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MHD Thermal Diffusion Effects On Mixed Convection And Mass Transfer Flow Past A Vertical Porous Plate In A Porous Medium With Heat Generation And Viscous Dissipation

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ABSTRACT

The problem of mixed convection and mass transfer flow over a vertical porous flat plate, in presence of heat generation and thermal diffusion taking in to account the viscous dissipation, in the presence of magnetic field, is studied numerically. The non-linear partial differential equations and their boundary conditions, describing the problem under consideration, are transformed into a system of ordinary differential equations by using usual similarity transformations. The non-linear momentum equation is linearized by the Quasi-linearization technique. The set of linear ordinary differential equations are solved by using the implicit finite difference scheme along with Gauss-sidel iterative method. The effects of suction parameter, heat generation parameter, Soret number and Magnetic field parameter and viscous dissipation are examined on the flow field of a hydrogen-air mixture as a non-chemical reacting fluid pair. The analysis of the obtained results showed that the flow field is significantly influenced by these parameters.

Keywords: MHD, thermal diffusion, heat generation, viscous dissipation, finite difference method.

1. INTRODUCTION

Magneto-hydrodynamic equations are ordinary electromagnetic and hydrodynamic equations modified to take into account the interaction between the motion of the fluid and the electromagnetic field. The formulation of the electromagnetic theory in a mathematical form is known as Maxwell's equation. In recent years, the study of convective heat transfer from surfaces embedded in porous media has received considerable attention in the literature. The interest for such studies is motivated by several thermal engineering applications, such as geothermal systems, the storage of nuclear wastes, oil extraction, ground water pollution and thermal insulation.

Combined heat and mass transfer by free-forced convection in a porous medium has attracted considerable attention in the last several decades, due to its many important engineering and geophysical applications. Combined convective cooling is one of the preferred methods for cooling computer systems and other electronic equipments due to its simplicity and low cost. Again, the demand for faster and denser circuit technologies and packages has been accompanied by increasing heat fluxes at the chip and package levels, the application of air cooling techniques, involving either free or forced convection, plays a significant role over the years. In an enclosure, the interaction between the external forced stream and the buoyancy driven flow induced by the increasing high heat flux from electronic modules leads to the possibility of complex flows. Therefore it is important to understand the heat transfer characteristics of combined convection in an enclosure. In many modern buildings, mechanical ventilation is provided as a means of room load removal and provision of good indoor air quality.

In a mixed convection, both natural convection and forced convection participate in the heat transfer process. The bulk fluid flow direction can be any of the three possible directions in a horizontal channel, forward, backward or upward. The forced flow can be in the same direction as the flow created by natural convection, and this flow condition is called assisting mixed convection. Where as, for the other case, forced flow direction is in an opposing direction to the flow that is created by buoyancy, and this flow condition is referred to as opposing mixed convection. But in some cases, convection from a horizontal heated enclosure, the forced flow is perpendicular to the buoyancy induced flow and this situation is called transverse mixed convection. It is known that a flow situation where both free and forced convection effects are of comparable order is called mixed convection. The study of such a mixed convection flow finds application in several industrial and technical processes such as nuclear reactors cooled during emergency shutdown, solar central receivers exposed to winds, electronic devices cooled by fans and heat exchangers placed in a low-velocity environment. The simplest physical model of such a flow is the two dimensional laminar mixed convection flows along a vertical flat plate and extensive studies have been conducted on this type of flow. Applications of this model can be found in the areas of reactor safety, combustion flames and solar collectors, as well as building energy conservation.

2. MATHEMATICAL ANALYSIS

A two-dimensional steady combined free-forced convective and mass transfer flow of a viscous, incompressible fluid over an isothermal semi-infinite vertical porous flat plate embedded in a porous medium is considered. The flow is assumed to be in the x-direction, which is taken along the vertical plate in the upward direction and the y-axis is taken to be normal to the plate. The surface of the plate is maintained at a uniform constant temperature T_w and a uniform constant concentration C_w, of a foreign fluid, which are higher than the corresponding values T_1 and C_1, respectively, sufficiently far away from the flat surface. It is also assumed that the free stream velocity U_1, parallel to the vertical plate is constant. Then under the boundary layer and Boussinesq's approximations, the governing equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{v}{k'}u-\frac{\sigma\beta_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho c_\rho}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q_0}{\rho C_\rho}(T-T_\infty) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}+D_T\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u, v are the velocity components in the x-and y-directions respectively, v is the kinematics viscosity, g is the acceleration due to gravity, ρ is the density of the fluid, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while C, C_w and C_∞ are the corresponding concentrations. Also, K' is the permeability of the porous medium, k is the thermal conductivity, C_p is the specific heat at constant pressure, Q_0 is the heat generation constant, D_M is the coefficient of mass diffusivity and D_T is the coefficient of thermal diffusivity.

For the flow there is no slip at the plate. For uniform plate temperature and concentration the appropriate boundary conditions for the above problem are as follows:

$$u=0,\ v=v_w(x),\ T=T_w,\ C=C_w, \text{ at } y=0 \tag{5a}$$

$$u=U_\infty,\ T=T_\infty,\ C=C_\infty \text{ as } y\to\infty \tag{5b}$$

In order to obtain similarity solution of the problem we introduce the following non-dimensional variables (see Schlichting [15], Rahman and Sattar [16].

$$\eta=y\sqrt{\frac{U_\infty}{vx}},\ \psi=\sqrt{vxU_\infty}\,f(\eta) \tag{6a}$$

$$\theta(\eta)=\frac{T-T_w}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_w}{C_w-C_\infty}$$

where ψ is the stream value function

Since $u=\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{\partial\psi}{\partial x}$ we have from

$u = U_\infty f'$ and $v = -\sqrt{\frac{\nu U_\infty}{x}}(f - \eta f')$ (6b)

Here prime denotes the differentiation with respect to η

Now substituting equations (6) in equations (2) – (4) we obtain

$$f''' + \frac{1}{2} ff'' - Kf' + g_s\theta + g_c\phi - Mf' = 0 \quad (7)$$

$$\theta'' + \frac{1}{2}\Pr f\theta' + \Pr Q\theta + \Pr Ecf''^2 = 0 \quad (8)$$

$$\phi'' + \frac{1}{2} Scf\phi' + SoSc\phi = 0 \quad (9)$$

The boundary conditions (5) then turn into

$$f = f_w,\ f' = 0,\ \theta = 1,\ \phi = 1 \text{ at } \eta = 0$$
$$f' = 1,\ \theta = 0,\ \phi = 0 \text{ as } \eta \to \infty \quad (10)$$

where $f_w = -2v_w(x)\sqrt{\frac{x}{\nu U_\infty}}$ is the suction parameter. The dimensionless parameters introduced in the above equations are defined as follows:

$K = \frac{\nu x}{K' U_\infty}$ is the local permeability parameter.

$Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$ is the local temperature Grashof number.

$Gm = \frac{g\beta^*(C_w - C_\infty)x^3}{\nu^2}$ is the local mass Grashof number.

$\mathrm{Re} = \frac{U_\infty x}{\nu}$ is the Reynolds number.

$g_s = \frac{Gr}{\mathrm{Re}^2}$ is the temperature buoyancy parameter.

$g_c = \frac{Gm}{\mathrm{Re}^2}$ is the mass buoyancy parameter.

$\Pr = \frac{\nu \rho c_p}{k}$ is the Prandtl number.

$Q = \frac{Q_0 x}{\rho c_p U_\infty}$ is the local heat generation parameter.

$Sc = \frac{\nu}{D_M}$ is the Schmidt number.

$M = \frac{\sigma\beta_0^2 x}{\rho U_\infty}$ is the magnetic field parameter.

$Ec = \frac{U_\infty^2}{C_p(T_w - T_\infty)}$ is the Eckert number and $So = \dfrac{D_T(T_w - T_\infty)}{v(C_w - C_\infty)}$ is the Soret number.

3. MEHOD OF SOLUTION:

Due to the coupled nature of the current system, the system of non-linear equations (7)-(9) with the associated boundary condition in (10) must be solved simultaneously by the implicit finite difference scheme in combination with Quassi-linearization technique. The effect of step size $\Delta\eta$ and the edge of the boundary layer η_∞ on the solution has been studied to optimize them, consequently, we have taken $\Delta\eta$ = 0.01 and $4 \le \eta_\infty \ge 8$ for computation. The results presented here are independent of step sizes and η_∞ at least up to the 5$^{\text{th}}$ decimal place. Applying the Quasi–linearization technique [17] to the non-linear equation (7)

4. SKIN-FRICTION COEFFICIENT, NUSSELT NUMBER AND SHERWOOD NUMBER

The parameters of engineering interest for the present problem are the local skin-friction coefficient, local Nusselt number and the local Sherwood number which indicate physically wall shear stress, rate of heat transfer and rate of mass transfer respectively.

The equation defining the wall skin-friction is

$$\tau_w = \mu(\frac{\partial u}{\partial y})_{y=0} = \mu U_\infty \sqrt{\frac{U_\infty}{vx}} f''(0) \tag{15}$$

Hence the skin-friction coefficient is given by

$$C_f = \frac{2\tau_w}{\rho U_\infty^2} \text{ or } \frac{1}{2} C_f (\text{Re})^{\frac{1}{2}} = f''(0) \tag{16}$$

Now the heat flux (q_w) and the mass flux (M_w) at the wall are given by

$$q_w = -k(\frac{\partial T}{\partial y})_{y=0} = -k\Delta T \sqrt{\frac{U_\infty}{vx}} \theta'(0). \tag{17}$$

$$\text{And } M_w = -D_M(\frac{\partial C}{\partial y})_{y=0} - - D_M \Delta C \sqrt{\frac{U_\infty}{vx}} \phi'(0) \tag{18}$$

Where $\Delta T = T_w - T_\infty$ and $\Delta C = C_w - C_\infty$

Hence, the Nusselt number (Nu) and Sherwood number (Sh) are obtained as

$$Nu = \frac{xq_w}{k\Delta T} = -(\text{Re})^{\frac{1}{2}} \theta'(0) \text{ or } Nu(\text{Re})^{-\frac{1}{2}} = \theta'(0) \tag{19}$$

$$\text{and } Sh = \frac{xM_w}{D_M \Delta C} = -(\text{Re})^{\frac{1}{2}} \phi'(0) \text{ or } Sh(\text{Re})^{-\frac{1}{2}} = -\phi'(0) \tag{20}$$

These above coefficients are then obtained numerically sorted in Table 1

Table.1 Numerical values of C_f, Nu & Sh for Pr = 0.71, Sc = 0.22, $g_s = g_c = 0.1$, $K = 0.05$

Q	f_w	So	C_f	Nu	Sh	
0.5	0.5	0.2	1.8032	0.1061	0.2897	
1.0	0.5	0.2	2.3278	-0.6634.	0.3458	
2.0	0.5	0.2	4.6053	-5.1783	0.5958	
2.0	1.0	0.2	4.5947	-4.7314	0.6071	
2.0	1.5	0.2	4.4997	-4.1596	0.6115	
2.0	1.5	0.8	4.5017	-4.1841	1.1368	
2.0	1.5	2.0	4.5048	-4.1872	1.1811	

5. RESULTS AND DISCUSSION

The effects of f_w on the velocity, temperature and concentration profiles are shown in figure.1(a), 1(b) and 1(c) respectively. It is seen from this figure that the velocity profiles decrease monotonically with the increase of suction parameter f_w indicating the usual fact that suction stabilizes the boundary layer growth. We see that both the temperature and concentration profiles decrease with the increase of f_w. Sucking decelerated fluid particles through the porous wall reduce the growth of the fluid boundary layer as well as thermal and concentration boundary layers. The effect of Q on the velocity, temperature and concentration profiles are shown in figures 2(a)-(c) respectively. From this figure we see that when the heat is generated the buoyancy force increases which induces the flow rate to increase giving rise to the increase in the velocity profiles. From figure it is observed that temperature increases significantly with the increase of Q. On the other hand, from Fig.2(c) it can be seen that the concentration profiles decrease with the increase of the heat generation parameter.

Fig.3(a) & (b) shows the variation of dimensionless velocity and concentration profiles for different values of So. It is seen from this figure that velocity profiles increase with the increase of So from which we conclude that the fluid velocity rises due to greater thermal-diffusion. From this figure it is noticed that the concentration profiles increase significantly with the increase of Soret number.

Figs 4(a)-(c) display dimensionless velocity, temperature and concentration profiles for different values of Magnetic field parameter M. The Hartmann number represents the importance of magnetic field on the flow. It is observed that the presence of magnetic field sets in Lorentz force which in turn results retarding force on the velocity field and therefore as Hartmann number increases, so does the retarding force and hence, the velocity profiles decreases. From Fig.4(b) it is noticed that the temperature profiles increases with the increase in Hartmann number because the magnetic filed retards the velocity of the fluid and therefore the temperature of the plate is higher. It is seen that the concentration of the fluid increases with the increase of Hartmann number. The figures 5(a) and (b) depicts the effects of viscous dissipation on velocity and temperature profiles respectively. It can be seen from the figures, with the increase of Eckert number the velocity and temperature profiles are increases.

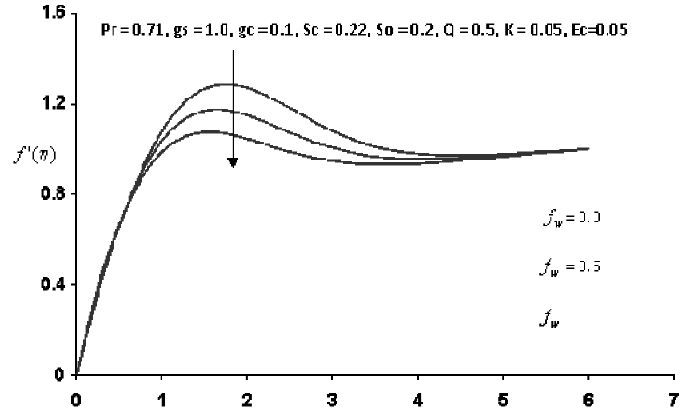


Fig.1(a) Velocity profiles for different values of f_w

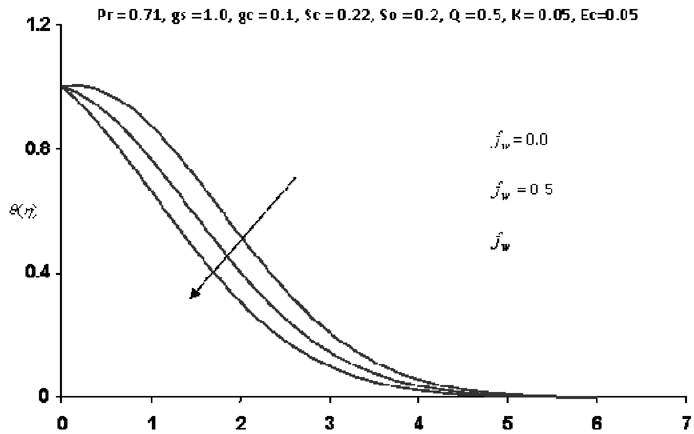


Fig.1(b) Temperature profiles for different values of f_w

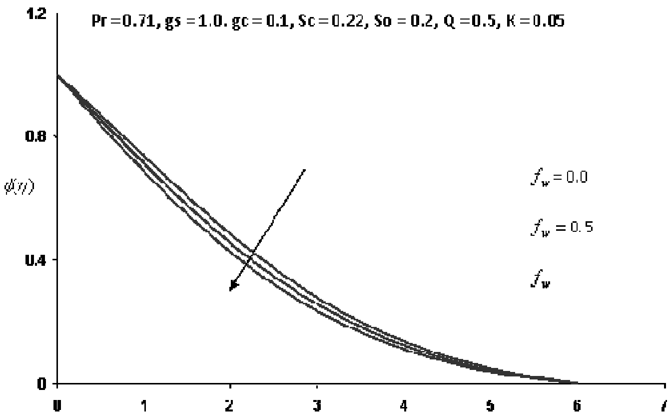


Fig.1(c) Concentration profiles for different values of f_w

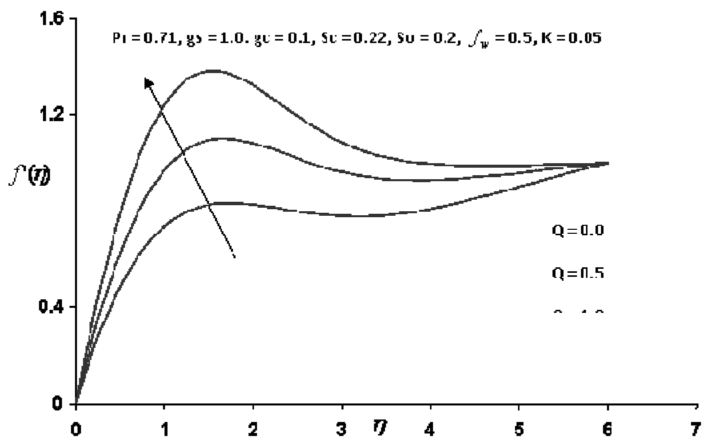


Fig.2(a) Velocity profiles for different values of Q

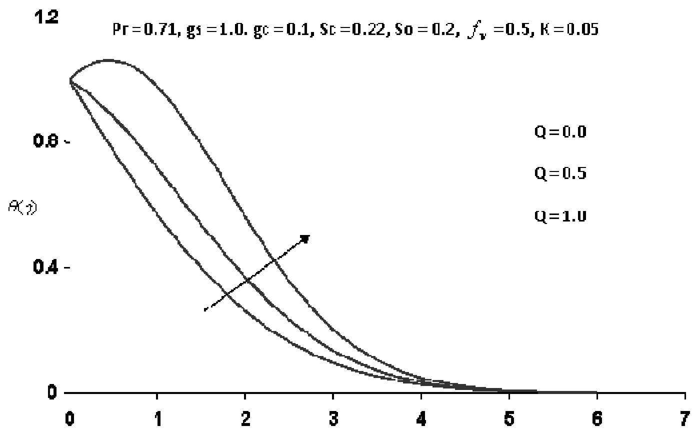


Fig. 2(b) Temperature profiles for different values of Q

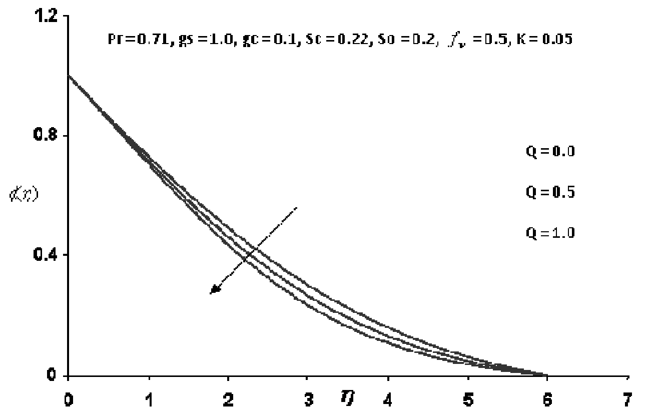
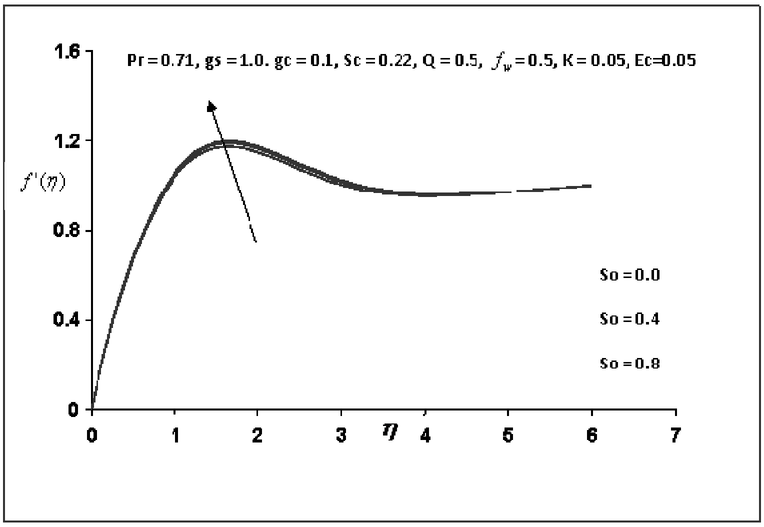


Fig.2(c) Concentration profiles for different values a Q



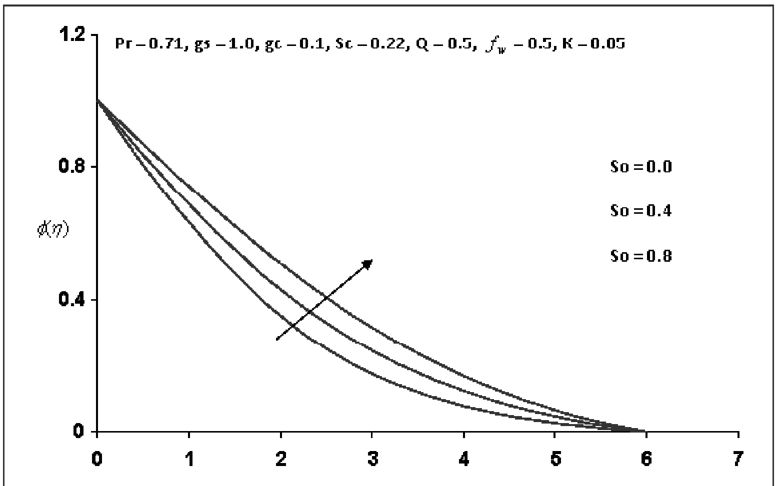


Fig.3(b) Concentration profiles for different values So

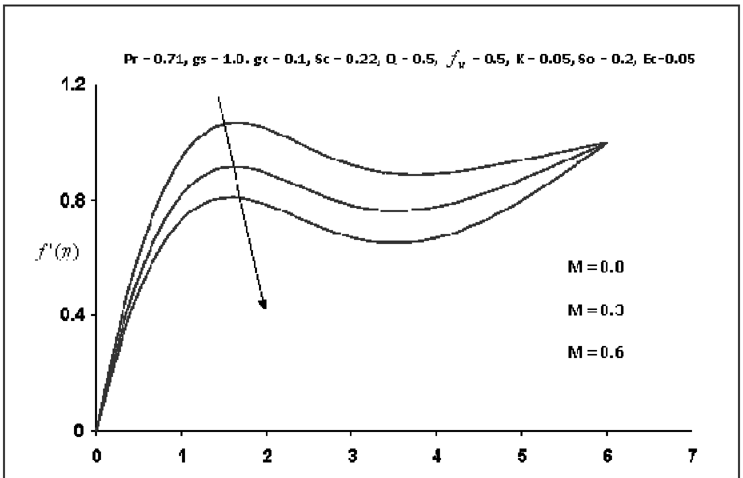


Fig.4(a) Velocity profiles for different value of Magnetic field parameter

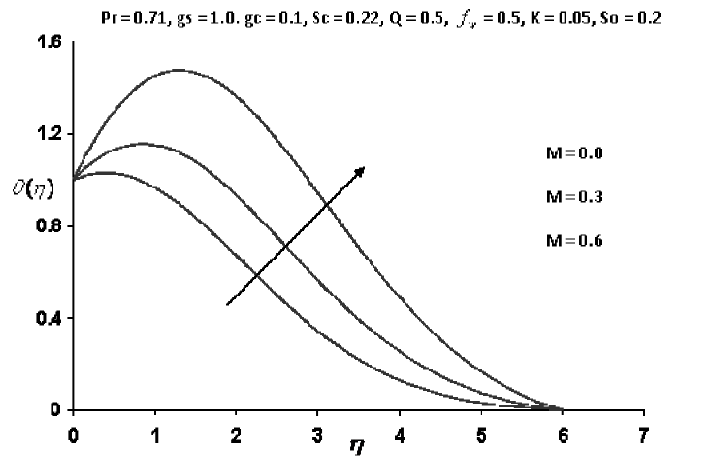


Fig.4(b) Temperature profiles for different value of Magnetic field parameter

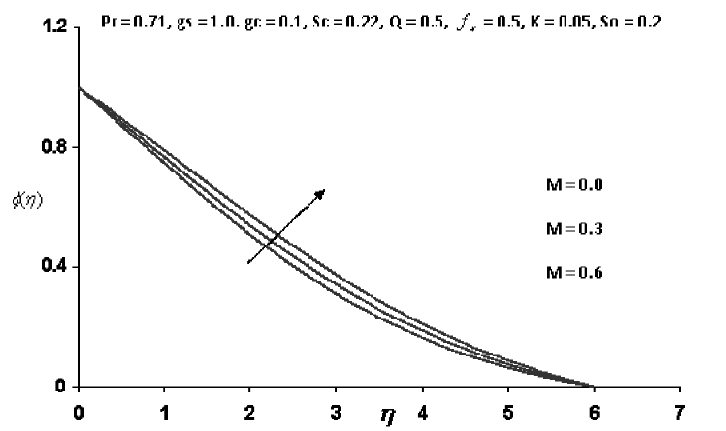


Fig.4(c) Concentration profiles for different value of Magnetic field parameter

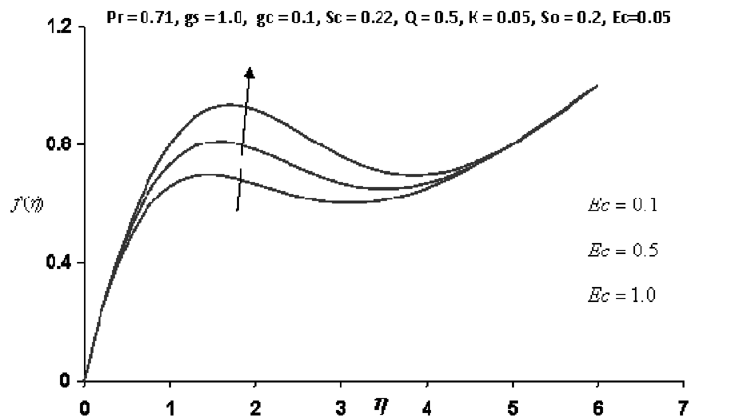


Fig. 5(a) Velocity profiles for different values of Ec when M = 0.5

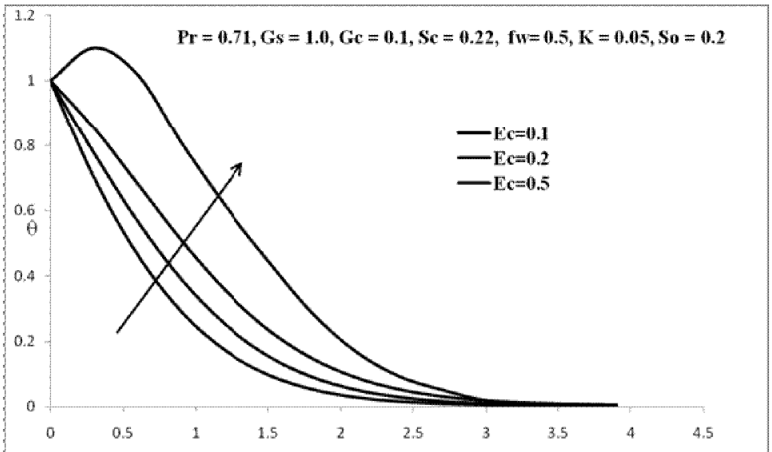


Fig. Temperature profile for different values of Eckert number Ec

5 CONCLUSIONS

In this paper we have studied the steady MHD two-dimensional combined free-forced convection and mass transfer flow over a semi-infinite vertical porous plate embedded in a porous medium in the presence of heat generation, thermal diffusion taking into the account of viscous dissipation. The effects of various parameters have been examined on the flow field for a hydrogen-air mixture as a non-chemical reacting fluid pair. From the present investigation the following conclusions may be drown:

1) Wall suction stabilizes the velocity, thermal as well as concentration boundary layer growth.
2) Both the velocity and temperature profiles increase whereas the concentration profile decreases with the increase of heat generation parameter.
3) Both the velocity and concentration profiles increase with the increase of Soret number.
4) The velocity profiles decrease with the increase of magnetic field parameter and where as both temperature and concentration profiles increase with the increase of magnetic field parameter,
5) In mixed convection regime, both the Skin-friction coefficient and Sherwood number increases whereas the Nusselt number decreases with the increase of both heat generation parameter and Soret number.
6) The viscous dissipation effects is to enhance the velocity and temperature profiles.

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