

Analytical Models for Variability Management in Manufacturing Supply Chain

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Abstract

To respond to rapid changes in customer demands, many manufacturers have adopted the Build-to-Order manufacturing strategy, which inherently requires to manage variabilities arising across supply chains. Manufacturers would fail to deliver quality service to their customers unless they effectively manage the inventory of various parts. In this paper, we propose several analytical models, which take into account this variability in Build-to-Order manufacturing environment, and show that how they can be used to manage it to achieve the desired service to their customers.

Keywords: Build-to-Order, Brownian Motions, Supply Chain Management

1. Introduction

Inspired by the success of Dell Computers, Build-to-Order (BTO) has become a powerful and growing trend in the electronics industry. One of the competitive advantages from BTO is product variety – being able to offer each customer his own personalized product. The goal of BTO is to achieve personalization at mass-production prices by eliminating finished goods inventory. But personalization means unpredictable variability in demand spread over a product mix as diverse as the market itself. The challenge of BTO is to manage that variability without the traditional cushion of finished goods inventory [1, 2, 3, 4, 5, 6, 7, 8].

In this paper, we propose analytical models for rationally managing the variability in supply chains inherent in the BTO model. We develop analytical models of managing a single part. This work extends successful approaches from financial management to the problem of managing variability in demand and order releases. In addition to providing sound strategies for rationally managing demand variability, this work also provides analytical models of the relationships between the variance in demand for a

part and the total cost of supplying it.

Manufacturers manage their parts supplies by holding a very small inventory and meet varying demands by changing the release quantities frequently. Harrison *et al.* [9] showed that a control band policy is optimal for the Stochastic Cash Management Problem. This form of policy is described by three parameters $\{r, R, M\}$ with $0 < r \leq R < M$. In their paper, when the cash fund falls to 0, the controller sells off assets to return it to r . When the cash fund rises to M , the maximum allowed, the controller invests the excess causing the balance to fall to R .

In translating this problem to one of managing release variability, we introduce two additional parameters: m representing the minimum inventory level and \bar{M} representing a physical limit on the maximum inventory level allowed. The minimum level protects the manufacturer from the disruptions to his operations that arise when parts are not available. In Section 5, we address the question of setting the minimum level m . The maximum inventory level \bar{M} represents a limit on the inventory the manufacturer can accommodate due to space or budget constraints.

In this setting, a control band policy is described by three parameters $\{r, R, M\}$ with $0 \leq m < r \leq R < M \leq \bar{M}$. When inventory threatens to fall below m , the minimum allowed, the manufacturer increases the release quantity to return it to r . When inventory rises to M , the maximum allowed, the manufacturer reduces the release quantity causing the inventory to fall to R . These parameters determine the trade-off between the disruptions to the supply chain caused by large and small order releases and the costs of inventory and space required to avoid them. In Section 3, we discuss models and techniques for making these trade-offs intelligently.

We first describe in Section 2, a discrete, stochastic inventory control model that can be used to manage a single part. In Section 3, we propose to solve a corresponding Brownian control problem. The Brownian model loses some of the fine structure of the discrete model as it only considers the mean and variance of the process. On the other hand, the fact that the Brownian model only requires the mean and variance parameters is attractive in practice, since detailed distributional information about the demand process is either not available or difficult to collect. Although the Brownian control problem is generally easier to work with, its solutions do not directly provide solutions to our original discrete control problem. In Section 4, we propose to derive implementable policies for the discrete model from our solutions to the Brownian problem. We complete the paper in Section 5 with some concluding remarks.

2. Discrete Inventory Control Model

We consider the following discrete time inventory control model. The manufacturer makes a single product from a single part. Furthermore, the manufacturer's production facility has limited capacity, and the assembly time is negligible. Customer demands for the manufacturer's product are random. We use D_t to denote the demand at the beginning of period t . This demand is fulfilled at the end of the period. For ease of exposition, we assume that the demand process $\{D_t : t = 1, 2, \dots\}$ is an independent, identically distributed sequence with mean μ and variance σ^2 .

The manufacturer manages a single supplier of the part. To ensure that it does not run out, the manufacturer specifies a minimum inventory level $m \geq 0$ for the part and strives

to maintain at least that amount on site. In Section 5, we discuss several approaches to setting this minimum inventory level. Since customer demand is random, the inventory level at the manufacturer's site will on rare occasions drop below the minimum level. These rare events may be brought on by temporary disruptions in the supplier's production or in transportation. In this section, we assume such disruptions will never occur or, if they do occur, that by expediting shipments or calling on alternative sources, the manufacturer can obtain the required parts before the end of the period. Since the costs of shutting down an assembly plant are so high, manufacturers and their suppliers go to great lengths to avoid it – including flying in parts from a competitor.

In addition to the minimum, the manufacturer also specifies a maximum amount of inventory it will allow for the part and strives to keep its inventory between these minimum and maximum values. If the inventory threatens to exceed the maximum level, the manufacturer may reduce or even temporarily stop his orders for the part. This min-max strategy is common in lean manufacturing systems, which promote 'visual inventory controls'. In fact, the manufacturer may set separate minimum and maximum levels for the part at different locations, e.g., one set in the central storage area or 'marketplace' and a separate set at the point-of-fit along the assembly line.

The starting point of this model is a new inventory management policy. The manufacturer and supplier agree on a fixed delivery or release frequency of f times per period. They also agree on a nominal release quantity of α parts per delivery. Thus, the nominal quantity of parts delivered per period is $\lambda = f\alpha$. Having a fixed release frequency is important when the manufacturer has many suppliers. It allows the manufacturer to coordinate and consolidate scheduled deliveries from many locations, which facilitates smaller shipments and less inventory while simultaneously holding down transportation costs. In Section 4, we return to the question of determining the frequency of releases.

Now we describe the cost structure for our inventory control model. We ignore the shipping charges associated with sending the nominal release quantity α and focus on the costs involved in deviating from this nominal amount. When the manufacturer orders an amount δ larger than the nominal quantity, i.e., when it releases an order for $\alpha + \delta$, it must pay $c_f + c_v\delta$. This charge includes the fixed costs c_f representing, for example, the cost of dispatching an additional truck, and the variable costs c_v representing those aspects of the costs that grow with the size of the release. When the manufacturer orders an amount δ less than the nominal quantity, i.e., when it releases an order for $\alpha - \delta$, it must pay $d_f + d_v\delta$. Naturally, we require that $\delta \leq \alpha$ so the release quantities are non-negative. Typically, downward adjustments are less expensive than upward adjustments, but nevertheless generate costs in the supply chain. For example, the supplier may be forced to carry the extra δ units in inventory for some time. Finally, we assume that inventory at the manufacturer's site incurs a holding cost of h per item per unit of time.

Let Z_t be the inventory at the beginning of period t and let T_n be the time at which the manufacturer makes the n th adjustment to its release quantity. We denote the size of the n th adjustment by δ_n . The long-run average cost is

$$\limsup_{n \rightarrow \infty} \frac{1}{T_n} \left(\sum_{i=1}^n \left((c_f + c_v \delta_i) \mathbb{1}_{\delta_i > 0} + (d_f + d_v |\delta_i|) \mathbb{1}_{\delta_i < 0} \right) + h \sum_{i=1}^{T_n} Z_i \right),$$

where, for a set A , 1_A is the indicator function of A . Our optimization is to choose a release frequency f , nominal release quantity α , and the release policy (adjustment times $\{T_n\}$ and sizes $\{\delta_n\}$) so that the expected long-run average cost is minimized subject to the constraint that the inventory at the manufacturer's site must remain between m and \bar{M} .

3. Impulse Control of Brownian Motion

In this section, we describe a related problem that deals with the impulse control of Brownian motion. Suppose that we are given a Brownian motion $B = \{B_t, t \geq 0\}$ with mean μ and variance σ^2 . Brownian motion is a continuous time process with independent increments. It has continuous sample paths that can take any real values. Here, μ is interpreted as the output rate and we are to choose a constant λ that serves as the input rate for the Brownian motion. Once λ is fixed, the uncontrolled Brownian motion $X = \{X_t, t \geq 0\}$ has drift $\lambda - \mu$ and variance σ^2 . We are to control the Brownian motion X so that it remains bounded between m and \bar{M} . The controls are exercised at discrete times by adjusting the Brownian motion upward or downward. The resulting controlled process is denoted by $Z = \{Z_t, t \geq 0\}$. The Brownian control problem is to find the constant λ and a non-anticipating policy $\{(T_n, \delta_n), n \geq 1\}$ that minimize the expected long-run average cost

$$\limsup_{n \rightarrow \infty} \frac{1}{T_n} \left(\sum_{i=1}^n \left((c_f + c_v \delta_i) \mathbb{1}_{\delta_i > 0} + (d_f + d_v |\delta_i|) \mathbb{1}_{\delta_i < 0} \right) + h \int_0^{T_n} Z(t) dt \right).$$

Here, T_n is the time of the n th adjustment and δ_n is its size. To reflect the condition in the discrete model that the release quantities remain non-negative, we impose the constraint that the magnitude of each downward adjustment may not exceed λ . The parameters c_f , c_v , d_f , d_v and h have the same interpretations as in the discrete model. We let $c(m, \bar{M}, \mu, \sigma^2)$ denote the average cost under an optimal policy.

Harrison *et al.* [9] introduced the term impulse control of Brownian motion to describe problems like our Brownian control problem. They showed that a control band policy is optimal for their Brownian control problem. This form of policy is described by three parameters $\{r, R, M\}$ with $0 < r \leq R < M$. Extending this result, we anticipate that similar policies described by the parameters $\{r, R, M\}$ where $m < r \leq R < M \leq \bar{M}$ are optimal for our problem. Under these policies, when inventory falls to m , the minimum allowed, the optimal policy introduces an impulse raising it to r . When inventory rises to M , the optimal policy pushes it down to R .

4. Interpretation of the Brownian Control Policy

Having solved the Brownian control problem with optimal parameters λ^* , r^* , R^* and M^* , the challenge of translating the results into an effective policy for the original discrete system remains. For this, we also propose a control band policy based on the Brownian solution obtained in Section 3.

We use λ^* , r^* , R^* and M^* computed in Section 3 to implement a control band policy for

the discrete model. We first set a frequency f^* and quantity α^* so that $\lambda^* = f^* \alpha^*$. If we see the net effect of nominal shipments in the rest of the period and the demand for the period will leave the inventory level below m , we increase the shipment to $\alpha^* + (r^* - m)$.

On the other hand, if we see the net effect of nominal shipments in the rest of the period and the demand for the period will leave the inventory level above M^* , we decrease the total shipments for the period by $(M^* - R^*)$. This total amount can be spread among the period's releases, but then each release will be penalized for fixed cost. Recall that α^* and f^* are chosen so that

$$M^* - R^* \leq \alpha^* f^* = \lambda^*.$$

Thus, at the end of the period, the inventory will be below M^* .

5. Minimum Inventory Level

The discrete inventory control model and, with it, the Brownian control model address the question of how to best maintain inventory between two given limits m and \bar{M} . They are based on the assumption that regardless of the situation the supplier can get enough parts to the manufacturer in time. In this section, we directly address the question of whether the supplier can do this. To help ensure that it can, the manufacturer typically maintains a minimum inventory level m on site.

In a BTO environment, the manufacturer needs to fulfill customer orders quickly. An important performance measure for the manufacturer is the so called fill rate, the percentage of orders fulfilled on time. Achieving a high fill rate requires ample production capacity and high availability of parts. The manufacturer should set a high minimum inventory level m to prevent part shortages, and thus to improve on-time delivery of customer orders. On the other hand, setting the minimum level too high, incurs the costs and space requirements of excess inventory and robs the supplier of flexibility. To reduce supply chain costs, it makes sense to keep the minimum inventory level m as small as possible while protecting the manufacturer's service level.

Occasionally the inventory level of a part will drop below the minimum level m . These exceptions, hopefully rare, do occur. There are many factors, some controllable and some not, that cause them. For example, they may be due to a sudden surge of customer orders, transportation delays, shipment errors or production problems at the supplier. A paper by Choi *et al.* [10] revealed that, in a capacitated production environment, the fill rate depends on which of these exceptions occurs. For example, maintaining an inventory above the minimum level m most of the time does not guarantee a high fill rate if the supplier's production is not reliable.

Suppose, for example, that a supplier cannot ship for k consecutive periods. This might arise because of extreme weather, quality problems, etc. The manufacturer's maximum production in a period is limited by its capacity, c – the number of units the manufacturer can produce in a period when all the necessary parts are available. This implies that c is the minimum inventory necessary for the production process to operate without interruption due to part shortages in a single period. Thus, keeping the inventory level above $c \times k$ allows the manufacturer to operate normally even during an interval of k consecutive periods without shipments. Setting the minimum inventory m this way, however, simply cost too much. We propose to explore cost effective methods

for setting minimum levels that adequately guarantee the manufacturer's target fill rate. Choi *et al.* [10] provided a bound, in a capacitated production model, to achieve the manufacturer's target fill rate. In addition to demand information and production capacity, the bound uses the following two measures from the supplier: \bar{Q} the average number of undelivered parts in a shortage and p the frequency of part shortages. Suppose that k periods with no-shipments is the only possible cause for part shortage. Then \bar{Q} and p can be computed just from the possibility of part shortage during the exceptional periods. For example, when $k = 2$, the number of parts necessary for the first and the second periods are $R_1 := \min\{D_1, c\}$ and $R_2 := \min\{(D_1 - c)^+ + D_2, c\}$, respectively. In this case,

$$p = \varepsilon_s(2 \times \mathbb{P}\{R_1 > m\} + \mathbb{P}\{R_1 + R_2 > m \mid R_1 \leq m\}) \quad \text{and}$$

$$\bar{Q} = \varepsilon_s(\mathbb{E}[(R_1 - m)^+] + \mathbb{E}[(R_1 + R_2 - m)^+]),$$

where ε_s is the fraction of periods that the part is below the minimum inventory level. The k used in the previous discussion depends very much on the transportation mode used. For example, it takes 14 days from Asia to North American by sea, and 2 days by air. Of course, the cost of different transportation modes differs significantly.

6. Concluding Remarks

Many manufacturers are striving to build vehicles to order. BTO requires that production be organized according to a pull system. Lean manufacturing, the standard pull system in the auto industry, promotes ordering only what is needed when it is needed. Each supplier must deliver the anticipated usage of each part on a regular schedule. When usage varies widely from day to day, this places heavy demands on the suppliers and the carriers that are magnified up the supply chain.

The paper shows analytical models for rationally managing the variability in automotive supply chains inherent in BTO. They develop analytical models of the economics of managing a single part. This work involves solving the Brownian control problem, translating those solutions into workable operating policies within the assembly plants and determining the quality of those policies. The models in this paper combines advances in stochastic control and deterministic optimization and focuses them on a pressing problem in the largest manufacturing industry.

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