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Variable Viscosity and Thermal Conductivity on Mhd Flow Over a Stretching Surface with Power Low Velocity

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Abstract:

Effects of variable viscosity and thermal conductivity and heat transfer on MHD flow over a stretching surface with power low velocity have been analyzed. The fluid viscosity and thermal conductivity are assumed to be vary as an inverse linear function of temperature. Using similarity transformation the governing equations of motion are reduced to ordinary ones, which are solved numerically for prescribe boundary conditions by using shooting method. Numerical results for the Velocity profile and the temperature profile as well as for the Skin friction and Nusselt number are shown graphically and in tabular form for various parametric conditions to show interesting aspects of the solution.

Keywords: Stretching surface, Skin friction, power low velocity, shooting method, variable viscosity and thermal conductivity.

1. INTRODUCTION:

The study of two dimensional boundary layer flow and heat transfer over a stretching surface with variable viscosity and thermal conductivity is very important as it has numerous and wide ranging applications in various fields like polymer processing industry in particular manufacturing process of artificial film and artificial fibers and in some applications of dilute polymer solution. It also finds very useful applications in geophysics, particularly, geothermal energy extraction and underground storage system.

Crane [1] first studied the studied the steady two dimensional boundary layer flow due to the stretching of a flat elastic sheet. Since then the problem has been extensively studied by taking into account many different physical features either separately or in various combinations.

The flow and heat transfer of an electrically conducting incompressible fluid past a porous wall stretching linearly was considered by Chakrabarti *et al.*[2]. Grubka et al.[3] studied the heat transfer characteristics of a continuous stretching surface with variable temperature. Dandapat *et al.*[4] analyzed the effects of variable viscosity, thermal conductivity and thermocapillarity on the flow and heat transfer in a laminar liquid film on a horizontal stretching sheet. Mukhopadhyay [5] presented solutions for unsteady boundary layer flow and heat transfer over a stretching surface with variable fluid viscosity and thermal diffusivity in presence of wall suction. Effects of variable viscosity and non-linear radiation on MHD flow with heat transfer over a surface stretching with a power low velocity was discussed by Devi *et al.*[6]. Elbashbeshy [7] have investigated the influence of a variable viscosity on the heat transfer over a stretching surface with internal heat generation. EL- Kabeir [8] discussed the heat transfer with temperature dependent viscosity in a viscous fluid over stretching sheet in presence of viscous dissipation and internal heat generation. Devi *et al.* [9] also analysed the effects of variable viscosity on non- linear MHD flow and heat transfer over a surface stretching with power low velocity. Khan *et al.*[10] investigated the effect of variable viscosity and thermal conductivity on a thin film over a shrinking/stretching sheet. Magnetohydrodynamic flow of an electrically conducting power- law fluid over a stretching sheet is investigated by Andersson *et al.*[11].

2. FORMULATION OF THE PROBLEM:

Consider a two dimensional steady laminar flow of a viscous incompressible fluid along a horizontal stretching surface with power low velocity $U(x)=ax^m$, where a is the constant and m is the index of power low velocity. Let (u,v) be the velocity components along (x,y) directions respectively, where x-axis is considered along the surface and y-axis perpendicular to it. The surface is issued from a thin slit at x=0, y=0 and subsequently being stretched. A variable magnetic field B(x) is applied normal to the horizontal stretching surface. The fluid properties are assumed to be constant, except for the fluid viscosity and thermal conductivity which are assumed to vary as an inverse linear function of temperature.

Under these boundary layer assumptions, the equations of motion take the forms:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \ldots\ldots \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\left[\frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} + \mu\frac{\partial^2 u}{\partial y^2}\right] - \frac{\sigma B^2(x)}{\rho}\, u \ldots\ldots \tag{2}$$

$$\rho C_p\left(u\,\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial K}{\partial y}\frac{\partial T}{\partial y} + K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma(uB(x))^2 \ldots\ldots \tag{3}$$

Here μ is the coefficient of viscosity of the fluid, K is the coefficient of thermal conductivity of the fluid, C_p is the specific heat at constant pressure and σ, T, ρ are the electrical conductivity, temperature, density of the fluid respectively.

The boundary conditions for the problem

$$u = U(x) = ax^m, v = 0, T = T_w(x)\ at\ y = 0 \ldots\ldots \tag{5a}$$

$$u \to 0, T \to T_\infty\ as\ y \to \infty \ldots\ldots \tag{5b}$$

where $T_w(x) = T_\infty + bx^n$ is the wall temperature, b is a constant and n is the index of power low variation of wall temperature. The special form for magnetic field $B(x) = B_0 x^{\frac{(m-1)}{2}}$ is chosen to obtain the similarity solution.

Following Lai and Kulacki [12], let us assume that

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \delta(T - T_\infty)]\dots\dots \tag{6}$$

or, $\frac{1}{\mu} = A\,(T - T_r)\dots$ (7)

here $A = \frac{\delta}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\delta}$ where, μ_∞ is the viscosity at infinity, A and T_∞ are constants and their values depend on the reference state and thermal property of the fluid. T_r is transformed reference temperature related to viscosity parameter, γ is a constant based on thermal property of the fluid and A<0 for gas, A>0 for liquid.

Similarly,

$$\frac{1}{K} = \frac{1}{K_\infty}[1 + \frac{1}{\xi}(T - T_\infty)]\dots\dots \tag{8}$$

$$\frac{1}{K} = B(T - T_k)\dots\dots \tag{9}$$

$$B = \frac{\xi}{K_\infty}, \text{ and } T_k = T_\infty - \frac{1}{\xi}\dots\dots \tag{10}$$

where B and T_k are constants and their values depend on the reference state and thermal properties of the fluid, i.e. ξ

Let us introduce following similarity transformation

$$u = \frac{\partial\psi}{\partial y} = U(x)f'(\eta) \text{ and } v = -\frac{\partial\psi}{\partial x} = \frac{1}{2}U(x)\frac{y}{x}f' - \frac{1}{2}\left(\frac{2\nu U(x)}{(1+m)x}\right)\dots\dots \tag{11}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_{w-T_\infty}}\dots\dots \tag{12}$$

where, $\eta = \left[\frac{(1+m)U(x)}{2\nu x}\right]^{\frac{1}{2}} y,\ \psi = \left[\frac{2\nu x U(x)}{(1+m)}\right]^{\frac{1}{2}} f(\eta)\dots\dots$ (13)

Using the transformations (6-13), in equations (1-3), it is seen that the equation of continuity satisfies identically and rest of the equations become:

$$f''' - \frac{1}{\theta-\theta_r}f''\theta' - \frac{\theta-\theta_r}{\theta_r}\left(ff'' - \beta f'^2 - f'\right) = 0\dots\dots \tag{14}$$

$$\theta'' - \frac{1}{\theta - \theta_k}\theta'^2 - Pr\frac{(\theta - \theta_k)}{\theta_k}\left(f\theta' - \frac{2n}{m+1}f'\theta\right)$$

$$+PrEc\frac{\theta_r}{\theta-\theta_r}\frac{(\theta-\theta_k)}{\theta_k}(f'')^2 - EcM\frac{(\theta-\theta_k)}{\theta_k}(f')^2 = 0\dots\dots \tag{15}$$

Here, $\theta_r = \frac{T_r - T_\infty}{T_{w-}T_\infty} = \frac{1}{\delta(T_{w-}T_\infty)}$ and $\theta_k = \frac{T_k - T_\infty}{T_{w-}T_\infty} = \frac{1}{\xi(T_{w-}T_\infty)}$ are dimensionless reference temperature corresponding to viscosity and thermal conductivity respectively.

It is to be noted that these values are negative for liquids and positive for gases when $(T_{w-}T_\infty)$ is positive (Lai and Kulacki [12]).

$Pr = \frac{\mu C_p}{K}$ is the Prandtl number

M = $\frac{2\sigma B_0^2}{\rho a(m+1)}$ is the Magnetic field parameter

Ec = $\frac{U^2}{C_p(T_w - T_\infty)}$ is the Eckert number

$\beta = \frac{2m}{1+m}$ is the stretching parameter

The transformed boundary conditions are

$\eta = 0, f' = 1, \theta = 1$...... (16a)

$\eta \to \infty, f' = 0, \theta = 0$... ... (16b)

The important physical quantities of our interest in this problem are skin friction coefficient C_f and heat transfer coefficient *Nu*, the Nusselt number for the surface defined by

$$C_f Re^{\frac{1}{2}} = \frac{2\tau_w}{\rho U(x)^2}$$

and $NuRe^{-\frac{1}{2}} = \frac{xq_w}{K(T_{w-}T_\infty)}$

where, τ_w is the shear stress and q_w is the heat transfer from the surface.

3. RESULTS AND DISCUSSIONS:

The diffential equations (14) and (15) together with the boundary conditions (16a) and (16b) are solved numerically using Runge-Kutta fourth order method in conjunction with Shooting technique. Calculations are carried out for different values of θ_r, θ_k, Ec, M, n, β.

The variation of temperature profile and velocity profile with the variation of θ_r is plotted in the graphs of figure 1 and figure 2 respectively. It is clear from figure 2 that increase of viscosity parameter θ_r retards the velocity and from figure 2 it is found that increase of viscosity parameter θ_r enhances the temperature.

Figure 3 shows the graphs for the variation of temperature profile with the variation of θ_k. From figure 3 it can be concluded that the increase of thermal conductivity parameter θ_k increases the temperature.

The variation of velocity profile with the variation of stretching parameter β is shown in the figure 4, where the graphs of figure 4 represents that the velocity increases with the increasing value of β.

The graphs of figure 5 display the results of temperature profile for various values of Ec. From figure 5 it is clear that when Ec increases temperature decreases. Figure 6 represents the graphs of the variation of temperature profile with n and from this figure it can be concluded that temperature decreases with the increasing values of n.

The variation of temperature profile and velocity profile with the variation of M is displayed in the figure 7 and figure 8 respectively. Figure 7 represents that temperature increases significantly with the increase of M and from figure 8, it can be seen that with the increase of M velocity decreases.

The coefficient of Skin friction C_f i.e. the wall shear stress and heat transfer coefficient *i.e.* Nusselt number Nu at $\eta = 0$ are presented in table 1 to table 4 for various combinations of parameters. From table 1 to table 4 it is clearly seen that the skin friction decreases with the increasing values of θ_k, θ_r, M and Ec whereas Nusselt number increases with the increase of θ_r and M but decreases with the increase of θ_k and Ec.

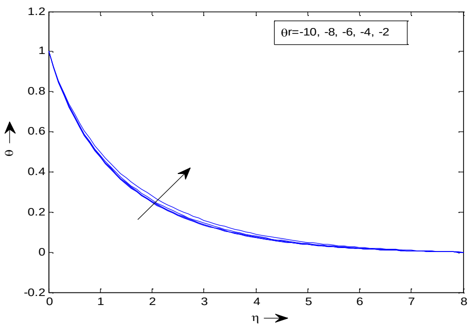

Fig: 1 Temperature profile against θ_r

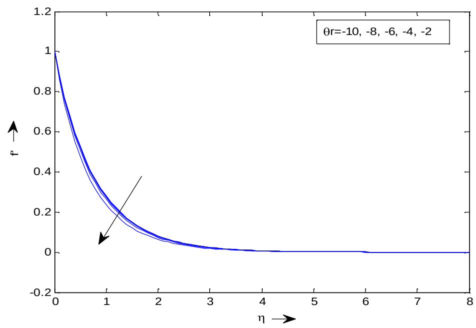

Fig: 2 Velocity profile against θ_r

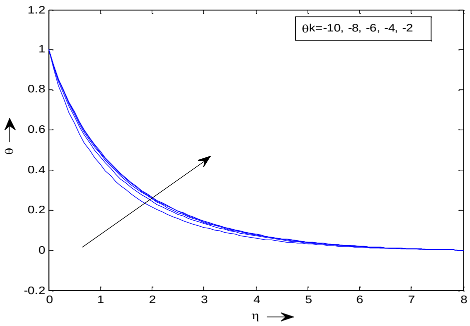

Fig: 3 Temperature profile against θ_k

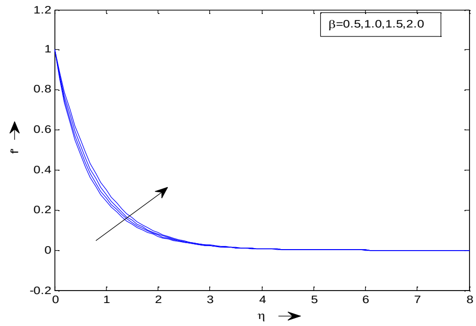

Fig: 4 Velocity profile against β

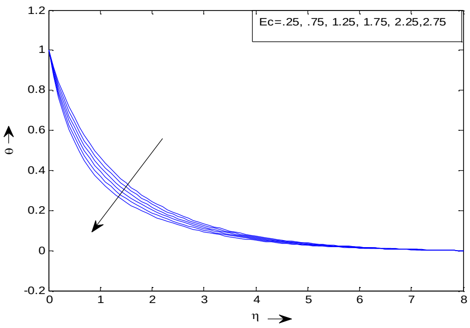

Fig: 5 Temperature profile against Ec

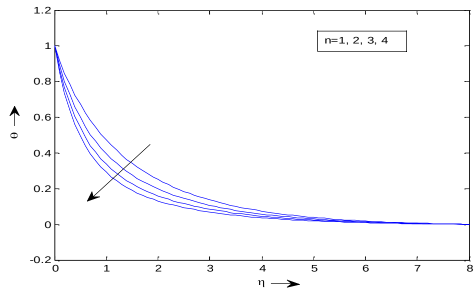

Fig: 6 Temperature profile against n

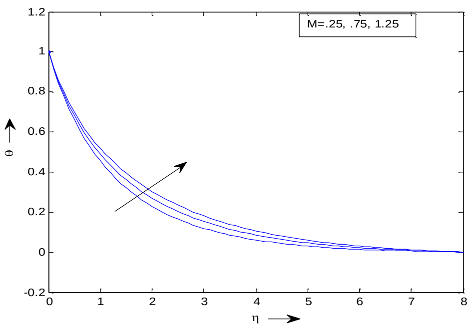

Fig: 7 Temperature profile against M

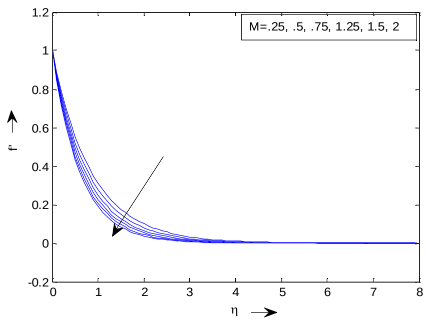

Fig: 8 Velocity profile against M

Table 1

$\theta_k \downarrow$ M $\rightarrow$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$
	.25		.5		.75		1	
-10	-1.1999	-0.8209	-1.3134	-0.8010	-1.4177	-0.7835	-1.5146	-0.7680
-8	-1.2001	-0.8351	-1.3136	-0.8151	-1.4179	-0.7974	-1.5148	-0.7818
-6	-1.2003	-0.8585	-1.3139	-0.8383	-1.4182	-0.8205	-1.5151	-0.8047
-4	-1.2009	-0.9046	-1.3145	-0.8840	-1.4188	-0.8659	-1.5158	-0.8498
-2	-1.2023	-1.0370	-1.3161	-1.0156	-1.4205	-0.9967	-1.5175	-0.9799

Table 2

$\theta_r \downarrow$ M $\rightarrow$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$
	.25		.5		.75		1	
-10	-1.1845	-0.8793	-1.2968	-0.8592	-1.3999	-0.8414	-1.4958	-0.8256
-8	-1.2005	-0.8771	-1.3141	-0.8567	-1.4184	-0.8388	-1.5154	-0.8229
-6	-1.2267	-0.8734	-1.3425	-0.8527	-1.4487	-0.8345	-1.5474	-0.8184
-4	-1.2772	-0.8664	-1.3971	-0.8450	-1.5071	-0.8264	-1.6093	-0.8099
-2	-1.4161	-0.8469	-1.5476	-0.8239	-1.6679	-0.8041	-1.7796	-0.7871

Table 3

$\theta_r \downarrow$ Ec $\rightarrow$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$
	.25		.5		.75		1	
-10	-1.2971	-0.8929	-1.2976	-0.9489	-1.2980	-1.0049	-1.2985	-1.0607
-8	-1.3145	-0.8914	-1.3151	-0.9490	-1.3157	-1.0065	-1.3162	-1.0639
-6	-1.3429	-0.8890	-1.3437	-0.9493	-1.3445	-1.0094	-1.3453	-1.0693
-4.	-1.3979	-0.8845	-1.3991	-0.9500	-1.4004	-1.0153	-1.4016	-1.0802
-2	-1.5492	-0.8727	-1.5520	-0.9536	-1.5548	-1.0338	-1.5575	-1.1133

Table 4

$\theta_k \downarrow$ Ec $\rightarrow$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$	f"(o)	$\theta'(o)$
	.25		.5		.75		1	
-10	-1.3138	-0.8330	-1.3143	-0.8864	-1.3149	-0.9395	-1.3154	-0.9926
-8	-1.3140	-0.8478	-1.3145	-0.9022	-1.3151	-0.9564	-1.3156	-1.0105
-6	-1.3143	-0.8721	-1.3148	-0.9283	-1.3154	-0.9844	-1.3160	-1.0403
-4	-1.3148	-0.9200	-1.3154	-0.9798	-1.3160	-1.0394	-1.3166	-1.0989
-2	-1.3165	-1.0580	-1.3171	-1.1283	-1.3178	-1.1985	-1.3184	-1.2685

4. CONCLUSIONS:

From the above analysis we can conclude as

1. The increasing values of viscosity retard the velocity but enhance the temperature.
2. The temperature increases and velocity decreases as the values of M increase.
3. The velocity increases with the increasing value of β.
4. Thermal conductivity enhances the temperature.
5. Eckert number, Ec retards the temperature.
6. Both the skin friction and Nusselt number decrease with the increase of θ_k.

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