} , emittance ϵ_n , energy spread σ_γ , and energy γ a using the 3D PIC code VSim for the FEL electron beam. Puffin code simulation in 3 dimensions in momentum space, 3D spatial dimensions, and 3D radiation fields apply to unaverage 3 dimensional model of the FEL and interaction of electron beams. The simulation implements a model of Puffin code uses for the applications of Trojan horse parameters in the future that resulting in stronger chirp electron pulse radiate coherent radiation, which leads to coherent effect [27]. The chirp request to remove before the Puffin code energy, which the energy chirp represents an obstacle for plasma acceleration stage in the electron beams quality for the FEL application. In the coming work of FEL to resolve the focusing channel and electron beam using the script in addition to assess the impact of the energy chirp of FEL power.

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