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Unsteady Mhd Flow of a Non-Newtonian Fluid Down and Open Inclined Channel with Naturally Permeable Bed

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Abstract:

This paper deals with unsteady MHD flow of a Walters fluid (Model B’) an open inclined channel of width “2a’ and depth ‘d’ under gravity, with naturally permeable bed, the walls of channel being normal to the surface of the bottom, under the influence of a uniform transverse magnetic field. The free surface is exposed to atmospheric pressure. A uniform tangential stress is applied at the free surface in the direction of flow. The naturally permeable bottom of the channel is taken at an angle β with the horizontal. Flow of fluid both in porous medium and in free fluid region is studied with the same pressure gradient. The exact solution of velocity distribution has been obtained by using Laplace transform and finite Fourier sine transform techniques. We have evaluated the velocity distribution and the flux of the fluid in different cases of time dependent pressure gradient g(t) viz., i) constant, ii) exponentially decreasing function of time and iii) cosine function of time. The effects of magnetic parameter ‘M’, viscoelastic parameter ‘K_0’, permeable parameter ‘K’, Reynolds number ‘R’ and time ‘t’ on velocity distribution ‘u’ and flux ‘C_f’ in three different cases are investigated.

Key words: Non-Newtonian fluid, Open inclined channel, Porous medium, Magnetic field.

INTRODUCTION

The flow of a liquid in an open inclined channel with a free surface has wide applications in the designs of drainage, irrigation canals, flood discharge channels and coating to paper rolls etc. Vanoni [11] has evaluated velocity distribution in open channels. Johnson [6] has studied the rectangular roughness in the open channel. SatyaPrakash [7] considered analytically a viscous flow down an open inclined channel with plane bottom and vertical walls under the action of gravity. The free surface was exposed to atmospheric pressure and bottom was taken as impermeable. Bakhmeteff [1], Henderson [5] and Chow [3] have discussed many types of open channel flows. Gupta etal [4] have studied the flow of a viscous fluid through a porous medium down an open inclined channel. VenkataRamana&Bathaiah [12] have studied the flow of a hydro magnetic viscous fluid down an open inclined channel with naturally permeable bed under the influence of a uniform transverse magnetic field. Unsteady laminar flow of an incompressible viscous fluid between porous and parallel flat plates have been investigated by Singh [8], taking (i) both plates are at rest (ii) Generalized plane coutte flow.

Rheology is the science of deformation and flow of matter. The aim of rheology is to predict the deformation on flow resulting from the application of a given force system to a body or vice versa. The subject of rheology is of technological importance as in many branches of industry, the problem arises of designing apparatus to transport or process substances, which can't governed by the classical stress, strain velocity relations. In the manufacture of rayon, nylon or other textiles fibers, viscoelastic effects are encountered when the spinning solutions are transported or forced through spinnerets and in the manufacture of lubricating greases and rubber. Further viscoelastic fluid occurs in the food industry, e.g. emulsions, pastes and condensed milk.

Non-Newtonian fluids have wide importance in the present day technology and industries. The Walters fluid is one of such fluid. The constitutive equations governing motion of Walters fluid (Model B') are

$$P_{ik} = -pg_{ik} + P_{ik} \quad 1.1$$

$$P'_{ik} = 2\eta_0 e^{ik} - 2K_0 e^{ik} \quad 1.2$$

The equation of motion and continuity are

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j v_{ij}\right) = -P_j + P_{ij} \quad 1.3$$

$$v_{ij\,=\,0} \quad 1.4$$

whereP_{ik} is the stress tensor, is the density, p is the pressure, g_{ik} is the metric tensor of a fixed coordinate system x', ^{i}is the velocity vector, in the contra variant form is

$$e^{ik} = \frac{\partial e^{ik}}{\partial t} + v_j e^{ik} - v_{kj} e^{ij} - v_{ij} e^{ik} \quad 1.5$$

It is converted derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{ik} + v_{ki} \quad 1.6$$

Here, h_0 is the limiting viscosity at small rates of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau) d\tau \quad 1.7$$

$$K_0 = \int_0^\infty \tau N(\tau) d\tau \quad 1.8$$

N(t) being the relaxation spectrum as introduced by Walters [13, 14] . This idealized model is a valid approximation of Walters's fluid (Model B') taking very short memory into account so that terms involving

$$\int_0^\infty \tau^\infty N(\tau) d\tau, n \geq 2 \quad 1.9$$

Have been neglected.

In this paper an attempt has been made to study the unsteady MHD flow of a Walters fluid (Model B') down an open inclined channel under gravity of width '2a' and depth 'd', with naturally permeable bed, the walls of the channel being normal to the surface of the bottom, under the influence of a uniform transverse magnetic filed. A uniform tangential stress is applied at the free surface in the direction of flow. The naturally permeable bottom of the channel is taken at an angle 'b' with the horizontal. We have evaluated the velocity distribution by using Laplace transform and finite Fourier Sine transform techniques. We have evaluated the velocity distribution and flux of the fluid in different cases of time dependent pressure gradient g (t), viz.,
(i) The fluid flows in the steady state for t 0
(ii) Unsteady state occurs at t > 0 and
(iii) Unsteady motion is influenced by time-dependent pressure gradient. The velocity distribution and flux have been obtained in some particular cases i.e., when

(i) g(t) = C

(ii) g(t) = C e^{-bt}

(iii) g(t) = C Cos bt; where C and b are constants.

The effects of magnetic parameter 'M', viscoelastic parameter K_0, porosity parameter K, Reynolds number 'R' and time 't' are investigated on the velocity distribution and the flux of the fluid.

2. FORMULATION & SOLUTION OF THE PROBLEM

The unsteady MHD flow of a Walters fluid (Model B') down an open inclined channel of width '2a' and depth 'd' under gravity, with naturally permeable bed, the walls of channel being normal to the surface of the bottom, under the influence of a uniform transverse magnetic field. A uniform tangential stress 'S' is applied at the free surface. The free surface is exposed to atmospheric pressure. The naturally permeable bottom of the channel is taken at an angle 'b' (o<bp/2) with the horizontal. The x-axis is taken along central line in the direction of the flow at the free surface, y-axis along the depth of the channel and the z-axis along the width of the channel. A uniform magnetic field of intensity 'H_O' is introduced in the y-direction. Therefore, the velocity and magnetic fields are given by and . The fluid being slightly conducting, the

magnetic Reynolds number much less than unity, so that the induced magnetic field can be neglected in comparison with the applied magnetic field [Sparrow &Cess(10)]. Fluid flow in porous medium is governed by Darcy's law and fluid in free flow region is governed by Naveir-stoke's equations.

In the absence of any input electrical field the equations of continuity and motion of the unsteady MHD Walters fluid (Model B') flowing down an open inclined channel of t>0 are

$$\frac{\partial u}{\partial x} = 0 \tag{2.1}$$

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \rho g\sin\beta + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - K_o\left(\frac{\partial^3 u}{\partial t\partial y^2} + \frac{\partial^3 u}{\partial t\partial z^2}\right) - \sigma\mu_e^2 H_o^2\, u \tag{2.2}$$

$$-\frac{\partial p}{\partial y} + \rho g\cos\beta = 0 \tag{2.3}$$

$$-\frac{\partial p}{\partial z} = 0 \tag{2.4}$$

The Darcy's equation for the flow in the porous medium is

$$Q = \frac{K}{\mu}\left(-\frac{\partial p}{\partial y} + \rho g\sin\beta\right) \tag{2.5}$$

Where

$\rho$is the density of the fluid.

g is the acceleration due to gravity

p is the pressure

$\mu$is the coefficient of viscosity

$K_0$is the non-Newtonian parameter

K is the permeability of the medium

Q is the velocity in the porous medium

$\mu_e$is the magnetic permeability

$\sigma$is the electrical conductivity of the fluid

$H_0$is the magnetic field

The boundary conditions are

$$t \le 0, u = u_0$$

$$t > 0, z = \pm a, u = 0$$

$$y = 0, \mu\frac{\partial u}{\partial y} = S$$

$$y = d, u = u_B$$

$$\left(\frac{\partial u}{\partial t}\right)_{y=d} = \frac{s_1}{\sqrt{K}}(u_B - Q) \tag{2.6}$$

Where

$u_0$is the initial velocity

$u_B$is the slip velocity

$s_1$is the dimensionless constant depending on the porous material

S is the uniform tangential stress

We introducing the following non-dimensional quantities.

$$u^* = \frac{u}{U} \quad u_B^* = \frac{u_B}{U} \quad Q^* = \frac{Q}{U}$$
$$Q_B^* = \frac{Q_B}{U} \quad K_0^* = \frac{K_0}{\rho d^2} \quad K^* = \frac{K}{d^2}$$
$$t^* = \frac{t\nu}{d^2} \quad y^* = \frac{y}{d} \quad p^* = \frac{p}{\rho U^2}$$
$$z^* = \frac{z}{d} \quad S^* = \frac{S}{\rho U^2}$$

In the view of the above non-dimensional quantities equations 2.2&2.5reduces to (dropping ‘ * ‘)

$$\frac{\partial u}{\partial t} = -R\frac{\partial p}{\partial t} + \frac{R}{F}\sin\beta + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - K_0\left(\frac{\partial^3 u}{\partial t\partial y^2} + \frac{\partial^3 u}{\partial t\partial z^2}\right) - Mu \qquad 2.8$$

$$Q = -KR\left(\frac{\partial p}{\partial t} - \frac{\sin\beta}{F}\right) \qquad 2.9$$

Where

$R = \frac{Ud}{\nu}$(Reynolds number)

$F = \frac{U^2}{gd}$ (Froude number)

$M = \frac{\sigma\mu_e^2 H_0^2 d^2}{\mu}$(Magnetic parameter)

The non-dimensional boundary conditions are

$t \le 0, u = u_0$

$t > 0, z = \pm L\left(=\frac{a}{d}\right), u = 0$

$y = 0, \frac{\partial u}{\partial y} = SR$

$y = 1, u = u_B$

$$\left(\frac{\partial u}{\partial t}\right)_{y=1} = \frac{s_1}{\sqrt{K}}\left(\frac{KR}{F}\sin\beta - KR\frac{\partial p}{\partial x} - u_B\right) \qquad 2.10$$

Assuming

$-R\frac{\partial p}{\partial x} + \frac{R}{F}\sin\beta = g(t)\, att > 0$

$= Patt \le 0 \qquad 2.11$

Substituting $z = \frac{2L\xi}{\pi} - L$in equation 2.8, we get

$$\frac{\partial u}{\partial t} = g(t) + \frac{\partial^2 u}{\partial y^2} + \frac{\pi^2}{4L^2}\frac{\partial^2 u}{\partial \xi^2} - K_0\left(\frac{\partial^3 u}{\partial t\partial y^2} + \frac{\pi^2}{4L^2}\frac{\partial^3 u}{\partial t\partial \xi^2}\right) - Mu \qquad 2.12$$

The boundary conditions 2.10 reduces to

$t \le 0, u = u_0$

$t > 0, \xi = 0, \pi andu = 0$

$y = 0, \frac{\partial u}{\partial y} = SR$

$y = 1, u = u_B$

$$\left(\frac{\partial u}{\partial t}\right)_{y=1} = \frac{s_1}{\sqrt{K}}[Kg(t) - u_B] \quad 2.13$$

Since u_0 is the initial velocity i.e., ≤ 0, by taking g(t)=P in equation 2.12, u_0 is given by

$$u_0 = u_B \frac{ch\alpha y}{ch\alpha} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1-\cos n\pi}{n}\left[\frac{P}{\alpha^2}\left(1 - \frac{ch\alpha y}{ch\alpha}\right) - \frac{SR}{\alpha}\frac{sh\alpha(1-y)}{ch\alpha}\right]\sin n\xi \quad 2.14$$

Where
sh = sinh
ch = cosh
th = tanh

$$u_B = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1-\cos n\pi}{n}\frac{A}{B}\sin n\xi \quad 2.15$$
$$B = \frac{s_1}{\sqrt{K}} + \alpha th\alpha$$
$$q = \frac{n\pi}{2L}2$$
$$\alpha^2 = q^2 + M$$

Now to solve equation 2.12, we take Laplace transform to equation 2.12 w.r.t. 't' defined as

$$\bar{u}(y,\xi,s) = \int_0^{\infty} u(y,\xi,t)\, e^{-st} dt, s > 0 \quad 2.16$$

We get

$$\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\pi^2}{4L^2}\frac{\partial^2 \bar{u}}{\partial \xi^2} - \frac{M+s}{1-K_0 s}\bar{u} = \frac{1}{1-K_0 s}\frac{PK_0 M}{\alpha^2}\frac{ch\alpha y}{ch\alpha} + \frac{PK_0 q^2}{\alpha^2} - \bar{g}(s) - u_0$$
$$+ \frac{SRMK_0}{\alpha^2}\frac{sh\,\alpha(1-y)}{ch\,\alpha} - u_B K_0\,\alpha^2 \frac{ch\,\alpha y}{ch\,\alpha} \quad 2.16$$
$$\bar{g}(s) = \int_0^{\infty} g(t)\, e^{-st}\, dt \quad 2.17$$

The transformed boundary conditions are

$$y = 0, \frac{\partial \bar{u}}{\partial y} = \frac{SR}{s}$$
$$y = 1, \bar{u} = \bar{u}_B$$
$$\left(\frac{\partial \bar{u}}{\partial t}\right)_{y=1} = \frac{s_1}{\sqrt{K}}\left[\frac{KP}{s} - \bar{u}_B\right] \quad 2.18$$

On taking the finite Fourier sine transform of equation 2.17 w.r.t. defined as

$$\bar{u}^*(y,N,s) = \int_0^{\infty} \bar{u}(y,\xi,t)\sin N\xi\; d\xi \quad 2.19$$

We get

$$\bar{u}^* = \sum_{N=1}^{\infty} \frac{1-\cos N\pi}{N}\frac{A}{B}\left\{\frac{1}{s} + \frac{1+K_0\,\alpha^2}{(1-K_0\,s)(\alpha^2-H^2)}\right\}$$
$$-\sum_{N=1}^{\infty} \frac{1-\cos N\pi}{N}\left\{-\frac{P}{s\alpha^2} + \frac{P}{\alpha^2 H^2}\frac{1-K_0\,q^2}{1-K_0\,s} + \frac{\bar{g}(s)}{H^2(1-K_0\,s)}\right\}\frac{ch\,Hy}{ch\,y}$$

$$+\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\left\{-\frac{1}{s}\left(\frac{P}{\alpha^2}\frac{ch\,\alpha y}{ch\,\alpha}+\frac{SR}{\alpha}\frac{sh\,\alpha(1-y)}{ch\,\alpha}\right)+\frac{P}{\alpha^2H^2}\frac{1-K_0\,q^2}{1-K_0\,s}+\frac{\bar{g}(s)}{H^2(1-K_0\,s)}\right\}$$

$$-\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\frac{A}{B}\frac{1+K_0\,\alpha^2}{(1-K_0\,s)(\alpha^2-H^2)}\frac{ch\,\alpha y}{ch\,\alpha} \qquad 2.20$$

The transformed boundary conditions are

$$y=0,\frac{\partial\bar{u}^*}{\partial y}=\sum_{N=1}^{\infty}\frac{SR}{s}\frac{1-\cos N\pi}{N}$$

$$y=1,\bar{u}^*=\bar{u}_B^*$$

$$\left(\frac{\partial\bar{u}^*}{\partial t}\right)_{y=1}=\sum_{N=1}^{\infty}\frac{s_1}{\sqrt{K}}\left[\frac{KP}{s}\frac{1-\cos N\pi}{N}-\bar{u}_B^*\right] \qquad 2.21$$

On inverting finite Fourier sine transform as given by Sneddon [9]

$$\bar{u}(y,\xi,s)=\frac{2}{\pi}\sum_{N=1}^{\infty}\bar{u}^*(y,N,s)\sin N\xi \qquad 2.22$$

In equation 2.20 we get

$$\bar{u}=\frac{u_B}{s}\frac{ch\,Hy}{ch\,H}-\frac{u_B}{s}\left(\frac{1+K_0\,\alpha^2}{1+MK_0}\right)\left(\frac{ch\,Hy}{ch\,H}-\frac{ch\,\alpha y}{ch\,\alpha}\right)$$

$$+\frac{2}{\pi}\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\left\{\frac{P}{s\alpha^2}\left(\frac{ch\,Hy}{ch\,H}-\frac{ch\,\alpha y}{ch\,\alpha}\right)+\left(\frac{P}{\alpha^2H^2}\frac{1-K_0\,q^2}{1-K_0\,s}+\frac{\bar{g}(s)}{H^2(1-K_0\,s)}\right)\left(1-\frac{ch\,Hy}{ch\,H}\right)-\frac{SR}{s\alpha}\frac{sh\,\alpha(1-y)}{ch\,\alpha}\right\}\sin N\xi \qquad 2.23$$

On inverting the Laplace transform as defined by Snedden [9]

$$u(y,\xi,t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\bar{u}(y,\xi,s)\,e^{st}\,dt \qquad 2.24$$

In equation 2.23 we obtain

$$u=u_B\frac{ch\,\alpha y}{ch\,\alpha}-\sum_{r=0}^{\infty}\frac{(-1)^r(2r+1)\pi\,u_Bd_t\cos a_rye^{-A_rt}}{a_r^2+\alpha^2}$$

$$+\frac{2}{\pi}\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\left\{\sum_{r=0}^{\infty}\frac{4(-1)^r\,P\cos a_rye^{-A_rt}}{(2r+1)\pi(a_r^2+\alpha^2)}+\int_0^1h(u)\;g(1-u)du-\frac{SR}{s\alpha}\frac{sh\,\alpha(1-y)}{ch\,\alpha}\right\}\sin N\xi \qquad 2.25$$

Where

$$a_r=(2r+1)\frac{\pi}{2}$$

$$A_r=\frac{a_r^2+\alpha^2}{1-K_0(a_r^2+q^2)}$$

$$h(u)=\sum_{r=0}^{\infty}\frac{4(-1)^r\cos a_rye^{-A_rt}}{(2r+1)\pi[1-K_0(a_r^2+q^2)]}$$

$$d_t=\frac{(1+K_0\,\alpha^2)-(a_r^2+\alpha^2)}{1-K_0(a_r^2+q^2)}$$

If we take the limit $K_0 \to 0, K \to 0$ *and* $M \to 0$in equation 2.25 then we get the velocity distribution in the case of viscous non-magnetic and impermeable bed. In this case the velocity distribution is

$$u = \frac{8L^2}{\pi^3}\left(-R\,\frac{\partial p}{\partial x} + \frac{R}{F}\sin\beta\right)\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N^3}\left(1 - \frac{ch\,qy}{ch\,q}\right) Sin\,N\xi \qquad 2.26$$

Which is in agreement with SatyaPrakash [7].

Now we evaluated the velocity distribution and flux in three different cases viz.,

Case 1

When g(t) = C, using in equation 2.25, we get

$$u = u_B\frac{ch\,\alpha y}{ch\,\alpha} + \sum_{r=0}^{\infty}\frac{(-1)^r(2r+1)\pi\, u_B d_t \cos a_r y e^{-A_r t}}{a_r^2+\alpha^2}$$
$$+ \frac{2}{\pi}\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\left\{\sum_{r=0}^{\infty}\frac{4(-1)^r(P-C)\cos a_r y e^{-A_r t}}{(2r+1)\pi\left(a_r^2+\alpha^2\right)}\right.$$
$$\left. + \frac{C}{\alpha^2}\left(1 - \frac{ch\,\alpha y}{ch\,\alpha}\right) - \frac{SR}{s\alpha}\frac{sh\,\alpha(1-y)}{ch\,\alpha}\right\}\sin N\xi \qquad 2.27$$

$$C_f = \int_{z=-L}^{L}\int_{y=0}^{1} u\, dy\, dz$$
$$= u_B\frac{th\,\alpha}{\alpha} + \sum_{r=0}^{\infty}\frac{2\,u_B d_t e^{-A_r t}}{a_r^2+\alpha^2}$$
$$+ \frac{4L}{\pi^2}\sum_{N=1}^{\infty}\left(\frac{1-\cos N\pi}{N}\right)^2\left\{\sum_{r=0}^{\infty}\frac{8\,(P-C)e^{-A_r t}}{(2r+1)^2\pi^2\left(a_r^2+\alpha^2\right)}\right.$$
$$\left. + \frac{C}{\alpha^2}\left(1 - \frac{th\,\alpha y}{th\,\alpha}\right) - \frac{SR}{\alpha^2}\left(\frac{ch\,\alpha-1}{ch\,\alpha}\right)\right\} \qquad 2.28$$

Case 2

When g(t) = C e^{-bt}, b>0, using in equation 2.25 we get

$$u = u_B\frac{ch\,\alpha y}{ch\,\alpha} + \sum_{r=0}^{\infty}\frac{(-1)^r(2r+1)\pi\, u_B d_t e^{-A_r t}\cos a_r y}{a_r^2+\alpha^2}$$
$$+ \frac{2}{\pi}\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\left\{\sum_{r=0}^{\infty}\frac{4(-1)^r\, P\, e^{-A_r t}\cos a_r y}{(2r+1)\pi\left(a_r^2+\alpha^2\right)} - \frac{SR}{\alpha}\frac{sh\,\alpha(1-y)}{ch\,\alpha} + \right.$$
$$\left.\sum_{r=0}^{\infty}\frac{4(-1)^r\, C\,\cos a_r y}{(2r+1)\pi\left[1-K_0\left(a_r^2+q^2\right)\right]}\frac{e^{-bt}-e^{-A_r t}}{A_r-b}\right\}\sin N\xi \qquad 2.29$$

$$C_f = u_B\frac{th\,\alpha}{\alpha} + \sum_{r=0}^{\infty}\frac{2\,u_B d_t e^{-A_r t}}{a_r^2+\alpha^2}$$
$$+ \frac{4L}{\pi^2}\sum_{N=1}^{\infty}\left(\frac{1-\cos N\pi}{N}\right)^2\left\{\sum_{r=0}^{\infty}\frac{8\,P\,e^{-A_r t}}{(2r+1)^2\pi^2\left(a_r^2+\alpha^2\right)}\right.$$
$$\left. + \sum_{r=0}^{\infty}\frac{8C}{(2r+1)^2\pi^2\left[1-K_0\left(a_r^2+q^2\right)\right]}\frac{e^{-bt}-e^{-A_r t}}{A_r-b} - \frac{SR}{\alpha^2}\left(\frac{ch\,\alpha-1}{ch\,\alpha}\right)\right\} \qquad 2.30$$

Case 3

When g(t) = C Cos bt, using in equation 2.25 we get

$$u = u_B\frac{ch\,\alpha y}{ch\,\alpha} + \sum_{r=0}^{\infty}\frac{(-1)^r(2r+1)\pi\, u_B d_t\, e^{-A_r t}\cos a_r y}{a_r^2 + \alpha^2}$$

$$+\frac{2}{\pi}\sum_{N=1}^{\infty}\frac{1-\cos N\pi}{N}\left\{\sum_{r=0}^{\infty}\frac{4(-1)^r\,P\,e^{-A_r t}\cos a_r y}{(2r+1)\pi\,(a_r^2+\alpha^2)}-\frac{SR}{\alpha}\frac{sh\,\alpha(1-y)}{ch\,\alpha}\right.$$

$$\left.+\sum_{r=0}^{\infty}\frac{4(-1)^r\,C\cos a_r y}{(2r+1)\pi\left[1-K_0(a_r^2+q^2)\right]}\frac{A_r\cos bt+b\sin bt-A_r\,e^{-A_r t}}{A_r{}^2-b^2}\right\}\sin N\xi \qquad 2.31$$

$$C_f=u_B\frac{th\,\alpha}{\alpha}+\sum_{r=0}^{\infty}\frac{2\,u_B d_t e^{-A_r t}}{a_r^2+\alpha^2}$$

$$+\frac{4L}{\pi^2}\sum_{N=1}^{\infty}\left(\frac{1-\cos N\pi}{N}\right)^2\left\{\sum_{r=0}^{\infty}\frac{8\,P\,e^{-A_r t}}{(2r+1)^2\pi^2(a_r^2+\alpha^2)}-\frac{SR}{\alpha^2}\left(\frac{ch\,\alpha-1}{ch\,\alpha}\right)\right.$$

$$\left.+\sum_{r=0}^{\infty}\frac{8C}{(2r+1)^2\pi^2\left[1-K_0(a_r^2+q^2)\right]}\frac{A_r\cos bt+b\sin bt-A_r\,e^{-A_r t}}{A_r{}^2-b^2}\right\} \qquad 2.32$$

3. CONCLUSIONS

We discuss velocity distribution and flux in three distinct time dependent pressure gradients viz.,

(i) g(t) = C

(ii) g(t) = C e^{-bt}

(iii) g(t) = C Cos bt

Figures 1 to 3, 4 to 6 and 7 to 9 are drawn to investigate the effects of velocity distribution 'u' against 'y' for different values of viscoelastic parameter K_0 magnetic parameter 'M' and time 't' respectively in three different cases (i), (ii) & (iii). We noticed that in all the three cases the velocity distribution increases with increase in M or t, where as it decreases with increase in K_0. Further we observed that the velocity distribution decrease upto y = 0.8 and then increase with increase in 'y' in figures 1 to 3. Also we noticed that the velocity distribution 'u' decrease with increase in 'y' in figures 4 to 9. Figures 10 to 12 are prepared to bring out the effects of Reynolds numbers R, viscoelastic parameter K_0 on the flux C_f of the fluid. We observed that C_f decreases with increase in R or K_0 respectively in all the three cases.

In figures 13 to 15, C_f is drawn against't' for different values of K_0 in cases (i), (ii) & (iii). We noticed that C_f decreases with increase in K_0, where as it increases with increase in 't' in all three cases.

From figure 16, one can conclude that C_f increases with increase in K or M in case (i). From figures 17 & 18 we conclude that C_f decreases with increase in 'M', where as it increases with increase in K respectively in cases (ii) & (iii).From tables 1 to 3, we see that the velocity distribution is plotted against 'y' fordifferent values of 'K' respectively in three different cases. We noticed that 'u' increase in 'K' where as decreases with increase in 'y' respectively in three different cases. From table 4 we conclude that the slip velocity u_B decreases with increase in M, where as it increases with increase in 'K'.

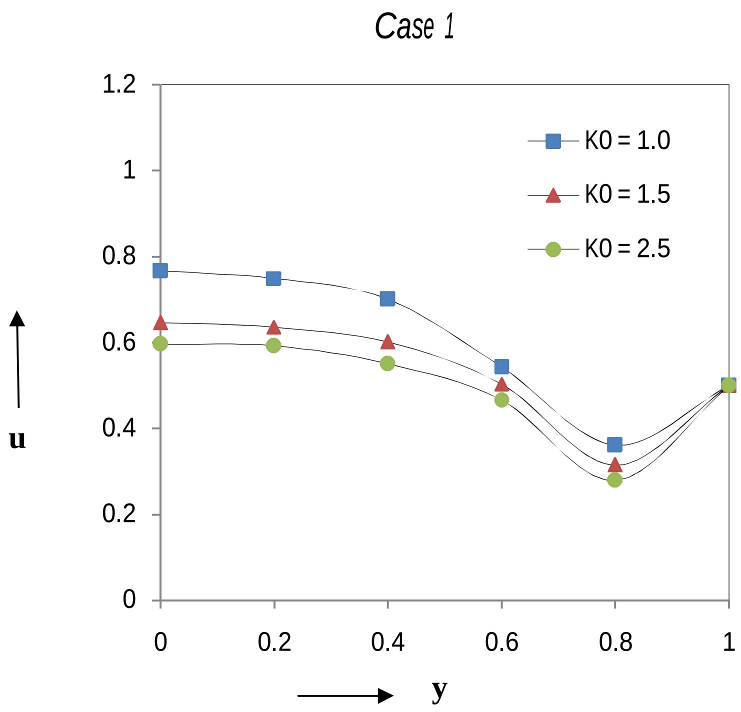


Fig .1 : u against y for different K_0

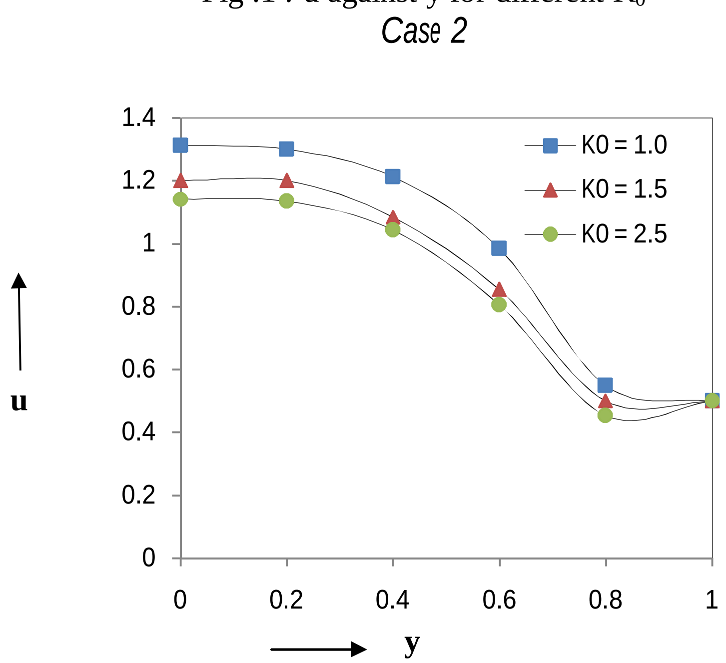


Fig.2 : u against y for different K_0

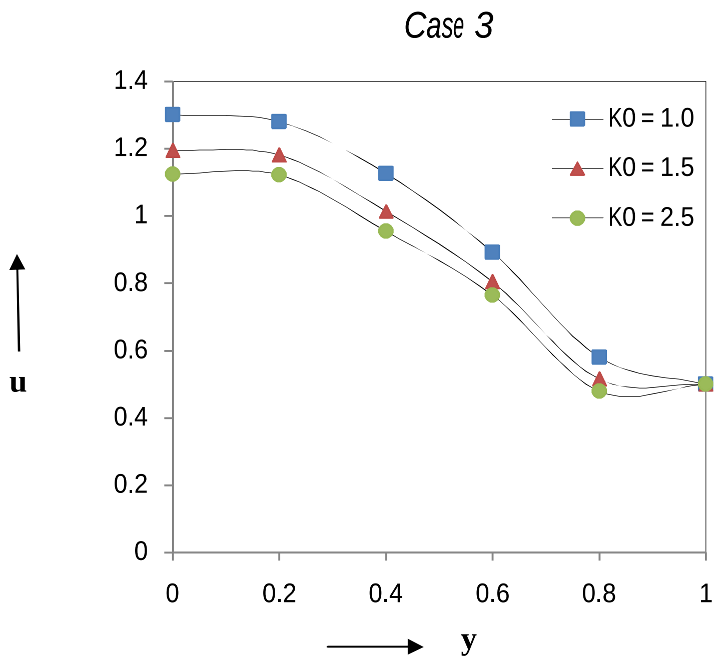


Fig.3: u against for different K_0

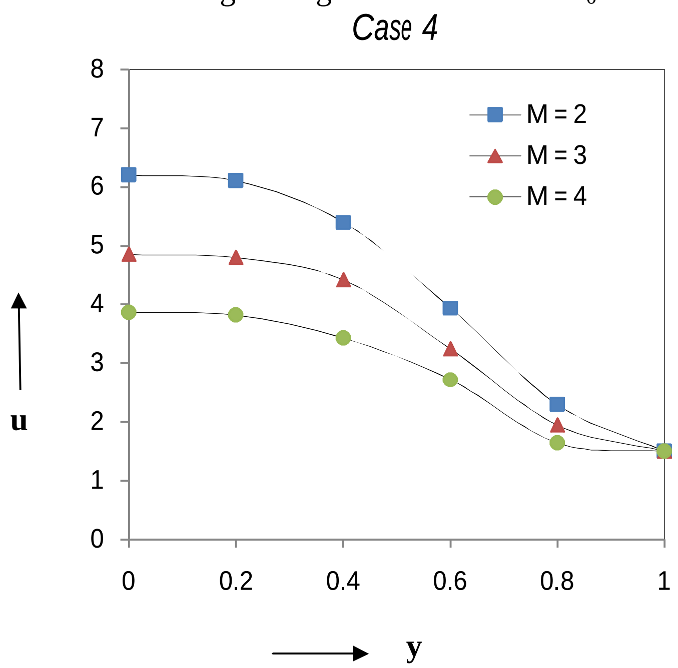


Fig.4 : u against y for different M

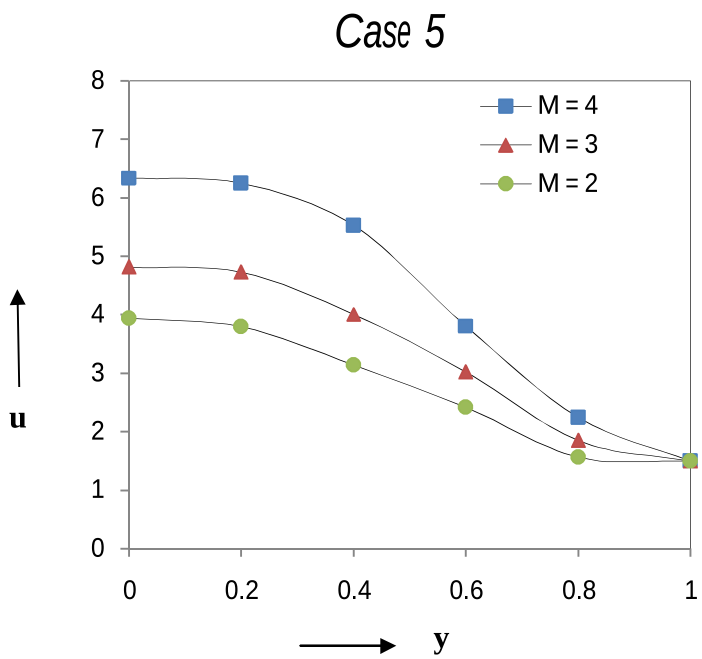


Fig.5: u against y for different M

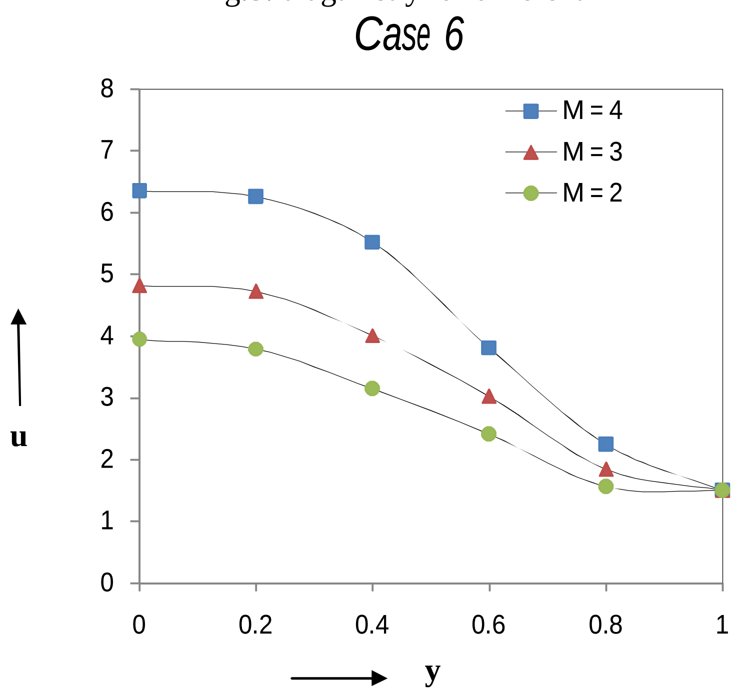


Fig.6 : u against y for different M

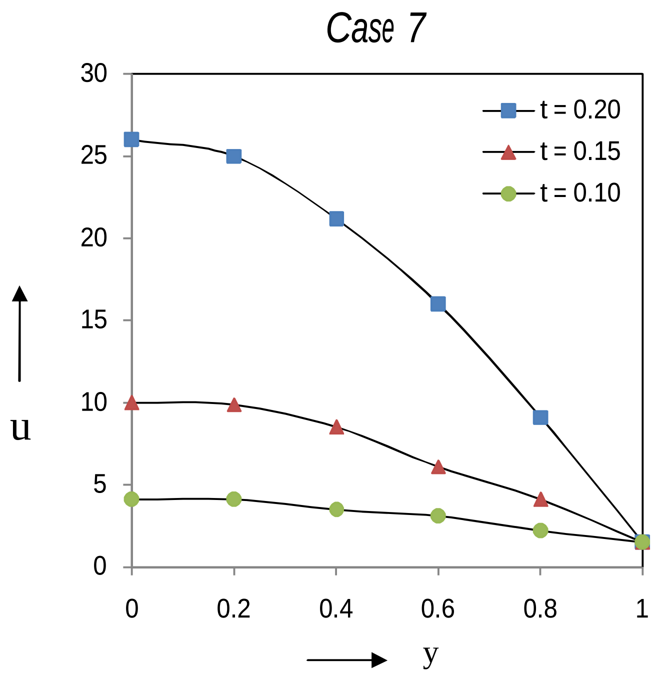


Fig.7:u against y for different t

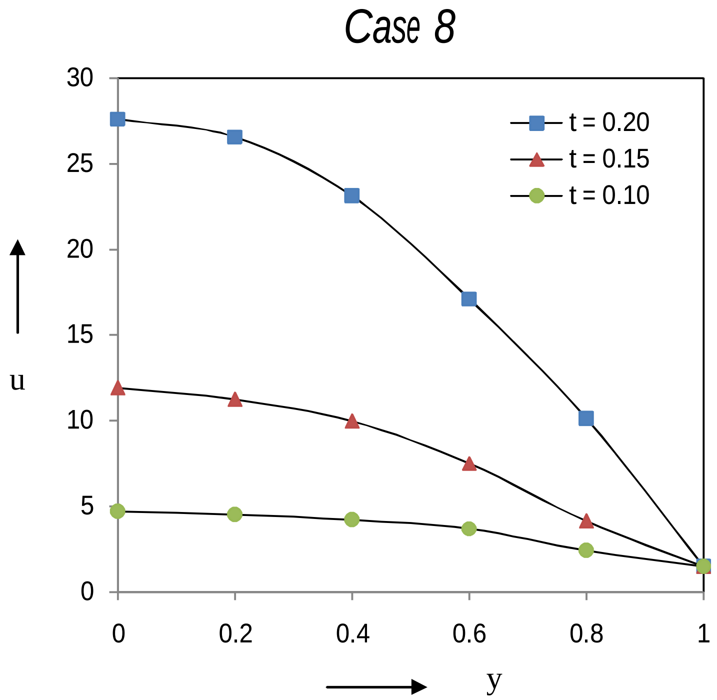


Fig. 8 : u against y for different t

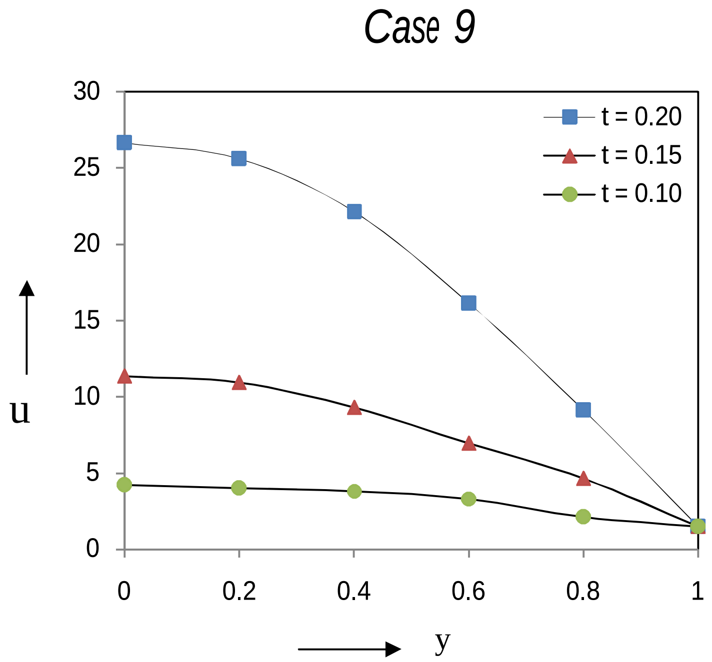


Fig.9 : u against y for different t

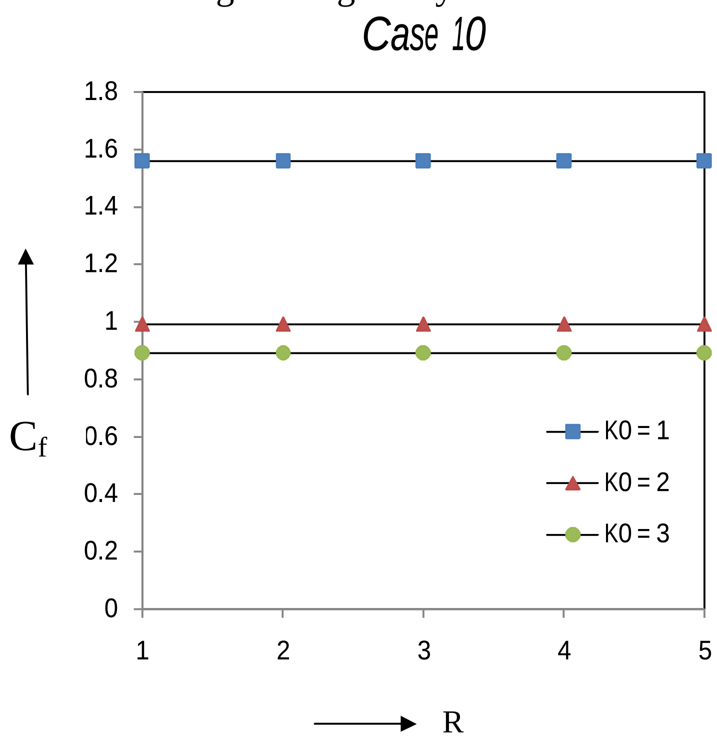


Fig.10: C_f against R for Different K_0

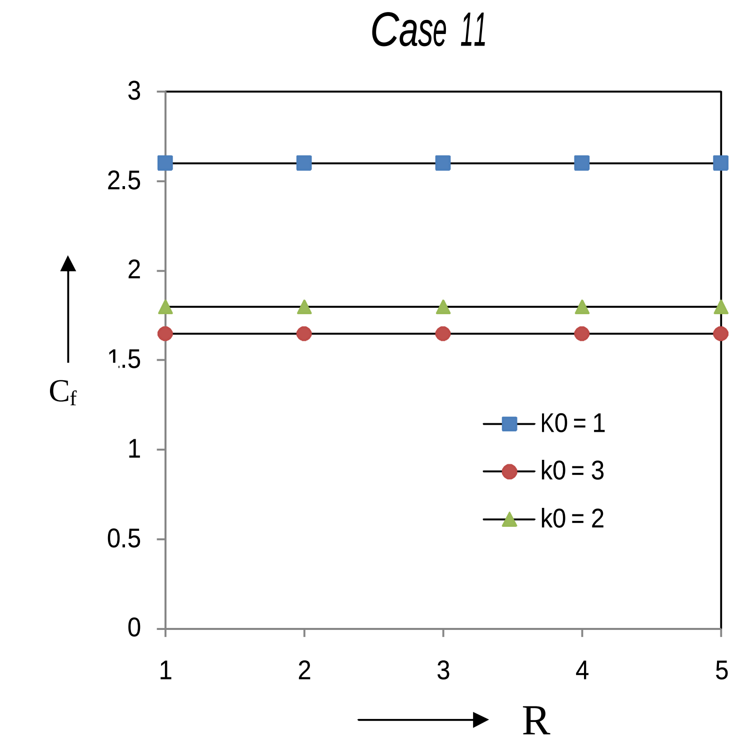


Fig.11: C_f against R for different K_0

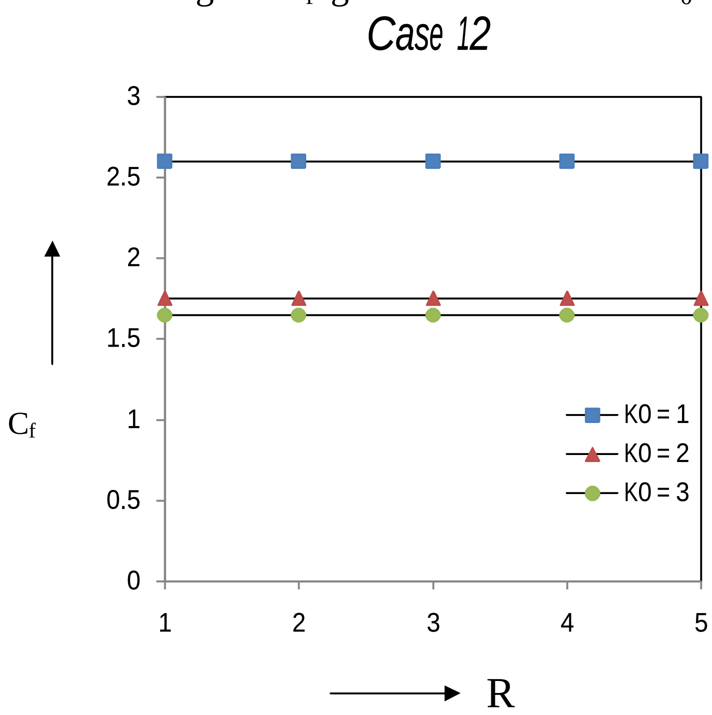
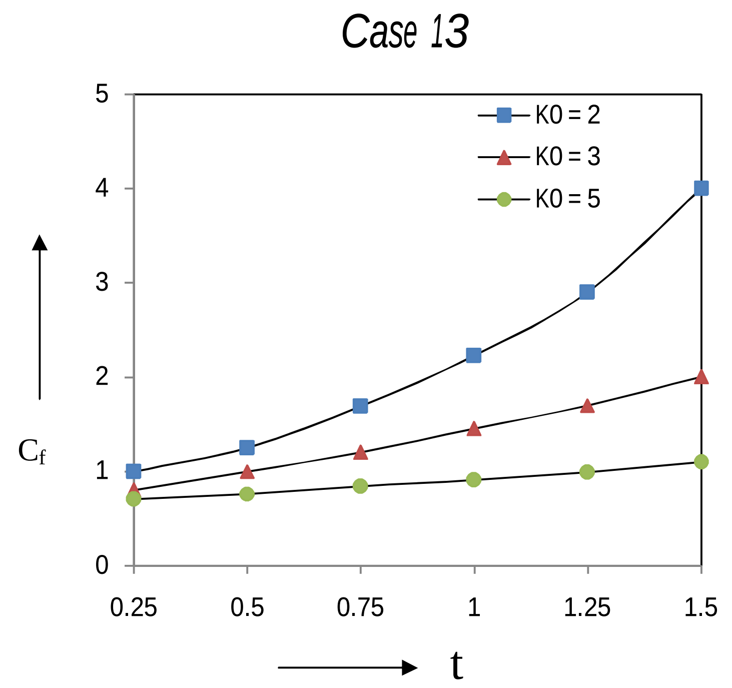


Fig.12 : C_f against R for different K_0



Fig;13 : C_f against t for different K_0

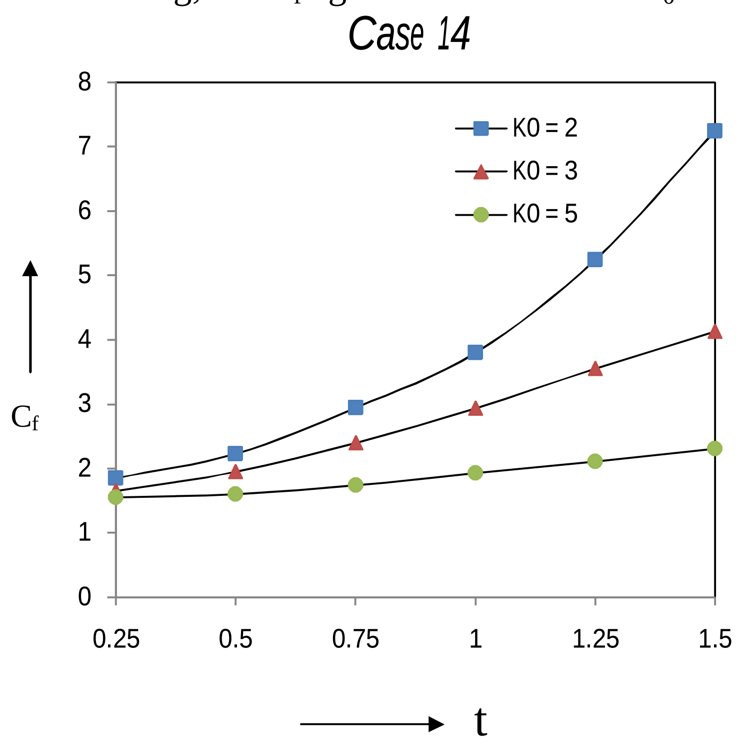


Fig.14 : C_f against t for different K_0

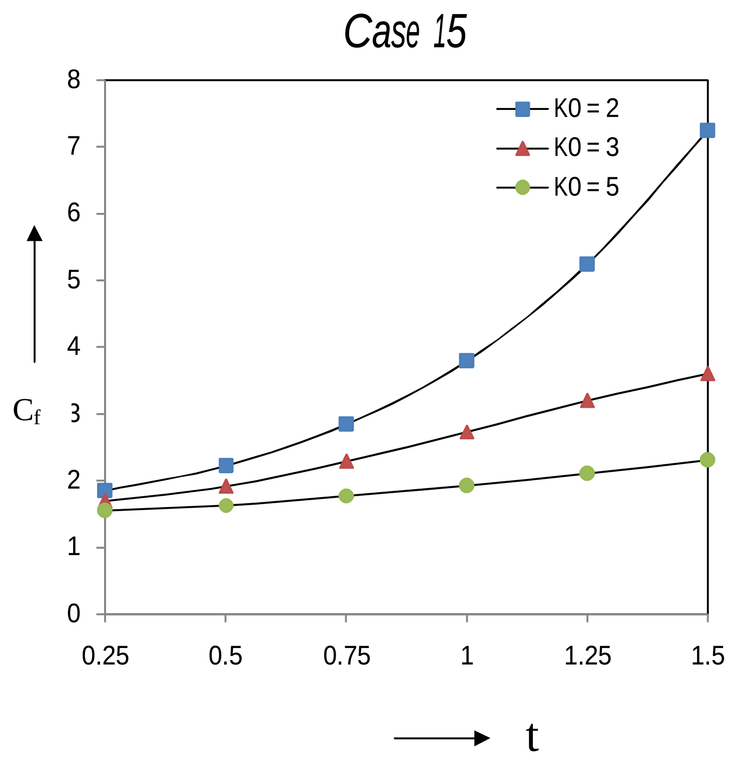


Fig.15 : C_f against t for different K_0

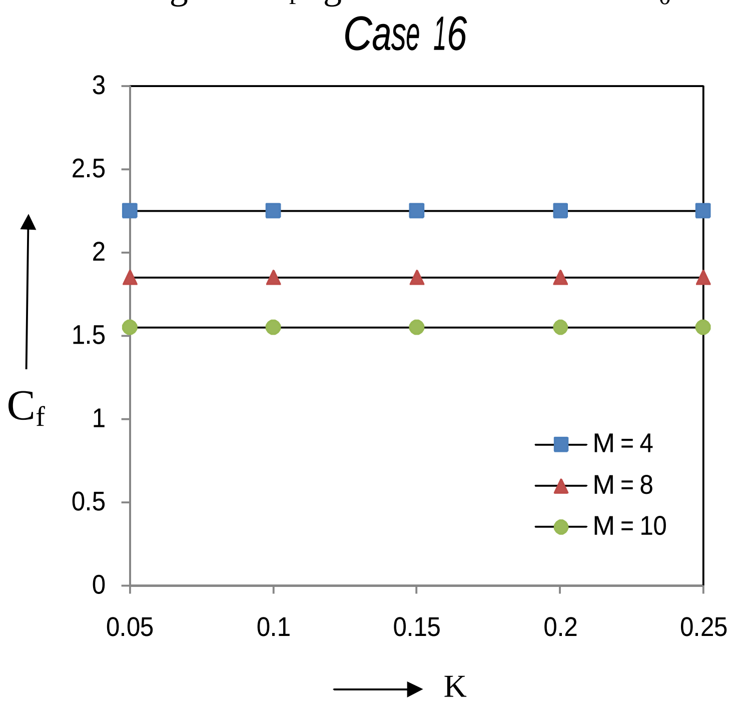


Fig.16 : C_f against K for different M

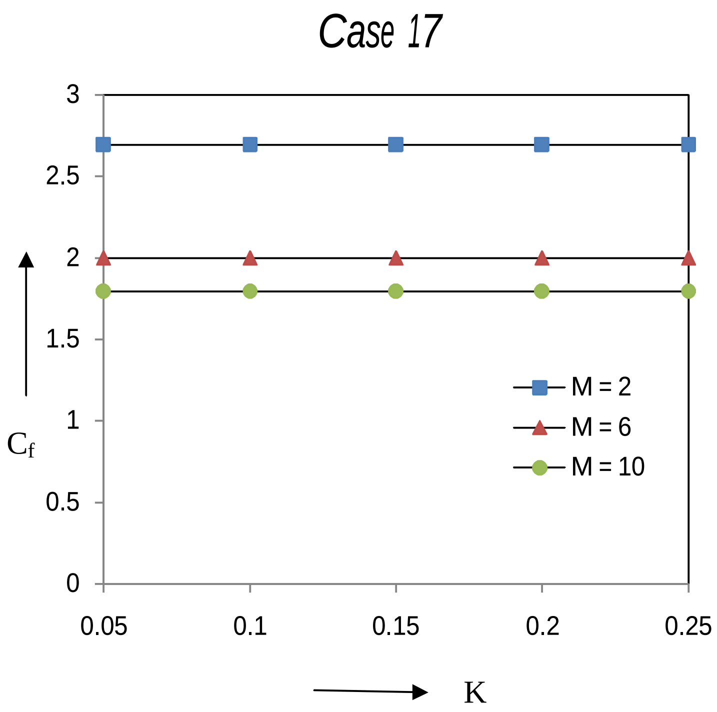


Fig.17 : C_f against K for different M

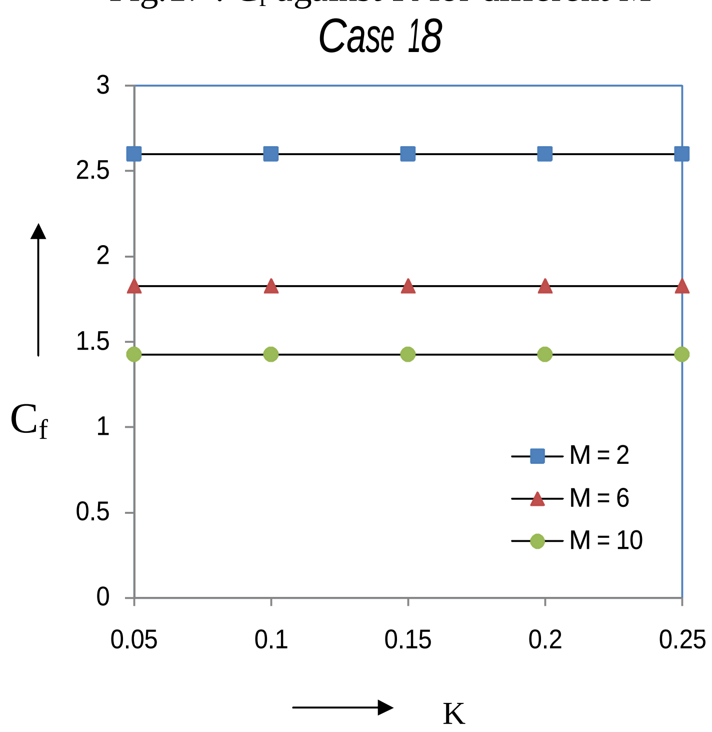


Fig.18 : C_f against K for different M

Table.1 *Case 1* u against y for different K

y	K=0.05	K = 0.10	K = 0.15	K = 0.20	K = 0.25
0.0	1.45647	1.45717	1.45756	1.45783	1.45804
0.2	1.39990	1.40060	1.40100	1.40127	1.40147
0.4	1.21344	1.21453	1.21453	1.21480	1.21501
0.6	0.91064	0.91173	0.91173	0.91201	0.91221
0.8	0.50257	0.50366	0.50366	0.50393	0.50414
1.0	0.42520	0.53390	0.53390	0.56084	0.58115

Table.2 *Case 2* u against y for different K

y	K=0.05	K = 0.10	K = 0.15	K = 0.20	K = 0.25
0.0	2.04097	2.04167	2.04206	2.04234	2.04254
0.2	1.96327	1.96397	1.96437	0.96464	1.96484
0.4	1.71223	1.71293	1.71333	1.71360	1.71381
0.6	1.29792	1.29862	1.29902	1.29929	1.29950
0.8	0.72571	0.72641	0.72680	0.72707	0.72728
1.0	0.42468	0.49414	0.53339	0.56033	0.58063

Table.3 *Case 3* u against y for different K

y	K=0.05	K = 0.10	K = 0.15	K = 0.20	K = 0.25
0.0	2.00489	2.00559	2.00599	2.00626	2.00647
0.2	1.92892	1.92962	1.93001	1.93029	1.93049
0.4	1.68290	1.68360	1.68400	1.68427	1.68447
0.6	1.27648	1.27718	1.27757	0.27784	1.27805
0.8	0.71434	0.71504	0.71570	0.71570	0.71591
1.0	0.42471	0.49417	0.56035	0.56035	0.58066

Table.4 u_B against K for different M

y	M = 5	M = 10	M = 15	M = 20	M = 25
0.05	0.42192	0.42183	0.42174	0.42165	0.42156
0.10	0.49066	0.49054	0.49041	1.49029	0.49017
0.15	0.52951	0.52936	0.52922	0.52907	0.52893
0.20	0.55616	0.55600	0.55585	0.55569	0.55553
0.25	0.57626	0.57609	0.57575	0.57575	0.57558

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