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Analysis of Peristaltic Flow in a Tube: Rabinowitsch Fluid Model

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Abstract

Applications of non-Newtonian fluids has taken a wide area in real life as well as in industrial system. The flow of non-Newtonian fluids in tubes and pipes plays very important role in day to day applications such as medical instruments, human body and machines. Due to several reasons, the study of such mechanisms with the Newtonian fluids has not been found satisfactory and several non-Newtonian fluid models have been taken into account by researchers from time to time. In the present analysis, the study of peristaltic flow in a tube has been carried out taking into account the non-Newtonian fluid : Rabinowitsch fluid model. Analytical solutions have been obtained for velocity and pressure gradient. Numerical solutions for velocity, streamlines and pressure gradient have been obtained and the physical behaviors thereof have been analyzed with the graphical presentation.

Keywords: Non-Newtonian fluid, Peristaltic flow, Rabinowitsch fluid Model.

1. Introduction

In the recent decades, the study of the peristaltic flows has become very important because of its numerous applications in day to day life as well as in engineering and industrial applications. Some examples of engineering and industrial applications are the flow of fuels and lubricants (mobilizing oils)-engines and machine parts; flow of pulps -food industries; flow of melts-plastics and polymer industries; sanitary fluid transport and transport of corrosive fluids-daily life; and some examples of peristaltic flow in medical science can be found in gall bladder, gastro-intestinal tract, female fallopian tubeand medicine injector equipment. A more detail can be also found in well recognized articles [1-6].

In recent years, the use of non-Newtonian fluids has become almost an essential need for the engineering, industrial systems and medical science, and therefore the study of different scientific mechanism with non-Newtonian fluids has also expanded is wings. It has been found that the use of high molecular weight polymer solutions (viscosity index improvers) can give rise to a non-Newtonian fluid with minimized sensitivity of change in shearing strain rate [7]. However, the use of additives changes the stress-strain relationship of the fluid.To study the flow properties of such fluids, many traditional non-Newtonian models such as couple stress, power law, micropolar and Casson models are employed. Amongst these models, Rabinowitsch fluid model [8] is an established model [9] to analyse the non-Newtonian behaviour of the fluid. The following stress-strain relation holds for Rabinowitsch fluid model for one dimensional fluid flow:

$$\bar{\tau}_{rz} + \kappa \bar{\tau}_{rz}^{3} = \bar{\mu} \frac{\partial \bar{w}}{\partial \bar{r}} \tag{1}$$

where $\bar{\mu}$ is the *initial viscosity* and κ is the non-linear factor responsible for the non-Newtonian effects of the fluid which will be referred to as coefficient of pseudoplasticity in this paper. This model can be applied to Newtonian lubricants for $\kappa = 0$, dilatant lubricants for $\kappa < 0$ and pseudoplastic lubricants for $\kappa > 0$. The advantage of this model lies in the fact that the theoretical analysis for this model is verified with the experimental justification by Wada and Hayashi [9]. *A*fter Wada and Hayashi, many researchers used this model to theoretically analyse the performance characteristics of bearing performance with non-Newtonian lubricants [10-12]. Recently, this model was used by Singh *et al.* [8, 13 -16] to investigate the performance of different types of hydrostatic,hydrodynamic and squeeze filmbearing systems. Therefore, by the two reasons – first, Rabinowitsch fluid model fits a wide range of viscosity data [9], and second, none of the investigators have studied the flow characteristics of peristaltic flow in a tube with the Rabinowitschfluid model, the present investigation is motivated.

2. Analysis

The Schematic diagram of a peristaltic flow through a uniform tube is shown in Fig. 1. The fluid in the system is taken as incompressible non-Newtonian Rabinowitsch type (pseudoplastic or dilatant) fluid under isothermal condition. Consider the sinusoidal wave trains propagate with constant speed *c* along the walls of the tubewhich let the fluid flow inside. The geometry of the wall surface $\left(\bar{r}_w\right)$(cf. Fig. 1) is defined as

$$\bar{r}_w = \bar{r}_i + \bar{a}\sin\left(\frac{2\pi}{\lambda}\left(\bar{z} - c\,\bar{t}\right)\right) \tag{2}$$

where, $\bar{r}_i$ is the tube radius at inlet, $\bar{a}$ is the wave amplitude, λ is the wavelength, c is propagation velocity and $\bar{t}$ is time.

The equations of motion of the flow in tube are

$$\frac{1}{\bar{r}}\frac{\partial(\bar{r}\,\bar{u})}{\partial\bar{r}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \tag{3}$$

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}\right) = -\frac{\partial\bar{p}}{\partial\bar{r}} - \frac{1}{\bar{r}}\frac{\partial(\bar{r}\,\bar{\tau}_{rr})}{\partial\bar{r}} - \frac{\partial\bar{\tau}_{rz}}{\partial\bar{z}} \tag{4}$$

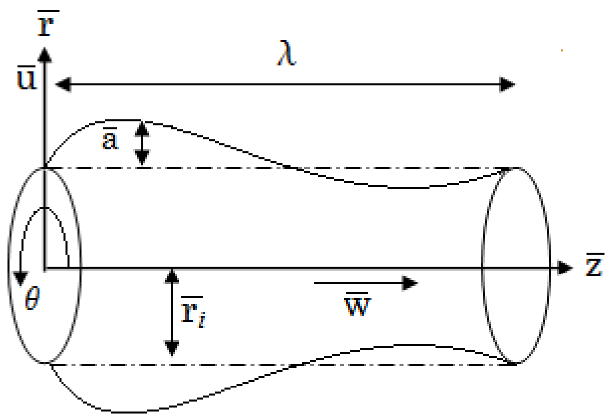
$$\rho\left(\frac{\partial\bar{w}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{w}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}\right) = -\frac{\partial\bar{p}}{\partial\bar{z}} - \frac{1}{\bar{r}}\frac{\partial(\bar{r}\,\bar{\tau}_{rz})}{\partial\bar{r}} - \frac{\partial\bar{\tau}_{zz}}{\partial\bar{z}} \tag{5}$$

and the related boundary conditions are

$$\frac{\partial\bar{w}}{\partial\bar{r}} = 0 \text{ at } \bar{r} = 0 \tag{6}$$

$$\bar{w} = 0 \text{ at } \bar{r} = \bar{r}_w \tag{7}$$

where, $\bar{u}$ and $\bar{w}$ are velocity components in radial and axial directions, respectively; ρ is the density of fluid, $\bar{\tau}_{ab}\,(a,b = r,z)$ are components of stress and $\bar{p}$ is pressure.



Schematic diagram of peristaltic flow in a circular tube.

Taking the coordinate transformation $\tilde{z} = \bar{z} - c\bar{t}$, $\tilde{w} = \bar{w} - c$ together with the dimensionless parameters

$$P=\frac{r_i^2\bar{P}}{c\lambda\mu},\ r=\frac{\bar{r}}{r_i},\ r_w=\frac{\bar{r}_w}{r_i},\ R_e=\frac{\rho c r_i}{\mu},\ t=\frac{c\bar{t}}{\lambda},\ u=\frac{\lambda\bar{u}}{r_i c},w=\frac{\tilde{w}}{c},$$
$$z=\frac{\tilde{z}}{\lambda},\quad \alpha=\frac{c\mu}{r_i}\kappa,\ \ \beta=\frac{k\lambda}{r_i},\ \ \delta=\frac{r_i}{\lambda},\quad \phi=\frac{a}{r_i},\quad \tau_{ab}=\frac{r_i}{c\mu}\tau_{ab} \tag{8}$$

the equations (1 -5) are transformed as

$$\tau_{rz}+\alpha\tau_{rz}^3=\frac{\partial w}{\partial r} \tag{9}$$

$$r_w=1+\phi\sin(2\pi z) \tag{10}$$

$$\frac{1}{r}\frac{\partial(r\,u)}{\partial r}+\frac{\partial w}{\partial z}=0 \tag{11}$$

$$0=-\frac{\partial p}{\partial r} \tag{12}$$

$$0=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r} \tag{13}$$

where, the terms of order δ and higher have been dropped with the assumption that $\delta<1$, that is, the wavelength (λ) is longer than the inlet radius (r_i) of the tube, which is a common situation in the creeping flow in medical sciencesuch asendoscopy. Further, the boundary conditions (6-7) are transformed to

$$\frac{\partial w}{\partial r}=0 \quad \text{at } r=0 \tag{14}$$

$$w=-1 \text{ at } r=r_w \tag{15}$$

Integrating the equation (13) under the condition that the shear stress τ_{rz} vanishes along the axial line $r=0$, the following is obtained

$$\tau_{rz}=\frac{1}{2}\frac{\partial P}{\partial z}r \tag{16}$$

Solving equation (9) together with equation (16) and boundary condition, equations (14-15), expression for axial velocity w is obtained as

$$w=\frac{\partial p}{\partial z}\left(\frac{r^2-r_w{}^2}{4}\right)+\alpha\left(\frac{\partial p}{\partial z}\right)^3\left(\frac{r^4-r_w{}^4}{32}\right)-1 \tag{17}$$

The instantaneous volumetric flow rate is

$$\tilde{Q}=2\pi\int_0^{\bar{r}_w}\bar{r}\,\bar{w}\,d\bar{r} \tag{18}$$

Using the time relaxed system $\tilde{w}=\bar{w}-c$, $\tilde{z}=\bar{z}-c\bar{t}$ and taking the time-mean flow $\bar{Q}=\frac{1}{T}\int_0^T\tilde{Q}\,d\bar{t}$ over the time $T=\lambda/c$, equation (18) can be written as

$$\bar{Q} = \bar{q} + \pi c\left(\bar{r}_i^{\,2} + \frac{1}{2}\bar{a}^2\right) \tag{19}$$

which can be written in dimensionless form as

$$Q = q + \frac{1}{4}\left(\phi^2 + 2\right) \tag{20}$$

where, $q = \dfrac{\bar{q}}{2\pi c r_i^2} = \int_0^{r_w} r\,w\,dr$; $Q = \dfrac{\bar{Q}}{2\pi c r_i^2}$.

With the help of equations (17-20), the equation of pressure gradient is obtained as

$$\frac{dp}{dz} + \frac{1}{6}\alpha\, r_w^{\,2}\left(\frac{dp}{dz}\right)^3 = -8\left(\frac{2Q + r_w^{\,2}}{r_w^{\,4}}\right) \tag{21}$$

For small values of $\alpha << 1$, equation (21) can be perturbed as follows

$$p = p_0 + \alpha\, p_1 \tag{22}$$

so that,

$$\frac{dp}{dz} = -8\left(\frac{2Q + r_w^{\,2}}{r_w^{\,4}}\right) + \frac{256}{3}\alpha\frac{(2Q + r_w^{\,2})^3}{r_w^{\,10}} \tag{23}$$

The pressure rise can be calculated from equation (23) as follows

$$\Delta p = \int_0^1 \frac{\partial p}{\partial z}\,dz \tag{24}$$

Now, for the purpose of analysis, equation (24) can be integrated using a standard numerical technique for integration.

3. Results and Discussion

To analyse the effects of non-Newtonian fluids on the flow behaviour, the numerical results for pressure rise and flow rate have been calculated and plotted in the figures (1-3). The influence of non-Newtonian fluids is estimated with the different values of pseudoplastic parameter α $(-0.1 \le \alpha \le 0.1)$.

Fig. 1 shows the dimensionless amount of pressure rise at wall for different values of dimensionless instantaneous flow rate Q. It is observed that the flow rate increases with the increase of flow rate in the case of pseudoplastic fluids and the vice versa for the dilatant fluids. The effect of moving from Newtonian to pseudoplastic fluid increases the pressure rise, which is more significant for higher flow rate.

Fig. 2 shows the variation of pressure gradient along the axial direction. The effect of non-Newtonian fluids is analysed by means of taking different values of α into account. The pseudoplastic fluids $(\alpha > 0)$ increases the pressure gradient while the case is reversed for dilatant fluids $(\alpha < 0)$.

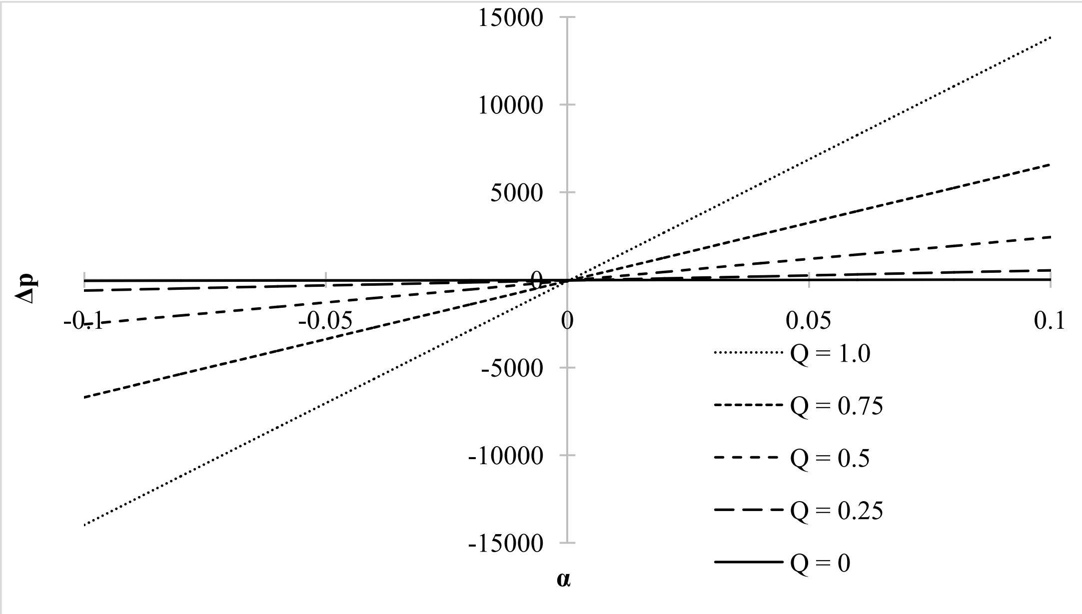


Fig. 1: Variation of pressure rise at wall with respect to α for different values of flow rate. $\phi = 0.5$.

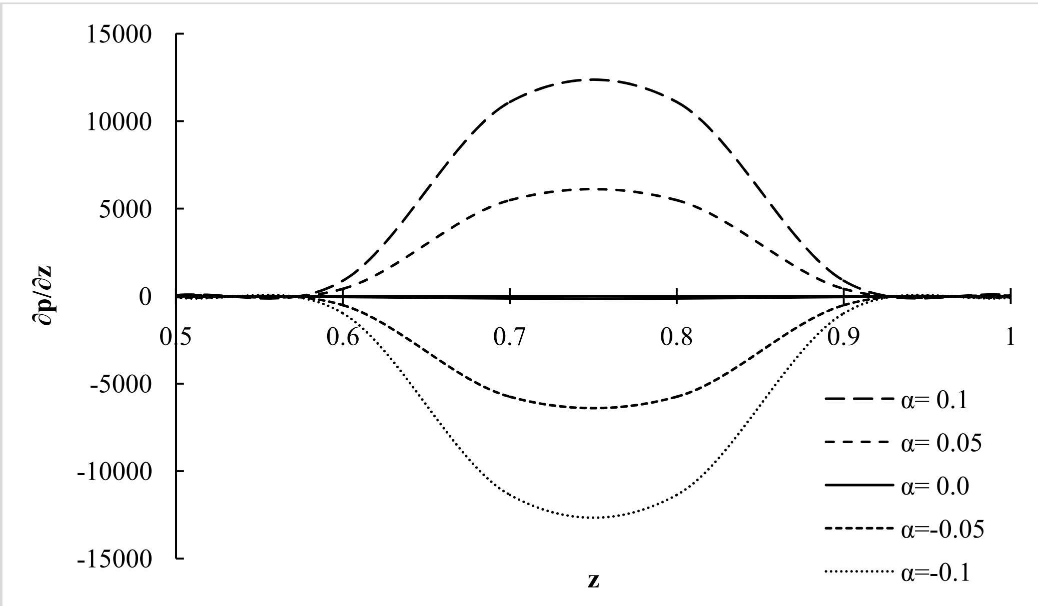


Fig. 2: Variation of pressure gradient along axial direction (z) for different values of α. $Q = 0.5$, $\phi = 0.5$.

Fig. 3 shows the fluctuation in velocity profile with respect to radial points for different values of α. The effect of non-Newtonian fluids on velocity profile is demonstrated at a particular point $z = 0.2$, which is just for the sake of the completeness and avoid the excess of plots.The similar variation holds for other values of z the overall effect has been observed for pressure rise.

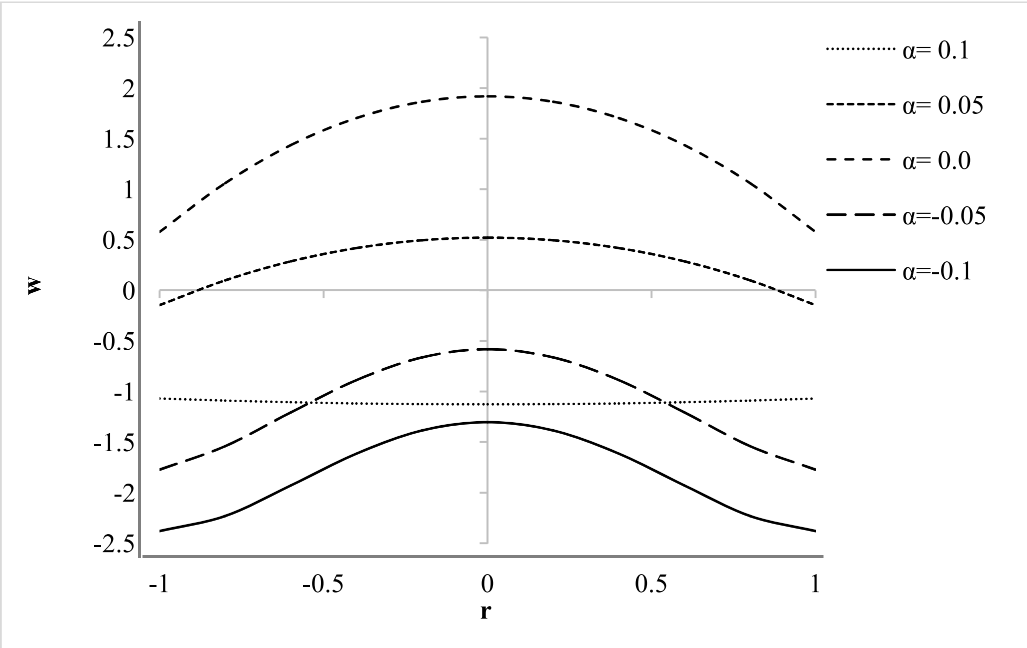


Fig. 3: Variation of dimensionless velocity (w) for different values of α. $Q = 0.5$, $\phi = 0.5$.

4. Conclusion

The influence of non-Newtonian pseudoplastic and dilatant type fluids on the nature of peristaltic flow in a tube has been analyzed by means of Rabinowitsch fluid model. Analytical expressions for radial velocity, pressure gradient and pressure rise at walls have been obtained. Both the pseudoplastic as well as dilatant fluids showed significant influence on the flow behavior, however the influence is observed to be dependent on instantaneous flow rate.

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