

Inventory Cost Optimization for Cement Bag's Vendor and n-Retailers by Applying Centralized Model

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Abstract

This paper concerns with a two-level inventory system with one-supplier, one-warehouse and n-retailers. The retailers are replenished from the warehouse. The problem consists of determining the optimal policy which minimizes the overall cost, that is, the sum of the replenishment costs, carrying cost and transportation costs. We study two situations: when the retailers make decisions independently and when the retailers are branches of the same firm. Solution methods to determine near-optimal policies in both cases are provided. Computational results on a live problem are reported.

Keywords: Total Inventory Cost, Centralized and Decentralized Models, Warehouse-Retailer System.

1. Introduction

The goal of many research efforts related to supply chain management is to propose mechanisms to reduce operational costs. Inventory holding and transportation costs are regarded as the most important operational costs in inventory management. Many researches in supply chain management only consider the inventory cost as a criterion to decide replenishment policy. In the replenishment process, beside the inventory cost, the transportation cost is a major cost factor which affects the shipment size. Thus in this research work the transportation cost is also considered to minimize the inventory cost.

This paper deals with a joint replenishment arrangement with a two-level inventory system having one supplier, one warehouse and multi buyer or retailer, facing a deterministic demand and selling a single product in the marketplace. We considered a situation involving a family of Wonder cement production at Nimbahera (Rajasthan), distribution warehouse at Neemuch (MP) and selling at Mandsaur (MP). All the relevant data collected from the warehouse and from a dealer of this field. There is a replenishment cost, carrying cost and transportation cost for warehouse as well as retailers due to economies of scale in transportation and distribution expenses.

In such a setting, it seemed as if it would be economically beneficial for both parties to enter into a joint replenishment arrangement. We proposed both centralized and decentralized decision models to determine the best solution to minimize costs. We proved the optimal properties of the models and numerically illustrated the benefits generated from such an arrangement. In addition, we have compared both the models to identify the best policy to minimize the total inventory cost for warehouse as well as for retailer.

2. Literature Review

Distribution/inventory systems consisting of one warehouse and N retailers are encountered frequently in practice, and several works in the literature have been devoted to these systems, see e.g. Graves and Schwarz (1977) [6], Williams (1981, 1983) [7][8], Roundy (1985) [10], Abdul-Jalbar et al. (2002, 2003) [4][1], Chen and Chen (2005) [5] and Armand Baboli et.al. (2008) [2].

Lei Zhao et.al. (2011) has proposed two transportation modes: a slow mode with low cost and long and stochastic lead time, and a fast mode with high cost and short and deterministic lead time. The manufacturer is subjected to a credit constraint that bounds both the in warehouse inventory and the number of outstanding orders [9].

Srinivasa Rao Y et.al. (2012) has proposed a two echelon supply chain system which consisting of one manufacturer and multiple retailers and a coordination model which maximizes the total profit is developed and analyzed for deteriorating item [11].

Alexandre Vieira Braga et.al. (2012) has aimed to examine issues related to the management of inventory, working capital, pricing process and costs in companies operating in the fruit canning segment in the Brazilian state of Rio Grande do Sul. Data analysis involved the use of descriptive statistical and correlation techniques between variables [3].

3. Model and Notations

In this paper we consider a two-level inventory system consisting of one warehouse and twelve retailers. We obtain the replenishment cost, carrying cost and transportation cost for all retailers and warehouse in two different cases; centralized and decentralized.

Assumptions made for the analysis are as follows:

- Demand rate is constant for the retailer.
- Ordering cost is fixed per order for a certain amount.
- Transportation cost is considered from supplier to warehouse and from warehouse to retailer separately.
- Shortage is allowed neither at the warehouse nor at the retailer.
- The lead time for an order to arrive at the retailer from warehouse is constant.
- The lead time for the warehouse is constant.

The main objective in both centralized and decentralized inventory model is:

$$\text{Minimize } TC = TC_r + TC_w \dots\dots \quad (1)$$

Where TC_r is the total optimal cost for retailer and TC_w is the total cost for the warehouse.

Table 1: Notations used in models

Sr. No.	Notations	Meaning
1	N	number of retailers (12 in our model)
2	R	retailer index ($r = 1, 2, \dots, 12$)
3	ρ	density of retailers (retailers per square km)
4	Q_r	replenishment order quantity in units of retailer r
5	Q_R	sum of all replenishment order quantities from retailer 1 to 12
6	F_r, F_w	fixed cost of order at retailer and warehouse
7	V_r, V_w	variable cost of order at retailer and warehouse
8	D_r	demand quantity of retailer
9	d_{wr}, d_{sw}	travelling distance from warehouse to retailer and from supplier to warehouse (km)
10	h_r, h_w	carrying charge in % at retailer r and at the warehouse
11	L_{wr}, L_{sw}	lead-time from warehouse to retailer and supplier to warehouse
12	S	truck capacity (bags)
13	I_w	Interval between two orders in the warehouse
14	T_w, T_s	fixed cost of transportation from warehouse to retailer and supplier to warehouse
15	t_w, t_s	variable cost of transportation from warehouse to retailer and supplier to warehouse

4. Decentralized Case

In the decentralized case, the retailer and the warehouse intend to optimize their own costs independently. Demand rate at the retailer and transportation time to the retailer is supposed to be constant. Thus to compute the total cost of the warehouse and each retailer, we used following methods:

Retailer's Model: In this model we consider the case that each retailer determines its own Economic Order Quantity and optimal cost. We assume that the retailer's costs include carrying cost (C_{Cr}), cost of replenishment (C_{Rr}), and transportation costs (C_{Tr}). The total cost to each retailer is defined as follows:

$$TC_r = C_{Rr} + C_{Cr} + C_{Tr}$$

here C_{Rr} , C_{Cr} and C_{Tr} can be determined as follows:

$$C_{Rr} = F_r \times D_r / Q_r$$

$$C_{Cr} = (Q_r/2 + \sigma_r L_{wr}) V_r h_r \text{ [here, } \sigma_r L_{wr} = \sqrt{(L_{wr} \times \sigma_r^2)}]$$

$$C_{Tr} = [T_w + t_w(Q_r/S)d_{wr}] \times (D_r / Q_r)$$

Warehouse Model: In this case we search for the optimal strategy of the warehouse (i.e., how often to place orders) that will minimize the costs of total inventory cost. Here the total cost is comprised of replenishment cost, carrying cost and transportation cost. Hence, the total cost of the warehouse can be defined as follows:

$$TC_w = C_{Rw} + C_{Cw} + C_{Tw}$$

here, C_{Rw} , C_{Cw} and C_{Tw} can be determined as follows:

$$C_{Rw} = F_w \times \text{No. of orders}$$

$$C_{Cw} = [\mu_w (I_w + L_{sw})/2 + \sigma_w (I_w + L_{sw})] \times V_w \cdot h_w \text{ [here, } \sigma_w (I_w + L_{sw}) = \sqrt{(I_w + L_{sw}) \times \sigma_w^2}]$$

$$C_{Tw} = T_s + t_s(\mu_w \cdot I_w/S)d_{sw} \times (\sum_{r=1}^N D_r / Q_r)$$

5. Centralized Case

In the centralized case the objective is to find the optimal cost for both the retailer and the warehouse. In this case, the warehouse and the retailers belong to the same firm. Therefore, the firm should pay all the costs, and the goal is to minimize (1), that is, the cost at the warehouse plus the costs at the retailers.

Therefore, the warehouse and the retailers must optimize their decision variables in a way to reduce the total cost of the system and should place their orders at the same time.

Retailer's Model: In this model we consider the case that all retailer determines its own Economic Order Quantity and optimal cost and place the orders jointly. The combined total cost for all retailers is defined as follows:

$$TC_r = C_{Rr} + C_{Cr} + C_{Tr}$$

here C_{Rr} , C_{Cr} and C_{Tr} can be determined as follows:

$$C_{Rr} = (F_r / Q_r) \times \sum_{r=1}^N D_r$$

$$C_{Cr} = (Q_r/2 + \sigma_r L_{wr}) V_r h_r \quad [\text{here, } \sigma_r L_{wr} = \sqrt{(L_{wr} \times \sigma_r^2)}]$$

$$C_{Tr} = [T_w + t_w(Q_r/S)d_{wr} + N\sqrt{(1/\rho) / (Q_r/S)}] \times (\sum_{r=1}^N D_r / Q_r)$$

Warehouse Model: The cost of warehouse in centralised model can be determined by:

$$T_{Cw} = C_{Rw} + C_{Cw} + C_{Tw}$$

here C_{Rw} , C_{Cw} and C_{Tw} can be determined as follows:

$$C_{Rw} = (F_w / Q_r) \times \sum_{r=1}^N D_r$$

$$C_{Cw} = (Q_r/2 + \sigma_w L_{sw}) V_w h_w \quad [\text{here, } \sigma_w L_{sw} = \sqrt{(L_{sw} \times \sigma_w^2)}]$$

$$C_{Tw} = [T_s + t_s(Q_r/S)d_{sw}] \times (\sum_{r=1}^N D_r / Q_r)$$

6. Numerical Analysis

We considered a situation involving a family of Wonder cement production at Nimbahera (Rajasthan), distribution warehouse at Neemuch (MP) and selling at Mandsaur (MP). All the relevant data like fixed costs, variable costs, lead time, carrying charges and travelling distances from retailers to warehouse and from warehouse to supplier etc are collected from the warehouse and from a dealer of this field in Mandsaur (MP). All these relevant data are put into the discussed model and the output costs of retailers and warehouse for each model are shown in the following tables.

Table 2. Model Evaluation of decentralized retailers

R	F _r	V _r	L _{wr}	S	T _w	t _w	D _r	Q _r	d _{wr}	C _{Rr}	C _{Cr}	C _{Tr}	TC _r
1	220	150	0.03	320	1000	180	1000	100	52	2200.00	23701.88	39250.00	65151.88
2	220	150	0.03	320	1000	180	700	105	53	1466.67	24076.88	27535.42	53078.96
3	220	150	0.03	320	1000	180	800	110	50	1600.00	24451.88	29772.73	55824.60
4	220	150	0.03	320	1000	180	1050	100	54	2310.00	23701.88	42393.75	68405.63
5	220	150	0.03	320	1000	180	750	95	52	1736.84	23326.88	29832.24	54895.95
6	220	150	0.03	320	1000	180	900	110	51	1800.00	24451.88	34000.57	60252.44
7	220	150	0.03	320	1000	180	800	90	55	1955.56	22951.88	33638.89	58546.32
8	220	150	0.03	320	1000	180	850	100	53	1870.00	23701.88	33840.63	59412.50
9	220	150	0.03	320	1000	180	900	90	52	2200.00	22951.88	36325.00	61476.88
10	220	150	0.03	320	1000	180	950	110	54	1900.00	24451.88	37492.61	63844.49
11	220	150	0.03	320	1000	180	800	105	51	1676.19	24076.88	30569.00	56322.11
12	220	150	0.03	320	1000	180	700	95	52	1621.05	23326.88	27843.42	52791.35
Total			-				10200	1210	-	22336.31	285172.56	402494.26	710003.11

Table 3. Model Evaluation of decentralized warehouse

F_w	V_w	S	L_{sw}	I_w	T_s	t_s	C_{Rw}	C_{Cw}	C_{Tw}
200	110	560	.03	.67	1500	160	1800	402640.66	422640.00

Table 4. Model Evaluation of centralized retailers.

F_r	V_r	S	L_{wr}	T_w	t_w	C_{Rr}	C_{Cr}	C_{Tr}
2640	150	600	.06	2500	280	22254.55	236796.56	397136.81

Table 5. Model Evaluation of centralized warehouse.

F_w	V_w	S	L_{sw}	P	T_s	t_s	C_{Rw}	C_{Cw}	C_{Tw}
600	500	700	.06	5	3000	350	5057.85	315728.75	152789.26

7. Computational Results

To build a better understanding of performance differences, we make table 6, 7 & 8 to compare the total cost of decentralized ordering model with the centralized ordering model and display the improvements.

Table 6: Cost table for decentralized model

Decentralized Cost	C_R	C_C	C_T	T_C
Retailer	22336.31	285172.56	402494.26	710003.11
Warehouse	1800.00	402640.66	422640.00	827080.66
Total	24136.31	687813.22	825134.26	1537083.79

Table 7: Cost table for centralized model

Centralized Cost	C_R	C_C	C_T	T_C
Retailer	22254.55	236796.56	397136.81	656187.92
Warehouse	5057.85	315728.75	152789.26	473575.86
Total	27312.4	552525.31	549926.07	1129763.78

Table 8: Cost difference table

Model	C_R	C_C	C_T	TC
Decentralized	24136.31	687813.22	825134.26	1537083.79
Centralized	27312.40	552525.31	549926.07	1129763.78
Difference	3176.09	135287.91	275208.19	407320.01

As shown, the effects of using the centralized ordering model in our inventory model immediately become noticeable. The difference between the total cost of decentralized and centralized ordering model is 407320 Rs.

8. Conclusion and Final Remark

We address the single-vendor multi-buyer system in which the vendor supplies an item to several buyers at a finite production rate. We have focused our attention on the decentralized and the centralized cases. We have implemented both the procedures and the computational results show that centralized ordering cost results in reduced total cost when compared to the decentralized ordering strategy. It can also be concluded that centralized model is more beneficial for warehouse in terms of money if we compare with the retailers.

Further directions for research may be focused on the one-warehouse and N-retailer system but considering shortages and variable lead times at the warehouse and/or at the retailers. Another relevant aspect of these systems consists of analyzing inventory policies for more general inventory/distribution systems with several warehouses and retailers.

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