# Hybrid GA-PSO Based Tuning of Unscented Kalman Filter for Bearings Only Tracking

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#### Abstract

The basic problem in Target tracking is to estimate the trajectory of a object from noise corrupted measurements and hence becoming very important field of research as it has wider applications in defense as well as civilian applications. Kalman filter is generally used for such applications. When the process and measurements are non linear extensions of Kalman filters like Extended Kalman Filter, Unscented Kalman Filters are widely used. UKF can give estimations up to second order characteristics of random process. Implementation of Kalman filter i, e tuning of filter has to be performed before applying to real time situations. It demands prior estimations of Noise covariance matrices which are left for engineering intuitions. This paper presents the nonlinear state estimation using unscented Kalman filter and tuning of the filter is done using bio-inspired hybrid algorithms like PSO GA and Hybrid GA-PSO.

**Keywords**: Unscented Kalaman Filter, Non linear Estimation, Target tracking, Tuning of filter, Hybrid GA-PSO Algorithm.

# 1. Introduction

In many tracking applications Kalman Filter (KF) is used to estimate the velocity, position and acceleration of a maneuvering target from noisy radar measurements at high data rates. Bearings only tracking is attracted many researches in these days due to its practical military and civilian applications [1-2]. When the process is to be estimated and measurement model is nonlinear, EKF is used in which, the process is approximated to first order term of the Taylor's expansion for calculating the mean and covariance of the random process [3]. This linearization however poses some problems

e.g. it can produce highly unstable filters if the assumptions of local linearity is violated. In this paper we simulate UKF (estimator) which generalizes sophisticatedly to nonlinear systems without the linearization steps required by the EKF.

The UKF uses deterministic sampling approach [4]. Approximating a Gaussian distribution is easier than approximating a nonlinear transformation so state distribution is Approximated by a Gaussian random vector. The Kalman filter demands priori information about the noise covariances from the user [5]. Initial process and measurement noise covariances play an important role in convergence of the filter. If the noise covariances are not chosen properly it may leads towards degradation of the filter performance [6]. A few techniques for determining the process and measurement noise covariances have been discussed in the literature [7], [8] and widely used tuning method is least squares approach.

The Standard Genetic Algorithm (SGA) is inspired by Charles Darwin's evolutionary theory of evolution. Typically Genetic Algorithm maintains a population of candidate solutions for problem at hand and makes it evolve by iteratively applying a set of stochastic operations [9]. Particle Swarm Optimization (PSO) is population based stochastic optimization technique inspired by social behaviour of bird flocking or fish schooling in searching for food [10]. PSO exploits a population of individuals to probe promising regions of the search space. In the context, the population is called a swarm and the individuals are called particles. Each particle moves with an adaptable velocity within the search space, and retains in its memory the best position it ever encountered. In the global variant of PSO the best position ever attained by all individuals of swarms is communicated to all the particles. PSO and GA are population based heuristic search technique which can be used to solve the optimization problems modelled on the concept of Evolutionary Approach. In standard PSO, the non-oscillatory route can quickly cause a particle to stagnate and also it may prematurely converge on suboptimal solutions that are not even guaranteed to be local optimum. So A Hybrid GA-PSO algorithm is proposed [11].

This paper implements GA,PSO and GA-PSO based tuning of UKF, in which process noise and measurement covariances are tuned based on biologically inspired evolutionary computing tools.

# 2. Problem Statement

In this paper target tracking environment is taken as shown in Fig. 1.

Multisensor target tracking is finding many applications these days because of its advantages like accurate target tracking and cheaper in cost.

Measurement processing generally includes a form of thresholding (measurement detection) process. Information loss during the thresholding has to be taken care and in very low SNR scenarios, thresholding might not be used, which leads to Track before Detect algorithms with high computation cost. Detections originate not only from targets being tracked, but also from thermal noise as well as from various objects such as terrain; clouds and these unwanted measurements are usually termed clutter. Target trackers (TT) are widely used in air defense, ground target tracking, and missile

defense. Target tracking have two portions: Data association algorithm section and estimation and prediction section. Data association is the process to match a measurement to a landmark .Gating is a technique for eliminating unlikely measurement-to-track pairings and the purpose of gating is to reduce computational expense by eliminating from consideration measurements which are far from the predicted measurement location [12]. Data association algorithms deal with situations where there are measurements of uncertain origin



Fig. 1: Sensor Placement and Tracking Scenario.

We want to track the aircraft position by using sensors in presence of process noise and measurement noise. The measurements are in polar coordinates (bearing  $\theta$ ) as we are using sensors (RADAR), which measure only the bearings (or angles) with respect positions of the sensors. Solving this problem is important, because more general multiple target tracking (MTT) problems can be partitioned to sub problems, in which single target is tracked separately. Basic problem is to estimate the target kinematic state (position and velocity) from noise corrupted measurements. Since the output of the filtering algorithm is required to be Cartesian position and velocity, the target Kinematic state can be described by the state vector defined in discrete time as

$$X_{k} = [x_{k}, y_{k}, v_{x_{k}}, v_{y_{k}}]^{T}$$
(1)

Where T denotes matrix transpose, xk, and yk are the Cartesian target coordinates at time index k and  $v_{x_k}$  and  $v_{y_k}$ , are their respective derivatives (velocities).

The state equation for the target motion could be approximated with a linear equation of the form

$$X_{k+1} = F_k x_k + G w_k \tag{2}$$

Where  $x_k$  is the state vector that contains state variables at time k, and  $w_k \sim N(0, Q_k)$  which is assumed as zero mean white Gaussian noise with covariance  $Q_k$  (called process noise).

The state equation for the two dimensional target motion could be approximated with a linear equation of the form

$$\begin{bmatrix} x_{k+1} \\ y_{k+11} \\ v_{x_{k+1}} \\ v_{y_{k+1}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_k & 0 \\ 0 & 1 & 0 & t_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ v_{x_k} \\ v_{y_k} \end{bmatrix} + \begin{bmatrix} t_k & 0 \\ 0 & t_k \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \end{bmatrix}$$
(3)

Comparing equations (2) and (3), then process noise covariance matrix can be written as

$$Q = E[w_k w_k^T] = \int_0^T G\sigma^2 G^T dt \qquad \dots (4)$$

Where  $\sigma$  is the standard deviation of the Gaussian random noise. When can be given as

$$= \begin{bmatrix} q_x * t_k^3 / 3 & 0 & q_x * t_k^2 / 2 & 0 \\ 0 & q_y * t_k^3 / 3 & q_y * t_k^2 / 2 \\ q_x * t_k^2 / 2 & 0 & q_x * t_k & 0 \\ 0 & q_y * t_k^2 / 2 & 0 & q_y * t_k \end{bmatrix}$$
 .....(5)

Where

 $q_x$  = level of power spectral density of X-directional noise

 $q_y$  = level of power spectral density of Y-directional noise The measurement model of the system can be written as

Where  $z_k$  measurement vector, and  $v_k \sim (0, R_k)$  which is assumed as zero mean white Gaussian noise with covariance  $R_k$  (called measurement noise covariance). Both noises are assumed to be uncorrelated. The measurement equation above relates the state  $x_k$  to the measurement  $z_k$  for above scenario sensors are placed at  $(s_x^1, s_y^1), (s_x^2, s_y^2)$ and measurement equation can be written as

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \tan^{-1}(y_k - s_y^1)/(x_k - s_x^1) \\ \tan^{-1}(y_k - s_y^2)/(x_k - s_x^2) \end{bmatrix} + \begin{bmatrix} s_d * randn \\ s_d * randn \end{bmatrix} \qquad \dots (7)$$

Where Sd is the standard deviation of the measurement noise.

Measurement noise covariance can be written as

$$R_k = E(v_k v_k^{T}) \dots (8)$$

This can be written as

$$R_k = dig(sd, sd) \dots (9)$$

The accuracy of the estimation depends on the priori measurement noise covariance matrix  $R_k$  and process noise covariance matrix Q which interns depends upon these spectral densities  $q_x$  and  $q_y$ . Selecting optimum parameters of these values gives optimum performance of the filter.

Trial and error approach to obtain these the above said three tuning parameters is tedious process and doesn't guarantee the accuracy of estimation in Mean Square Error (MSE) sense. Choosing optimum Parameters of noise covariance matrices, "i.e." is tuning the filter is a challenging task for Kalman filter designer. In this paper another approach of tuning the Unscented Kalman Filter based on the swarm intelligence and hybrid approach is proposed.

# 3. Unscernted Kalman Filter

The UKF is a recursive minimum-mean square-error (MMSE) estimator. It is based on the unscented transform (UT). The UT is a method for calculating the statistics of a random variable, which undergoes a nonlinear transformation. State distribution is approximated by Gaussian random vector and is represented by a set of deterministically chosen sample points called sigma points. These sigma points can capture the true mean and covariance of the Gaussian Random Variables, and when propagated through the true nonlinear system, capture the posterior mean and covariance accurately to the 3rd-order Taylor series expansion for any nonlinearity. High order information about the distribution can be captured using only a very small number of points as problems of statistical convergence are not an issue. Using UT, UKF captures the mean and covariance in the prior and posterior densities accurately.Let L-dimension state vector  $\hat{x}_{k-1}$  with mean  $\hat{x}_{k-1|k-1}$  and covariance P <sub>k-1|k-1</sub> be approximated by 2L+1 weighted samples or sigma points. Then one cycle of the UKF is as follows.

Sigma point calculation: Compute the (2L+1) sigma points as follows:

Assume the following initialization parameters.

 $\hat{\mathbf{x}}_{k-1|k-1}$  and covariance  $P_{k-1|k-1}$  are known. One step Prediction of  $\hat{\mathbf{x}}_{k-1|k-1}$  and its covariance  $P_{k-1|k-1}$  can be calculated by unscented transformation.

$$X_{k-1|k-1}^{0} = \hat{x}_{k-1|k-1}$$

$$W_{0}^{m} = \lambda/(L+\lambda)$$

$$\lambda = \alpha^{2}(L+\kappa) - L$$
(10)

Where  $\alpha$  determines the spread of sigma points around the mean and is usually set to a small positive value,  $\kappa$  is a secondary scaling parameter( for fine Tuning the higher order moments of approximation) which is usually set to 3-L.

$$X_{k-1|k-1}^{i} = \hat{x}_{k-1|k-1} + (\sqrt{(L+\lambda)P_{k-1|k-1}})_{i}, i = 1, 2, \dots, n$$

 $W_i^m = 1/2(L+\lambda), i = 1,...,L$  $X_{k-1|k-1}^i = \hat{x}_{k-1|k-1} - (\sqrt{(L+\lambda)P_{k-1|k-1}})_i, i = L+1,...,2n \quad W_{i+L}^m = 1/2(L+\lambda), i = 1,...L$ 

Whaere  $\left(\sqrt{(L+\lambda)P_{k-1|k-1}}\right)_i$  is the i<sup>th</sup> row or column(depending on the matrix square root form, if  $P = A^T A$  then the sigma points are formed from the rows of A. However, if the matrix square root is of the form  $P = AA^T$ , the columns of A are used of the matrix square root of  $(L+\lambda)P_{k-1|k-1}$   $W_i$  is the normalized weight associated with the i<sup>th</sup> point. Note that Cholesky decomposition is needed for the matrix square root.

Propagation: Propagate each sigma point through the nonlinear function

$$X_{k|k-1}^{i} = f(X_{k-1|k-1}^{i}),$$
(11)

Obtain the mean and covariance of the state by

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2L} W_i^m X_{k|k-1}^i,$$
(12)

$$P_{k|k-1} = Q_{k-1} + \sum_{i=0}^{2L} W_i^c \Big[ X_{k|k-1}^i - \hat{x}_{k|k-1} \Big] \times \Big[ X_{k|k-1}^i - \hat{x}_{k|k-1} \Big]^r$$

**Update:** Calculate the measurement sigma points  $Z_{k|k-1}^i$  using h(.) and update the mean and Covariance by

$$Z_{k|k-1}^{i} = h(X_{k|k-1}^{i}),$$
(13)

$$\hat{z}_{k|k-1} = \sum_{i=0}^{2L} W_i^m Z_{k|k-1}^i$$
(14)

$$\widetilde{\nu}_k = z_k - \hat{z}_{k|k-1},\tag{15}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \widetilde{v}_k, \tag{16}$$

$$P_{k|k} = P_{k|k-1} - K_k P_{zz} K_k^T,$$
(17)

$$P_{zz} = R_k + \sum_{i=0}^{2L} W_i^c \left[ Z_{k|k-1}^i - \hat{z}_{k|k-1} \right] \times \left[ Z_{k|k-1}^i - \hat{z}_{k|k-1} \right]^r, \quad (18)$$

$$P_{xz} = \sum_{i=0}^{2L} W_i^{\ c} \left[ X_{k|k-1}^{\ i} - \hat{x}_{k|k-1} \right] \times \left[ Z_{k|k-1}^{\ i} - \hat{z}_{k|k-1} \right]^{r}, (19)$$
$$K_k = P_{xz} P_{zz}^{-1}$$
(20)

The other advantage of the UKF over the EKF is that it can estimate the mean and covariance of the state accurately to second order for any nonlinearity (3<sup>rd</sup> for Gaussian inputs). Unlike Extended Kalman Filter, no Jacobians or Hessians are calculated. The computational complexity in UKF is same as EKF.

# 4. Tuning of UKF

The Tuning of the filter is referred as estimation of the noise covariance matrices. It has been shown previously that the performance of an EKF process depends largely on the accuracy of the knowledge of process covariance matrix and measurement noise covariance matrix. Incorrect apriori knowledge of noise covariance may lead to performance degradation and it can even lead to practical divergence. Hence, intelligent method of estimation of these matrices becomes very important. Measurements can be performed before the operation of the filter under various noise conditions and measurement noise covariance can be obtained off line.

# 4.1 Genetic Algorithm

Genetic algorithm is a powerful evolutionary computing tool developed by Goldberg. Its main principal is "Select The Best, Discard The Rest" as adopted by naturally in the environment. Two important elements required for any problem before a genetic algorithm can be used for a solution are:

1)Method for representing a solution (encoding)

Ex: string of bits, numbers, character

2)Method for measuring the quality of any proposed solution, using fitness function

Ex: Determining RMSE

The space of all feasible solutions (it means objects among those the desired solution is) is called search space (also state space). Each point in the search space represent one feasible solution. Each feasible solution can be "marked" by its value or fitness for the problem. The whole process can be categorized into following sub processes.

i)Initialization

Initially many individual solutions are randomly generated to form an initial population, covering the entire range of possible solutions (the search space).Each point in the search space represents one possible solution marked by its value (fitness)

ii)Selection

A proportion of the existing population is selected to bread a new breed of generation.

iii)Reproduction

Generate a second generation population of solutions from those selected through geneticoperators: crossover and mutation.

iv)Termination- A solution is found that satisfies pre-specified criteria

v)Fixed number of generations.-The highest ranking solution fs fitness has reached. Disadvantages of genetic algorithm

i)Computation complexity is high ii) More training time is required iii) Conversion from binary to decimal

iv) Chances of falling to local minima

#### 4.2 Particle swarm optimization

PSO is population based stochastic optimization technique inspired by social behavior of bird flocking or fish schooling etc[8]. The swarm of particles indicates estimation of multiple parameters involved in the problem. We can begin with initializing a random swarm of particles. During each iteration fitness of the particle is evaluated with the help of fitness function.

The trajectory of the particle is dependent on three factors: its previous position, pbest and gbest. Greater the strain of particle in searching food, smaller is the acceleration coefficients. The inertial weight factor w signifies the importance of the particle's previous position in further search.

Velocity updation

$$v_i(t+1) = w.v_i(t) + c_1 rand(pbest(t) - x(t)) + c_2 .rand.(gbest(t) - x_i(t))$$
 (20)

Position updation

$$P = P + V \dots \dots \dots \dots \tag{21}$$

Where

P - Instantaneous position of the particle, V - Instantaneous velocity of the particle, *Pbest* - positional best of the particle, *gbest* – global best position of the swarm of the particles, W – Inertial weight factor, C1, C2 – acceleration coefficients

Thus each particle tends to move towards gbest to reach food early. If gbest has less number of values then the particles will reach food early. The algorithm comes to an end when all the particles converge at the gbest i.e. food position [8]. In our problem i.e. attaining minimum possible value for steady state error signal is considered as global optimum.



Fig. 2: Flow chart for PSO Algorithm

### 4.3 Hybrid GA-PSO Algorithm

One advantage of PSO over GA is its Algorithmic simplicity. Another clear difference between PSO and GA is the ability to control convergence in PSO. The main problem with PSO is that it prematurely converges (Van den Bergh and Engelbrecht 2004) to stable point, which is not necessarily global extreme To overcome the limitations of PSO and GA, hybrid algorithm is proposed. Such approach is expected to combine merits of PSO and GA in tuning gain parameters. The merit of GA lies in its genetic operator, crossover and mutation. By applying crossover operation, information can be swapped between two particles to have the ability to fly to the new search area and mutation operator increases the population. The total numbers of iterations are equally shared by GA and PSO. First half of the iterations are run by GA and the solutions are given as initial population of PSO. Remaining iterations are run by PSO [11].

### 4.2.A. Applying ga-pso in filter tuning

We refer to filter tuning as a process of obtaining parameters of a filter such as values of matrices Q and R for UKF that give the best filter performance in Mean Square Error (MSE) sense. Typically this kind of problems of designing a filter with optimal tuning parameters was left up to engineering intuition, and trial and error method that do not guarantee best filter performance due to large number of parameters to be tuned. A straightforward way of tackling this problem is to employ global optimization method that minimizes function of MSE position error with respect to filter parameters. There are several issues associated with such an approach. First, each time we need a value of MSE during global optimization procedure we have to run UKF on all available data. This requires a significant computational time since for example in order to find a global minimum of a smooth function of 3 parameters; we need to compute the function value many times [13-14].

Here in this problem we are tracking with constant velocity and with small manoeuvre such as to relate practical problem. Therefore we have two power spectral densities of the corresponding continuous process noise, one parameter of measurement noise standard deviations (bearing). So a total of three parameters have to be optimized. Taking the extreme worst cases of these three parameters, we precede according to the above mentioned optimisation algorithms.

## 5. Simulations and Results

Here we consider a target scenario in which a moving target in the scene and two angular sensors for tracking it. The sensors are placed at  $(s_x^1, s_y^1, ) = (-1m, -2m)$  and  $(s_x^2, s_y^2, ) = (-1m, -2m)$ . The measurement noise standard deviation is taken as Sd= 0.5 radians and spectral densities of the process noise is consider as q x=0.1 and qy =0.1 to generate data as show in Fig. 4 below. The simulations are performed using industry standard MATLAB and EKF/UKF Toolbox.

GA-Parameters	Initialization of PSO	
GA type used= Standard Genetic	Size of the swarm " no of birds=30;	
Algorithm	Maximum number of "birds steps=30;	
Number of Variables =3	Dimension of the problem $=3$ ;	
Initial Population size $= 25$	PSO parameter $C1 = 2.05$	
Length of Chromosome $= 20$	PSO parameter $C2 = 2.05$	
Probability of Crossover $= 0.8$	pso momentum or inertia $w = 0.45$	
Probability of Mutation= 0.2	search space for $Sd = 0$ to 0.1	
	search space for $qx = 0$ to 0.01	
	search space for $qy=0$ to 0.05	

The following Fig. gives Radar sensor measurements



Fig. 6: Filter estimations with conventional and Hybrid Tuning.

From the the above Fig. we can say that the Hybrid GA-PSO tuned Kalman filter performing better comparered to conventional tuning.



From the above error analysis we can say that Tuned filter in which the covariance matrices are estimated using Nature Inspired Algorithms are giving minimal error.

Parameter	Conve ntional UKF	GA- Tuned UKF	PSO- Tuned UKF	GA-PSO Tuned UKF
Measurement noise standard deviation(Sd)	0.05	0.03	0.0140	3.298e- 09
q <sub>x</sub>	0.1	0.834	0.0062	0.0840
qy	0.1	0.753	0.0418	0.2824
RMSE (m)	0.099	8.672e- 04	0.0587	1.0772e- 05
Computational complex city	0.0015 6 (Secon ds)	0.622 (Seconds )	0.424 (Secon ds)	

From the above Table shows the RMSE of conventional tuned UKF and Nature inspired Algorithms based tuned UKF. We can see that Hybrid GA-PSO based tuned UKF is giving Minimum RMS error. In case of Computational complexcity it is always trade of between computations and RMS error. This can be overcome by latest High speed Digital Signal Processors.

# 6. Conclusion

The paper presents tuning Procedure for UKF. A comparison was made between two on linear filtering algorithm standard UKF, nature inspired evolutionary algorithm based Tuned UKF for maneuvering target tracking. Since the measurement covariance can be determined in different environments, like off-line, we can get standard deviation for different conditions. Then, Hybrid GA-PSO Tuned UKF can be applied for fine tuning of noise covariance matrices. The results are shown for conventional tuned and Hybrid GA-PSO-tuned UKF and we can say Tuned filter gives better performance. Computation complexity can be overcome by Hi speed DSP processors available.

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