International Journal of Industrial Electronics and Control.
ISSN 0974-2220 Volume 5, Number 2 (2013), pp. 1-10

http://www.irphouse.com

# A Simplified Mathematical Model for Inductance Profile of Switched Reluctance Motor

**M.S. Naruka[1], D.S. Chauhan[2] and S.N. Singh[3]**

*[1]Department of Electrical Engineering, IEC-Group of Institutions, Greater Noida, UP, INDIA.*
*E-mail: msnaruka@rediffmail.com*
*[2]Uttarakhand Technical University, Uttarakhand, INDIA.*
*[3]Department of Electrical Engineering, IIMT Engineering College, Greater Noida, UP, INDIA.*

***Abstract***

*This paper presents a new analytical representation and simulation of the phase inductance of Switched Reluctance Motor (SRM) using Matlab environment. This simulation method has many advantages: it is free from complicated mathematical expressions, can be applied widely, and saves run time. In this simulation, we are using 6/4 SRM, moreover, this simulation method can be easily realize various SRM model. The detailed modeling process, computer simulation and test results are presented.*

*Keywords— SRM, Torque, constant current mode, pulse mode, linear model.*

## *I: INTRODUCTION*

The concept of Switched reluctance machines (SRM) was established in the third decade of $18^{th}$ century in the first locomotive developed by Davidson to propel a locomotive on the Glasgow-Edinburgh railway near Falkirk [1]. However, the full potential of the motor could not be utilized because only mechanical switches alone were available at that time. The interest in fast-acting power semiconductor switches was revived with the coming of SRMs in the year of 1970's when Professor Lawrenson's of University of Leeds and his group established the fundamental design and operating principles of the machine [2]. Nowadays an induction motor dominates in about 85% of industry work replacing DC motors. This paper presents a description of SRM and a simplified model for inductance profile of SRM.

A new practical and simple approach to model the SRM was developed based on three assumptions one operates in linear region of B-H curve, second the inductance offered by the winding will remains constant at its minimum value during the unaligned position and third the reluctance of iron is negligible with respect to air gap. This paper describes the inductance profile against rotor angle that can be implemented in various categories of models.

### *II: CHARACTERISTICS OF SRM*

SRMs is a doubly salient pole machine having salient poles at both the stator and the rotor. It is singly excited reluctance motor due to its characteristic of having only one member that carries winding on the stator side. The rotor is made of either stacked steel laminations or a solid piece of soft iron, and does not require magnets or winding [3]. Fig. 1 shows typical 3 phase 6/4 SRM. The SRM motion is produced because of variable reluctance in the air gap between the rotor and the stator.

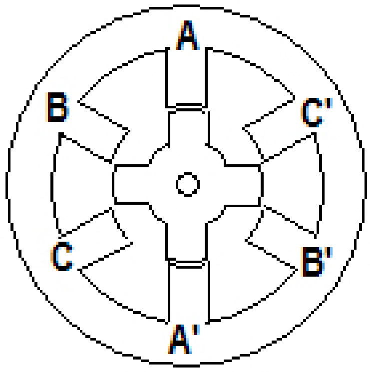


**Fig.1: 6/4 SRM with phases 'A', 'B', and 'C'.**

SRM drive has many advantages over DC & 3-phase AC drives specially in the field of transportation system. This emphasized the need of design, development and performance simulation of the machine with longer utilization in industry. For 6/4 SRM presented in this paper has 6 stator poles and 4 rotor poles. The number of phases are equal to the half the number of stator poles. When a current is applied to a stator phase winding, a magnetic field is created. A magnetic flow path appears around the stator, along the active phase poles, across the air gap, and along the rotor structure.

The principle of operation of SRM is based upon the fact that a piece of magnetic material is always tends to align itself in the minimum reluctance position when placed in a magnetic field [4]. When a rotor pole aligned with a stator pole, there is no torque because of the fact that the field lines are orthogonal to surfaces. In this position, the inductance is maximal since reluctance is minimal. Therefore SRM has been shown to offer highly efficient, reliable, robust and easy to manufacture. However there are some limitation with SRM also like its requires expensive position sensors and produces vibrations with acoustic noise as well as high torque ripples [5].

The operation of SRM is highly affected by the non linear characteristics of magnetic uses with high level of saturation occurring cyclically as the rotor continuously moves from unaligned to aligned positions with reference to energized stator phase. These poses two difficult tasks of first calculating the winding inductance in the machine design and second the mathematical representation for analysis and control purposes.

The mathematical model for inductance profile is developed for analysis of the machines. This helps in understanding the working, in the formulation of control strategies & in achieving the high accuracy in performance simulation.

***III: MODELING & PERFORMANCE CALCULATION***

The various investigators working in the field of modelling & performance calculation in SRM drive has presented the following four models [6, 7, 8, 9]:

(a) Linear model in which magnetic saturation is neglects and the characteristics is approximately the straight lines.
(b) Piece wise linear model in which the characteristic is approximate by two straight lines, one for linear region and other for saturation.
(c) Non-Linear model using functions which fit the characteristic in the whole plane.
(d) Tabulation method in which the use the data files which store the characteristic in the form of a table.

In the present work, the linear model has been considered for the development of inductance profile for switched reluctance motor.

In fig. 2 the schematic representation of SRM section showing one stator pole and rotor pole while rotor moves, it passes through a condition of unaligned with stator pole in region ($\theta$ < O) alignment with stator pole in region $\{0 < \theta < (\beta_s+\beta_r)\}$ and again a condition of unaligned in region $\{\theta > (\beta_s+\beta_r)\}$

As the rotor passes through the region of alignment, the phase inductance increases for ($\theta \leq \beta_s$), remains constant for ($\beta_s < \theta < \beta_r$) and decreases for $\{\beta_r < \theta < (\beta_s+\beta_r)\}$

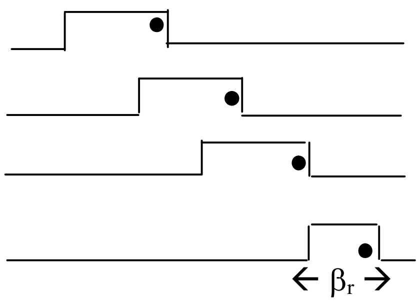


**Fig. 2: SRM developed view**

The following simplifying assumption has been made in analysis. First the motor operates in linear region of the B – H curve. Second the inductance of the winding in unaligned position remains constant at minimum value $L_{min}$ and starts rising at $\theta = 0$ and third is that the reluctance of iron is negligible with respect to the air-gap.

Let the area of overlap of stator & rotor poles is ‘A’ then based on the above assumptions, the instantaneous inductance of phase winding is obtained as

$$L = \frac{\phi}{i} = \frac{Ni}{Si} = \frac{N}{S} = \frac{N}{\frac{l}{\mu_0 A}} = \frac{\mu_0 AN}{l} \quad \text{-------------- (1)}$$

Where, S = Reluctance of air-gap
i = Instantaneous phase current
l = length of air gap
N = No. of turns per phase

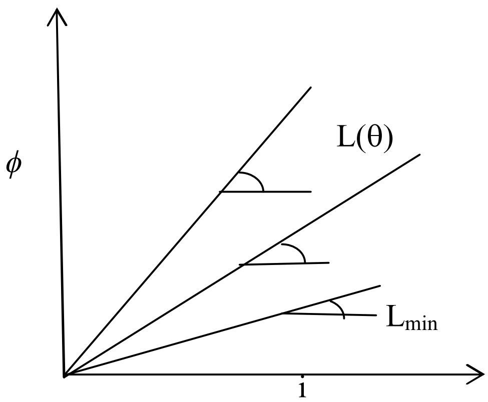


**Fig. 3: ϕ - i characteristics**

In a realistic machine, referring to Fig: 3, the inductance of motor phase winding can be calculated as follow:

(A) For $\theta \le 0$

In this region air-gap is max$^{m}$ and thus the inductance is min$^{m}$

The length of air gap $l$ = 2 (R-r) --------------- (2)

Area will be $A = \left( R - \frac{R-r}{2} \right) \beta r L$

Area $A = \left( \frac{R+r}{2} \right) \beta r L$ --------------------------- (3)

Where R: radius of stator bore
r: inner core radius of rotor
L: stack length

By combining equations (1), (2) & (3)

$$L(\theta) = L_{\min} = \frac{\mu_o AN}{l} = \mu_o \frac{\left[ \left( \frac{R+r}{2} \right) \beta r L \right] N}{2(R-r)}$$

$$= \frac{\mu_o \beta r L N (R+r)}{4(R-r)} \quad \text{-------------------- (4)}$$

(B) For $0 < \theta \leq \beta_s$

In this region $l = 2g$ , where g is the air gap length

$$A = (R - \frac{g}{2})\theta L$$

Hence $$L(\theta) = \frac{\mu o\left(R - \frac{g}{2}\right)\theta LN}{2g}$$

$$= \left[\frac{\mu o(2R - g)LN}{4g}\right]\theta \quad \text{--------------- (5)}$$

(c) For $\beta_s < \theta \leq \beta_r$

In this region poles are aligned and inductance in max$^{m}$ and is given by

$$L(\theta) = L_{\max} = \left[\frac{\mu o(2R - g)LN}{4g}\right]\beta_s \quad \text{----------------- (6)}$$

(d) For $\beta_r < \theta \leq (\beta_s+\beta_r)$

In this region inductance decreases linearly with θ.Angle of overlap in given by $\beta_s - (\theta - \beta_r) = (\beta_s+\beta_r) - \theta^0$ then equation

$$L(\theta) = \left[\frac{\mu o(2R - g)LN}{4g}\right](\beta s + \beta r - \theta) \quad \text{---------- (7)}$$

(e) For $\theta > (\beta_s+\beta_r)$

The inductance becomes constant at min$^{m}$ value

$L(\theta) = L_{min}$ --------------(8)

In summary

$$L(\theta) = \begin{cases} = \dfrac{\mu_o \beta r LN(R + r)}{4(R - r)}, & \theta \leq 0 \\ = \left[\dfrac{\mu o(2R - g)LN}{4g}\right]\theta, & 0 < \theta \leq \beta s \\ = \left[\dfrac{\mu o(2R - g)LN}{4g}\right]\beta_s, & \beta s < \theta \leq \beta r \\ = \left[\dfrac{\mu o(2R - g)LN}{4g}\right](\beta s + \beta r - \theta), & \beta r < \theta \leq (\beta s+\beta r) \\ = \dfrac{\mu_o \beta r LN(R + r)}{4(R - r)}, & \theta > (\beta s+\beta r) \end{cases}$$

-------------------------- (9)

This model can be represented by a set of $\lambda$ - i characteristic at different rotor positions as shown in fig 2. The slope of these lines given the instantaneous inductance.

*Calculation of Torque:* The expression for torque for a linear model of switched reluctance motor in given by

$$T(\theta,i) = \frac{1}{2} i^2 \frac{dL}{d\theta} \quad \text{----------------------- (10)}$$

Putting the values of L ($\theta$) from equation (9) in to eq (10)

$$T(\theta,i) = \frac{1}{2} i^2 \left| \begin{array}{ll} 0, & \theta \leq 0 \\ \left[ \dfrac{\mu o(2R-g)LN}{4g} \right], & 0 < \theta \leq \beta s \\ 0\ , & \beta s < \theta \leq \beta r \\ -\left[ \dfrac{\mu o(2R-g)LN}{4g} \right] & ,\beta r < \theta \leq (\beta s+\beta r) \\ 0, & \theta > (\beta s+\beta r) \end{array} \right. \quad \text{---------------------(11)}$$

Where i in the instantaneous value of current.

During steady state the current in allowed to flow in region $0 < \theta \leq \beta_s$ for forward motoring and in region $\beta_r < \theta \leq (\beta s+\beta r)$ (for reverse motoring operation). Since the motor operation in identical in both directions (if direction of angle measurement in same as the direction of rotation) only one region will be selected for further elaboration.

Selecting the interval $0 < \theta \leq \beta s$

$$T = \frac{1}{2} i^2 \frac{\mu o(2R-g)LN}{4g} \quad \text{----------------------------- (12)}$$

There may be two modes of operation of switched reluctance motor;

(A). Constant current mode: At low speed, the motional EMF in the motor winding is very small and current, if allowed to rise freely, will cross the rated value which is prevented by chopping the main switch. Therefore during this mode, the current can be approximated by a rectangular DC pulse of value $I_{rated}$ and eq. (12) changes to

$$T = \frac{1}{2} I^2{}_{rated} \frac{\mu o(2R-g)LN}{4g} \quad \text{----------------------- (13)}$$

(B). Single pulse mode: At higher speeds, the current will reach a peak due to the capacitor voltage and then will decrease to settle at some value near or below the rated and chopping is not required. This mode of operation is known as single pulse mode. But due to presence of the current sensor, main switch is turned OFF when current reaches peak and hence the decreasing current follows equation

$$i(t) = I_m e^{-(t/\tau)} \quad \text{---------------------} \ (14)$$

Where $\tau = \dfrac{L(\theta)}{R_x}$

$$R_x = R + \left(\frac{dL}{d\theta}\right)w$$

$$L(\theta) = L_{\min} + \left(\frac{dL}{d\theta}\right)\theta$$

Changing eq (14) in terms of θ i.e. replacing t by θ/w

$$i(\theta) = I_m e^{-(\theta / w\tau)}$$

$$= I_m e^{\left[\dfrac{-\theta R_x}{w\left(L_{\min} + \dfrac{dL}{d\theta}\theta\right)}\right]} \quad \ldots\ldots\ldots\ldots \ (15)$$

Putting

$$I_m = \sqrt{\frac{c}{L_{\min}}}\sqrt{V_m(V_m + V)}\left\{1 - \frac{1}{2Q}\cos^{-1}\left[\frac{1}{1 + \dfrac{V_m}{V}}\right]\right\}$$

Where Q= w $L_{min}/R_x$

In eq. (15) and then putting the value of instantaneous current in eq. (12), the expression for torque becomes.

$$T = \frac{\mu_0(2R-g)LN}{2X4g}\left[\frac{V_m C(V_m+V)}{L_{\min}}\right]\left\{1 - \frac{1}{2Q}\cos^{-1}\frac{1}{1+V_m/V}\right\}^2 e^{\left[\dfrac{-2\theta R_x}{\left(L_{\min}+\dfrac{dL}{d\theta}\theta\right)w}\right]} \quad \text{-----} \ (16)$$

Equation (17) holds for a very short duration till current comes down to $I_{set}$ and then equation of the current will be changed to

$$i(t) = \frac{V}{Rx} + \left[I_{set} - \frac{V}{R_x}\right]e^{-(t/\tau)} \quad \text{------------------------}(17)$$

Which is modified in terms of angle $\theta$ as

$$i(\theta) = \frac{V}{Rx} + \left[ I_{set} - \frac{V}{R_x} \right] e^{-(\theta/_{w\tau})} \text{ ---------------------- (18)}$$

Putting the value of $R_x = R + \frac{dL(\theta)}{d\theta} w$ and $\tau = \frac{L(\theta)}{Rx} = \frac{L(\theta)}{(R + (\frac{dL(\theta)}{d\theta})w}$

Equation (18) becomes

$$i(\theta) = \frac{V}{(R + \frac{dL(\theta)}{d\theta} w)} + \left[ Iset - \frac{V}{R + \frac{dL(\theta)}{d\theta} w} \right] e^{-\left\{ \frac{(R + \frac{dL(\theta)}{d\theta} w)\theta}{wL(\theta)} \right\}} \text{ ---------------------- (19)}$$

At higher speeds R <<< $\frac{dL(\theta)}{d\theta}$ therefore putting $K_L$ to $\frac{dL}{d\theta}$ in equation (19)

$$i(\theta) = \frac{V}{wK_L} + \left[ Iset - \frac{V}{wK_L} \right] e^{-\left\{ K_L \frac{\theta}{L(\theta)} \right\}} \text{ ------------ (20)}$$

Combining equation (13) & (20)

$$T = \left\{ \frac{\mu o(2R - g)LN}{8g} \right\} \left[ I_{set} e^{\frac{K_L\theta}{L(\theta)}} + \frac{V}{K_L w} (1 - e^{\frac{K_L \theta}{L(\theta)}}) \right]^2 \text{ ------------------------- (21)}$$

Eq. 21 holds for the maximum period therefore dictates the nature of characteristics of the drive. For the purpose of calculation of torque at different instants to study its variation with rotor angle, and that with speed, flow chart can be made & used for computer programming.

## *IV: MATLAB SIMULATION & RESULT*

The proposed mathematical modeling of inductance profile is implemented in MATLAB environment. The output of MATLAB programming is shown in Fig. 5. It is similar to the actual inductance profile of the switched reluctance motor shown in Fig. 4.

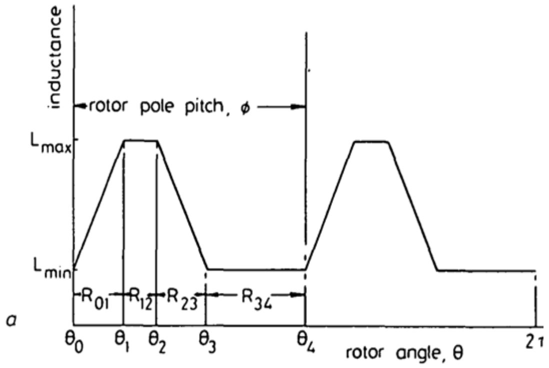


**Fig. 4: Actual view of theoritical inductance profile of SRM**

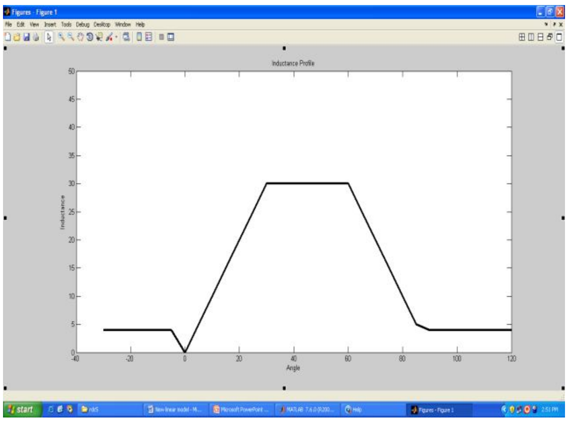

**Fig. 5: Simulated inductance profile of SRM**

### *V. CONCLUSION*

A new linear model discussed in previous section has been simulated using MATLAB implementation is presented in this paper. Validity of results obtained through MATLAB simulation is found in agreement with results computed from the analytical expressions gives a fairly good estimation of its actual values. Flow diagrams are used for computer programming. We consider this simulation method would be a mile stone for analyzing and developing design program tools of SRM.

**REFERENCES:**

**[1]** J. Bartos Frank, *"Forward to the past with SR Technology"*, Control Engineering International,Nov/Dec 1999,

**[2]** T.J.E Miller, *"Optimal design of Switched Reluctant Motors"*,IEEE Trans. Ind. Electronics, Vol.49, no.1, Feb. 2002, pp-15-27.

**[3]** T.J.E Miller, *"Electronic control of Switched Reluctant Machines"*, Bostan: Newnes, 2001.

**[4]** A Fitzgerald, C Kingsley, S Usmans, *"Electric Machinery"*, Newyork: McGraw Hill, 1990.

**[5]** Roux Christophe, M.M Morcos, *"A simple Model for Switched Reluctant Motors"*, IEEE Power Engineering Review, vol. 20, issue 10, Oct. 2000, page(s):49.

**[6]** P.N.Materu, & R.Krishnan,*"Steady state analysis of the variable speed switched reluctance motor drives"* IEEE Trans.on Industrial Electronics; vol 36, No-4, nov 1989 ,pp 523-529.

**[7]**. S.Giuseppe Buja, & I Maria Valla., *"Control characteristics of the SRM drive, Part-I: Operation in linear region"*IEEE on industrial Electronics, Vol-38, No-5, Oct 1991 pp 313-321.

**[8].** Silverio Bolognani; S.Giuseppe Buja, & I Maria Valla,., *"Switched reluctance motor performance analysis Based on An improved modeling of its magnetic characteristics"*, Electric Machines & Power Systems, 19:425-438, 1991.

**[9].** *R.* Arumugam, J.F. Lindsay, & R. Krishnan,*"A comparison of the performance of two different types of switched reluctance motors"*, Electric Machines Power Systems, 12, 1987, pp 281-286,.