Selection Procedure of Bayesian One Suspension Plan

M. Latha* and S. Jeyabharathi*

*Associate Professor, Government Arts College, Udumalpet-642126, Tamil Nadu, India **Ph.D Research Scholar, Karpagam University, Eachanari Post, Coimbatore-641021, Tamilnadu sanowjey100@yahoo.com)

ABSTRACT

This paper concerned with the selection of suitable proportion defective values for Bayesian One suspension plan with Beta binomial model. Tables also constructed for the utility values of Beta Binomial model and the single sampling plan considered as a base plan with c = 1, which was compared to c=0 plan and also compared to the conventional sampling plan.

KEY WORDS Bayesian Acceptance Sampling, One Suspension Plan, Beta binomial distribution.

INTRODUCTION

A Suspension rule is a procedure used to decide when to suspend inspection of a production process, where product is submitted for inspection in lots. The decision to suspend is based on the observed sequence of lot acceptances and rejections. A suspension rule, which is designated (j, k), $2 \le j \le k$ is a rule of suspending inspection based on finding 'j' lot rejections in k or less lots. Given j and k, atleast j lots must be inspected before a decion is possible upon the beginning of a new process or from the last suspension. A suspension system is a combination of suspension rule and a single lot-by-lot sampling plan or pair of plans. When a single sampling is used with a suspension rule is called One Suspension Plan system.

Cone and Dodge (1962) have first shown the effectiveness of a small sample lot-by-lot sampling system can be greatly improved by using cumulative results as a basis for suspending inspection. Troxell (1972) has applied the

suspension principle to acceptance sampling system to suspend inspection on the basis of unfavorable lot history, when small sampling plans are necessary are desirable. Latha (2002) has student the suspension Bayesian one plan rule for single sampling plan with C = 0.

CONDITIONS FOR APPLICATION OF ONE SUSPENSION PLAN

- 1. Production process is steady, so that the results on current and preceding lots are broadly indicative of a continuous process.
- 2. Samples are taken from lots in the order of their production.
- 3. Inspection is performed close to the production source.
- 4. Inspection is based on attributes, with quality measured in terms of fraction defective.
- 5. A single sample of size n is taken from each sampled lot.

OPERATING PROCEDURE

- 1. For the product under consideration establish a reference Quality Levels, (RQL). The RQL represents the desired quality at delivery considering the needs of service and cost of production.
- 2. Consider the established RQL, select a suspension system.
- 3. Apply the suspension rule to the first, second ...kth lot then to each successive group of k lots.
- 4. If any lot is rejected, declare the lot nonconforming and dispose it in accordance with standard procedures.
- 5. If any lot, the suspension rule occurs, declare the current lot nonconforming and also declare the process nonconforming.

AVERAGE RUN LENGTH (ARL)

According to Troxell (1972), the average run length of the suspension rule (j, k) designated ARL (j, k) can be calculated in the following manner.

ARL (j,k) = (Total number of inspected lots between two rejections)× (Expected number of rejections until suspension)

Troxell (1972) has suggested that, if the two reference. Quality levels RQL 1 and RQL 2 are too restrictive, RQL_2/RQL_1 is too low, and the sampling plan procedure in this case is to increase the acceptance number by $C \ge 1$. The present work is the study of Bayesian one suspension plan with C=1.

ONE PLAN SUSPENSION SYSTEM, WITH BAYESIAN SINGLE SAMPLING PLAN WITH C = 1

Based on Hald (1981), the APA function of Beta binomial model is given as

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$$\bar{\mathbf{P}} = \sum_{x=0}^{c=0} (s+x-1)C_{s-1} (t+n+x-1)C_{t-1}$$

$$(s+t+n+2-1)C_{s+t-1}$$

with parameters s and t and mean $\mu = \frac{s}{s+t}$, when c=1 the above equation is reduced as

$$P = (t + n - 1)C_{t-1} + S(t + n)C_{t-1}$$

$$(s + t + n + 1)C_{(s+t-1)}$$

When s = 1, equation (2) reduces to

$$\overline{\mathbf{P}} = \frac{(1-\mu)}{1+\mu(n-1)} + \frac{(n+2)\mu(1-\mu)}{[1+\mu(n+1)][1+n\mu]}$$

When s = 2,

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$$\overline{\mathbf{P}} = \frac{(2-\mu)(2-2\mu)}{[2+\mu(n-2)][2+\mu(n-1)]} + \frac{2(n+2)\mu(2-\mu)(2-2\mu)}{[2+\mu(n+1)][2+n\mu][2+\mu(n-1)]}$$

When s = 3,

$$\overline{\mathbf{P}} = \frac{(\mathbf{3}-\mu)(\mathbf{3}-2\mu)(\mathbf{3}-3\mu)}{[\mathbf{3}+\mu(n-1)][\mathbf{3}+\mu(n-2)][\mathbf{3}+\mu(n-3)]} + \frac{\mathbf{3}(n+2)\mu(\mathbf{3}-\mu)(\mathbf{3}-2\mu)(\mathbf{3}-3\mu)}{[\mathbf{3}+\mu(n+1)][\mathbf{3}+n\mu][\mathbf{3}+\mu(n-1)][\mathbf{3}+\mu(n-2)]}$$

When s = r ,equation 1.2 reduces to

$$\overline{\mathbf{P}} = \frac{(r-r\mu)[r-(r-1)\mu]_{m-m}(r-\mu)}{[r+\mu(n-1)][r+\mu(n-2)]_{m-}[r+\mu(n-r)]} + \frac{(n+2)(r\mu)(r-r\mu)[(r-(r-1)\mu][r-(r-2)\mu]_{m-}(r-\mu)}{[r+\mu(n+1)][r+n\mu][r+(n-1)\mu]_{m-}[r+\mu[n(r-1)]]}$$

Equation 2 is equated to the Troxell Table values of probability of acceptance for given n, ARL (j,k) and s through which μ , the average value of product quality, is found out which are listed in Figure.1 and Figure.2

ARL 5 10 50 100 200 2 0.5710 0.4292 0.2766 0.2434 0.2198 3 0.4789 0.3389 0.2006 0.1717 0.1512 4 0.4125 0.2795 0.1560 0.1309 0.1132 5 0.3622 0.2376 0.1270 0.1050 0.0896 6 0.3228 0.2065 0.1069 0.0874 0.0737 7 0.2911 0.1825 0.0921 0.0746 0.0624 8 0.2651 0.1635 0.0808 0.0650 0.0539 9 0.2433 0.1480 0.0720 0.0575 0.0474 10 0.2248 0.1352 0.0648 0.0516 0.0423 11 0.2090 0.1245 0.0590 0.0467 0.0381 12 0.1952 0.1153 0.0541 0.0427 0.0347 13 0.1831 0.1074 0.0499 0.0363 0.0294			r		r	
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10 0.2248 0.1352 0.0648 0.0516 0.0423 11 0.2090 0.1245 0.0590 0.0467 0.0381 12 0.1952 0.1153 0.0541 0.0427 0.0347 13 0.1831 0.1074 0.0499 0.0392 0.0318 14 0.1724 0.1004 0.0463 0.0363 0.0294	0	0.2031	0.1055	0.0808	0.0050	0.0559
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11 0.2090 0.1245 0.0590 0.0467 0.0381 12 0.1952 0.1153 0.0541 0.0427 0.0347 13 0.1831 0.1074 0.0499 0.0392 0.0318 14 0.1724 0.1004 0.0463 0.0363 0.0294	10	0 2248	0 1252	0.0619	0.0516	0.0422
12 0.1952 0.1153 0.0541 0.0427 0.0347 13 0.1831 0.1074 0.0499 0.0392 0.0318 14 0.1724 0.1004 0.0463 0.0363 0.0294	10	0.2248	0.1552	0.0048	0.0310	0.0425
12 0.1952 0.1153 0.0541 0.0427 0.0347 13 0.1831 0.1074 0.0499 0.0392 0.0318 14 0.1724 0.1004 0.0463 0.0363 0.0294	11	0.000	0.1045	0.0500	0.0467	0.0201
13 0.1831 0.1074 0.0499 0.0392 0.0318 14 0.1724 0.1004 0.0463 0.0363 0.0294	11	0.2090	0.1245	0.0590	0.0467	0.0381
13 0.1831 0.1074 0.0499 0.0392 0.0318 14 0.1724 0.1004 0.0463 0.0363 0.0294	10	0.1050	0.1150	0.0541	0.0405	0.00.15
14 0.1724 0.1004 0.0463 0.0363 0.0294	12	0.1952	0.1153	0.0541	0.0427	0.0347
14 0.1724 0.1004 0.0463 0.0363 0.0294						
	13	0.1831	0.1074	0.0499	0.0392	0.0318
15 0.1629 0.0944 0.0432 0.0338 0.0273	14	0.1724	0.1004	0.0463	0.0363	0.0294
15 0.1629 0.0944 0.0432 0.0338 0.0273						
	15	0.1629	0.0944	0.0432	0.0338	0.0273

Figure.1 VALUES OF μ FOR GIVEN n and ARL s=1 ARL (2, 2)

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K			n	n	
ARL	5	10	50	100	200
n 🔪					
2	0.5069	0.3779	0.2494	.2228	0.2044
3	0.4140	0.2911	0.1769	0.1539	0.1379
4	0.3498	0.2360	0.1354	0.1155	0.1016
5	0.3027	0.1982	0.1090	0.0916	0.0794
6	0.2667	0.1706	0.0909	0.0754	0.0647
7	0.2383	0.1497	0.0777	0.0639	0.0543
,	0.2000	0.1177	0.0777	0.0000	0100 10
8	0.2153	0.1333	0.0678	0.0554	0.0467
Ũ	0.2100	011000	0.0070	0.0000	0.0107
9	0.1964	0.1201	0.0601	0.0487	0.0408
,	0.1701	0.1201	0.0001	0.0107	0.0100
10	0.1805	0.1093	0.0539	0.0435	0.0362
10	0.1005	0.1075	0.0557	0.0155	0.0502
11	0.1670	0.1002	0.0489	0.0392	0.0325
11	0.1070	0.1002	0.0-07	0.0372	0.0525
12	0.1554	0.0925	0.0447	0.0357	0.0357
12	0.1554	0.0925	0.0447	0.0337	0.0337
13	0.1452	0.0860	0.0411	0.0328	0.0270
15	0.1432	0.0800	0.0411	0.0528	0.0270
1.4	0.1262	0.0002	0.0201	0.0202	0.0249
14	0.1363	0.0802	0.0381	0.0303	0.0248
1.7	0.1005	0.0752	0.0255	0.0201	0.0000
15	0.1285	0.0752	0.0355	0.0281	0.0230

Figure.2 .VALUES OF μ FOR GIVEN n and ARL s=2 ARL (2, 2)

RESULTS

Example: 1

From the Figure.1, for the given ARL (2, 2) =10, n=10, c=1and for s=1, the value of μ is given by μ =0.1352 where as for c=0, μ =0.055508 and in conventional sampling the fraction defective is given by p=0.04518.

Example: 2

From the Figure.2 for the given ARL (2, 2) =50, n=9, c=1and for s=2, the value of μ is given by μ =0.0601 where as for c=0, μ =0.018794 and in conventional sampling the fraction defective is given by p= 0.01812.

CONCLUSION

Bayesian Acceptance Sampling is a technique which deals with the procedures in which decision to accept or reject the lot or process is based on the examination of past history or knowledge of samples. The present work is concerned with the selection of suitable proportion defective values for Bayesian One suspension plan using Beta binomial model with c=1 and which are compared to the c=0 and with conventional sampling plan. It is observed that, compared to c=0 and conventional sampling plan, c=1 will be more advantageous to the producer.

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