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Performance Evaluation for Super Alloy (Ni-Cr) Designed Structural Assembly of Gas Turbine Using Component Mode Synthesis

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Abstract

A prototype model for gas turbine has been fabricated underthe experimental and theoretical investigation are carried out. The configuration of the model consists of (1) Blades (2) Disc / Hub (3)Rotor / Shaft. Among the parts Blades and Disc are made of Ni – Cr Alloy. And rotor is made of mild steel. Blades are twisted aerofoil shape.In this the mass moment of inertia, torsional stiffness and polar moments of inertia are calculated for the integral unit of gas turbine with the using of the principles of mechanical vibrations. By employing CMS approach, the design parameters such as (1) Natural frequency (2) Damping factor (3) Logarithmic decrement are obtained by applying this methodology, an attempt is made to identify the parameters for the stability criteria. They are comprehensively studied and their effect on individual parameters onthe stability is analyzed.A through discussion on the results obtained in this investigation are presented in this paper. Along with the Experimental results have also been compared with those of theoretical calculations.

Keywords: Torsional stiffness, Damping factor, FFT analyzer, natural frequency, Twisted aerofoil blade, Holzer's method.

NOMENCLATURE

ω_n = Natural frequency

ε = Damping factor

δ = Logarithmic decrement

FFT = Fast Fourier Transform

X_a = Minimum amplitude

X_b = Maximum amplitude

K_t = Tensional stiffens

J = Mass Moment of Inertia

G = Shear Modulus

I_p= Polar moment of Inertia

L = Length

M = Mass

r = Radius

d = Diameter

r_O = outside radius of Hub.

r_i = inside radius of Hub.

P = the distance between x and $\bar{x}$

q = the distance between y and $\bar{y}$

BBW = Bottom Base Width

TBW = Top Base Width

d1 = Inside diameter of hub

d2 = out side diameter of hub

INTRODUCTION

A gas turbine, also called a combustion turbine, also called a combustion turbine, is a type of rotary machine, used to produce power. It is the heart of power plant that produces electric current. It converts natural gas or other liquid fuels energy to mechanical energy.Prediction of natural frequencies of vibration is of considerable importance at the design stage of a turbine blade. The theoretical solution for natural frequencies of a stationery uniform cantilever beam is well known. The actual problem of a turbine blade is rather complicated and one has to consider several factors, like (1)Speed of rotation (2)disc radius(3)Twist of blade(4)Taper of the blade(5)Tip mass(6) Shear deflection and rotary inertia.In order to analyse the vibrations that are induced, experimentally mathematical as well as simulation is carried out in ANSYS 15.0 soft ware. The numerical approach is made by using the CMS approach.

A super alloy (or) high performance alloy is an alloy that exhibits several key characteristics excellent mechanical strength, resistance to thermal creep deformation. It has good surface stability and resistance to corrosion or oxidation. The crystal structure is face centred cubic austenitic. Super alloys develop high temperature strength through solid solution strengthening. Oxidation of corrosion resistance is provided by elements such as aluminium and chromium. The primary applications of such alloys are turbine engines, both aerospace and marine. Ni-based super alloys are excellent high temperature materials and have proven veryuseful. Ni-based super alloys are superior hot corrosion, oxidation and wear resistance compared to Ni-based super alloys. Ni-based alloys are used in load-bearing structures. Also we can say super alloy has high melting point. In this way it is high temperature sustainable. Super alloy is also good having reliability (or) durability. In this way super alloy is superior when compared to aluminium alloy.

In general the turbine blade materials used are Nickel chromium alloy, Inconel, titanium, Aluminium alloy. The natural frequency is found for each material at different modes using both numerical & simulation approach.

Gas turbines have a wide range of applications including air crafts, power plants, marine application etc., So there is a immediate need to minimize the vibrations induced in the structural assembly of gas turbine.

LITERATURE REVIEWON CMS

1. M.H.Liu and G.T.Zheng proposed and improved technique of the component mode synthesis for non classically damped systems, with the second order approximation. It is based on free interface vibration modes and residual attachment modes with dynamic effects of the trunctuated modes.

2. Klaus-jurgen Bathe and jian Dong has presented an approach to improve component mode synthesis solution using subspace iterations to obtain frequency and mode shape prediction of controlled accuracy. In traditional CMS method the obtained values have an error and this error can be minimized by using subspace iteration methods.
3. Rainer Nordmamn and Ericknopf has outlined the effected of natural frequencies if a rotor with flexible blades are predicted by coupling and modal properties of two or more subsystems: The rotor with rigid blades and the violated flexible blade rows.
4. C.FARH AT and M. GERADINT has constructed a substructure Interface impedance operator and present a spectral analysis that demonstrates that the method of Craig and Bampton (CB) is the most natural CMS method and then consider the CB method for assesmbling hetreogeneous substructures and recast it into a hybrid vibrational formulation.
5. Robert J Kuether and Matlhew S Allen formulated the efficiency of craig-Bampton approach for two example problems, one that couples two geometrically nonlinear beams at a shared rotational degree of freedom and another that couples an axial spring element to the axial DOF of a geometrical nonlinear beam.
6. A Batailly, M Legrand, P.Cartraud and C.Pierre Investigated the general behaviour of such approach in the presence of contact nonlinearities. It will be shown that in our contact case a good accuracy can be obtained from a reduced models with very limited number of modes.
7. Adam butland, Dr. Peteravitable extends the methodology to include test data for the components models. A typical test case is shown to illustrate the results. The results shown that this technique can be very accurate in the development of reduced order test verified Craig-Brampton CMS models.
8. G.A.Choy and Yang proposed the combine optimization algorithm, immune genetic algorithm for multi optimization problem by introducing the capability of the immune system to the GA and also applied to minimize the total weight of the short and the transmitted forces at the bearings.
9. Rao and Kolla etal presented the optimum value of efficiency and may of the axial flow gas turbine stage using genetic algorithm approach.
10. Dr.S.Vjayarangam, V.Alagappam and I.Rajendren presented a formulation and technique using GA for design optimization of leaf springs in which dimensions of leaf spring are optimized.
11. Oh and Kim developed a conceptual design optimisation code to minimize the fluid dynamic losses in mixed flow pump Impellers.

12. William Carnegie discussed the vibrate of pretwisted Cantilever blade and pred the frequencies of vibration.
13. Rao and Srinivas investigated the flean torsional vibrations of a linearly tape twisted flexible blade, rotationally constrained out an arbitrary position along the length of blade using neural networks to identify the location of the modal an optimum point for a given blade taper ration and root flexibility parameter so as to maximixe the fundamental frequency.
14. Rao and Srinivas considered the implementation of neural networks to system identification and vibration suppression of bladed rotor time records of displacement and velocity at a point on the blade two novel methods are employed for the stage optimization problem and sufficiently accurate results are arrived when compared with available data.

1. **Turbine Hub:-**

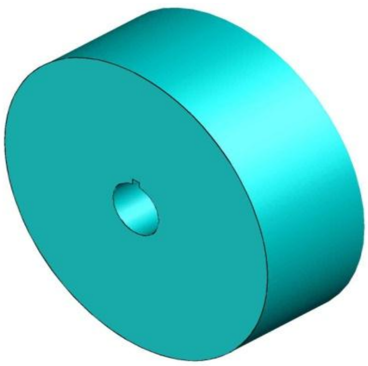


Figure-1

Material	: Nickel- Chromium	
Length	=	85 mm
Mass	=	2.4 Kg
Shear Modulus (G)	=	162 N/mm^2
Density	=	8470 Kg/m^3

- $Polar\ Moment\ of\ Inertia\ I_p = \frac{\pi}{32}[d_1^4 - d_2^4]$

$$= 17.155 \times 10^6 mm^4$$

$$\boldsymbol{I_p = 17.155 \times 10^6 mm^4}$$

- $Torsional\ Stiffness \qquad K_t = \frac{GI_p}{L}$

$$= \frac{162\ X\ 17.15\ X\ 10^6}{85}$$

$$= 32.7\ \times 10^6 N.mm$$

From the component mode synthesis Torsional stiffness value of hub we have divided into two parts

- $K_{t1} = K_{t2} = \frac{32.7\ \times 10^6 N.mm}{2}$

$$\boldsymbol{K_{t1} = K_{t2} = 16.35\ N.mm}$$

- $Mass\ momment\ of\ Inertia\ J = \frac{1}{2} m(r_1^2 - r_2^2)$

$$= 3.84 \times 10^3 Kg.mm^2$$

From the component mode synthesis mass moment of inertia is divided into two parts we have

- $J_1 = J_2 = \frac{3.84 \times 10^3 Kg.mm^2}{2}$

$$\boldsymbol{J_1 = J_2 = 1.92\ \times 10^3 Kg.mm^2}$$

$$\boldsymbol{J_1 = J_2 = 1.92\ \times 10^3 Kg.mm^2}$$

3.1.1 TURBINE ROTOR

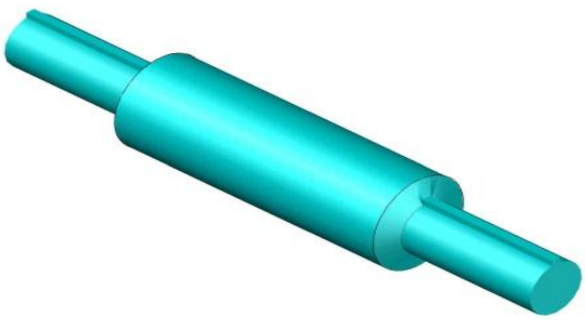


Figure-2

Material used	–	mild steel		
L	=	400 mm		
M	=	2.245 Kg		
G	=	78 X 10^9 N/m^2	=	78 X 10^3 N/mm^2

➢ $Polar\ Moment\ of\ Inertia\ \ I_p = \frac{\pi}{32} d^4$

$$\boldsymbol{I_p = 15.71\ \times 10^3 mm^4}$$

➢ $Torsional\ Stiffness \qquad K_t = \frac{GI_p}{L}$

$$= \frac{78 \times 10^3 \times 15.71 \times 10^3}{400}$$

$$= 3.06\ N.mm$$

With reference of component mode synthesis Torsional stiffness value of rotor is divided into two parts we have

➢ $K_{t3} = K_{t4} = \frac{3.06\ N.mm}{2}$

$$\boldsymbol{K_{t3} = K_{t4} = 1.53\ \times 10^6\ N.mm}$$

➢ $Mass\ momment\ of\ Inertia\ \ J = \frac{1}{2} mr^2$

$$= 112.25\ kg.mm^2$$

According to component mode synthesis mass moment of inertia of rotor is divided into two components we have

➢ $J_3 = J_4 = \frac{112.25}{2}\ kg.mm^2$

$$\boldsymbol{J_3 = J_4 = 56.125\ kg.mm^2}$$

3.1.2 Twisted Aerofoil/Blade :-

ρ	=	8470 Kg/m^3
G	=	162 N/m^2
b	=	60 mm
l	=	200 mm
q	=	30 mm

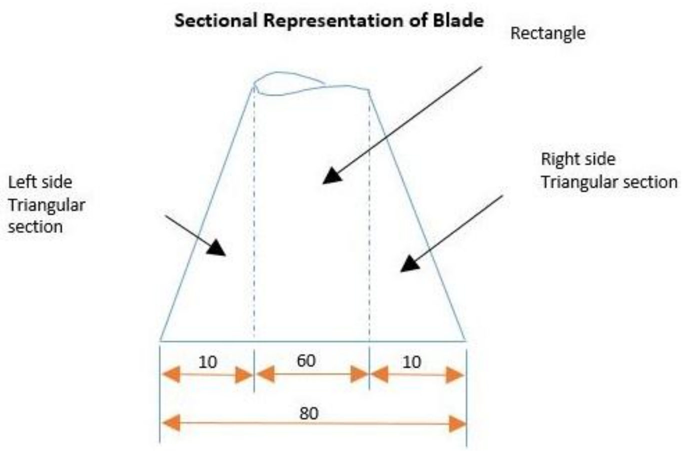


Figure-3

From trapezoid section of a blade to establish the mathematical expression we have

➢ $Polar\ Moment\ of\ Inertia\ I_p = \frac{d^3(a^2+4ab+b^2)}{36(a+b)}$

$$= \frac{200^3(60^2 + 4\,(60)\,(80) + 80^2)}{36(80 + 60)}$$

$$\boldsymbol{I_p = 46.35 \times 10^6\ mm^4}$$

- $Torsional\ Stiffness\quad K_t = \frac{G.I_p}{L}$

$$= \frac{162 \times 46.35 \times 10^6 \times 10^{-12}}{200 \times 10^{-3}}$$

$$K_t = 37.54 \times 10^6\ N.mm\ (For\ Single\ Blade)$$

$$K_t = 225.261 \times 10^{-3}\ N.mm\ (For\ Six\ Blade)$$

By the application of component mode synthesis Torsional stiffness of a blade is divided into two parts we have

$$\boldsymbol{K_{t5} = K_{t6} = 112.6 \times 10^6 N.mm}$$

Left Side Triangular Section of blade:-

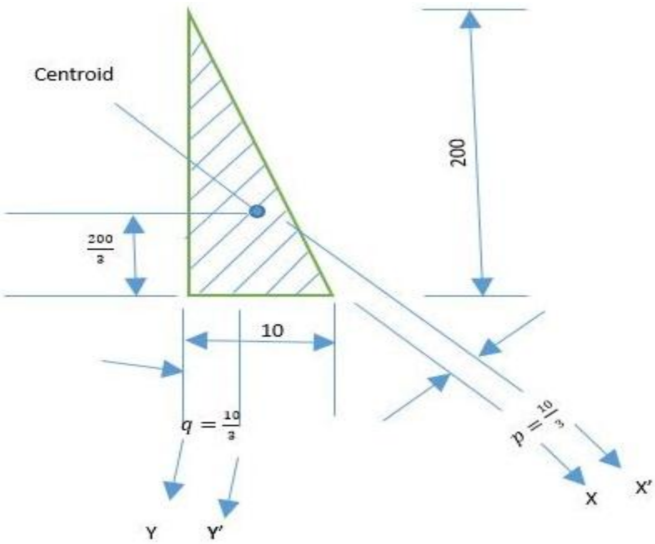


Figure-4

- $Mass\ momment\ of\ Inertia\quad J^I = m\left[\frac{h^2+l^2}{18} + q^2 + r^2\right]$

$$= 0.693\left[\frac{10^2 + 200^2}{18} + \left(\frac{10}{3}\right)^2 + \left(\frac{200}{3}\right)^2\right]$$

$$J^I = 4.63 \times 10^3 Kg.mm^2$$

Right Side Triangular Section of blade:-

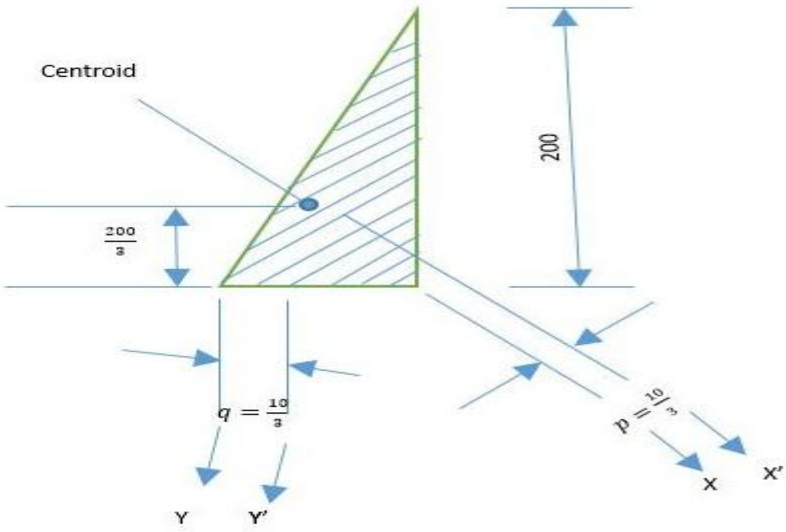


Figure-5

- $Mass\ momment\ of\ Inertia J^{II} = m\left[\frac{h^2+l^2}{18} + q^2 + r^2\right]$

$$= 0.693\left[\frac{10^2 + 200^2}{18} + \left(\frac{10}{3}\right)^2 + \left(\frac{200}{3}\right)^2\right]$$

$$J^{II} = 4.63 \times 10^3 Kg.mm^2$$

Rectangular Section of blade:-

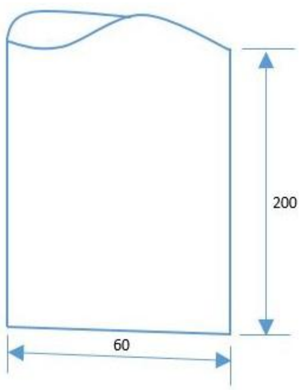


Figure-6

- $Mass\ momment\ of\ Inertia\ J^{III} = m\left[\frac{h^2+l^2}{18} + q^2 + r^2\right]$

$$= 0.693\left[\frac{60^2 + 200^2}{18} + 30^2 + 100^2\right]$$

$$J^{III} = 9.2 \times 10^3 Kg.mm^2$$

Mass moment of Inertia of Blade

- $J = Left\ Triangle\ Section + Right\ Triangle\ Section + Recangle$

$$= 18.46\ \times 10^6\ Kg.mm^2 (For\ Single\ Blade)$$

$$J\ = 110.76\ \times 10^3\quad Kg.mm^2 \qquad (For\ Six\ Blade)$$

Mass moment of inertia of blade is divided into 3 parts we have

$$J = \frac{110.76 \times 10^3}{3} Kg.mm^2$$

$$\boldsymbol{J_5 = J_6 = J_7 = 36.72 \times 10^3\ Kg.mm^2}$$

Table: 1

<table>
<tr><th></th><th>HUB</th><th>ROTOR</th><th>BLADE</th></tr>
<tr><td>Polar Moment of Inertia (I_p) mm^4</td><td>$17.155\ \times\ 10^6$</td><td>$15.71\ \times 10^3$mm^4</td><td>46.35×10^6</td></tr>
<tr><td rowspan="2">Torsional Stiffness(K_t) $N.mm$</td><td>16.35×10^6</td><td>$1.53 \times 10^6\ N.mm$</td><td>112.6×10^6</td></tr>
<tr><td>16.35×10^6</td><td>$1.53 \times 10^6\ N.mm$</td><td>112.6×10^6</td></tr>
<tr><td rowspan="3">Mass Moment of Inertia (J) $Kg.mm^2$</td><td rowspan="2">1.92×10^3</td><td rowspan="2">56.125×10^6</td><td>36.92×10^3</td></tr>
<tr><td>36.92×10^3</td></tr>
<tr><td>1.92×10^3</td><td>56.125×10^6</td><td>36.92×10^3</td></tr>
</table>

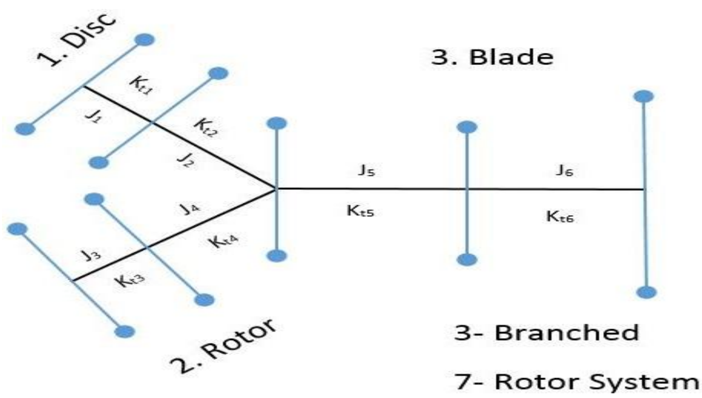


Figure-7

DETERMINATION OF NATURAL MODE VALUES

1. **Turbine Hub :**

- $Torsional\ Stiffnes\ K_t = \begin{bmatrix} K_{t1} & -K_{t1} \\ -K_{t1} & K_{t1}+K_{t2} \end{bmatrix}$

$$= \begin{bmatrix} 16.35\times10^6 & -16.35\times10^6 \\ -16.35\times10^6 & 32.7\times10^6 \end{bmatrix} N.mm$$

- $Mass\ Moment\ of\ Inertia\ J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$

$$= \begin{bmatrix} 1.92\times10^3 & 0 \\ 0 & 1.92\times10^3 \end{bmatrix} Kg.mm^2$$

The Characteristic Equation has $[K_t - J_w^2]\{X\} = 0$

Then $|K_t - J_w^2| = 0$

- $\begin{vmatrix} 16.35\times10^3 - 1.92\times10^{-3}\omega^2 & -16.35\times10^3 \\ -16.35\times10^3 & 32.7\times10^3 - 1.92\times10^{-3}\omega^2 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $534.645\times10^6 - 31.392\omega^2 - 62.784\omega^2 + 3.686\times10^{-6}\omega^4 - 267.32\times10^6 = 0$

$Let\ \omega^2 = \lambda$

- $\mathbf{3.686\times10^{-6}\lambda^2 - 94.176\lambda + 267.325\times10^6 = 0}$

Use this formula for finding roots fro above Quadratic equation $\lambda_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\lambda_{1,2} = \frac{-(-94.176)\pm\sqrt{(-94.176)^2-(4\times3.686\times10^{-6})(267.325\times10^6)}}{2\times3.686\times10^{-6}}$$

- $\lambda_{1,2} = 22.27\times10^6,\ \ 3.25\times10^6$
  - $\omega_1^2 = \lambda_1 = 22.3\times10^6$
  - $\omega_2^2 = \lambda_2 = 3.25\times10^6$

✓ Substitute $\omega_1^2 = \lambda_1 = 22.3\times10^6$ in $[K_t - J_w^2]\{X\} = 0$ then

- $\begin{bmatrix} 16.34\times10^3 - (22.3\times10^6\times1.92\times10^{-3}) & -16.34\times10^3 \\ -16.34\times10^3 & 32.69\times10^3 - (22.3\times10^6\times1.92\times10^{-3}) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} -26.418\times10^3 & -16.34\times10^3 \\ -16.34\times10^3 & -10.06\times10^3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$-26.4148 \times 10^3 X_1 - 16.34 \times 10^3 X_2 = 0$$

$$-16.34 \times 10^3 X_1 - 10.06 \times 10^3 X_2 = 0$$

❖ $\boldsymbol{If\ X_2 = 1\ then\ X_1 = 0.618}$

✓ Substitute $\omega_2^2 = \lambda_2 = 1.04 \times 10^{10}$ in $[K_t - J_w^2]\{X\} = 0$ then

$$\begin{bmatrix} 16.34 \times 10^3 - (3.25 \times 10^6 \times 1.92 \times 10^{-3}) & -16.34 \times 10^3 \\ -16.34 \times 10^3 & 32.69 \times 10^3 - (3.25 \times 10^6 \times 1.92 \times 10^{-3}) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

➢ $\begin{bmatrix} 10.1 \times 10^3 & -16.34 \times 10^3 \\ -16.34 \times 10^3 & 26.45 \times 10^3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$10.1 \times 10^3 X_1^I - 16.34 \times 10^3 X_2^I = 0$$

$$-16.34 \times 10^3 X_1^I + 26.45 \times 10^3 X_2^I = 0$$

❖ $\boldsymbol{If\ X_1^I = 1\ then\ X_2^I = 0.618}$

2. Rotor :-

➢ $Torsional\ Stiffnes\ K_t = \begin{bmatrix} K_{t3} & -K_{t3} \\ -K_{t3} & K_{t3} + K_{t4} \end{bmatrix}$

$$= \begin{bmatrix} 153.15 \times 10^4 & -153.15 \times 10^4 \\ -153.15 \times 10^4 & 306.3 \times 10^4 \end{bmatrix} N.mm$$

➢ $Mass\ Moment\ of\ Inertia\ J = \begin{bmatrix} J_3 & 0 \\ 0 & J_4 \end{bmatrix}$

$$= \begin{bmatrix} 0.56 \times 10^{-4} & 0 \\ 0 & 0.56 \times 10^{-4} \end{bmatrix}$$

The Characteristic Equation has $[K_t - J_w^2]\{X\} = 0$

Then $|K_t - J_w^2| = 0$

➢ $\begin{bmatrix} 153.15 \times 10^4 - 0.56 \times 10^{-4}\omega^2 & -153.15 \times 10^4 \\ -153.15 \times 10^4 & 306.3 \times 10^4 - 0.56 \times 10^{-4}\omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

➢ $46909.84 \times 10^8 - (85.764 + 171.528)\omega^2 + 0.313 \times 10^{-8}\omega^4 - 234.54 \times 10^8 = 0$

$Let\ \omega^2 = \lambda$

❖ $\boldsymbol{0.313 \times 10^{-8}\lambda^2 - 257.292\lambda + 23455.8 \times 10^8 = 0}$

Use this formula for finding roots fro above Quadratic equation $\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\lambda_{1,2} = \frac{-(-257.292) \pm \sqrt{(-257.292)^2 - (4 \text{ X } 0.313 \times 10^{-8} \times 23455.8 \times 10^{8})}}{2 \times 0.313 \times 10^{-8}}$$

- ❖ $\omega_1^2 = \lambda_1 = 7.17 \times 10^{10}$
- ❖ $\omega_2^2 = \lambda_2 = 1.04 \times 10^{10}$

Substitute $\omega_1^2 = \lambda_1 = 7.17 \times 10^{10}$ in $[K_t - J_w^2]\{X\} = 0$ then

$$\begin{bmatrix} 153.15 \times 10^4 - (0.56 \times 10^{-4} \times 7.17 \times 10^{10}) & -153.15 \times 10^4 \\ -153.15 \times 10^4 & 306.3 \times 10^{-4} - (0.56 \times 10^{-4} \times 7.17 \times 10^{10}) \end{bmatrix} \begin{Bmatrix} X_3 \\ X_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

➢ $\begin{bmatrix} -2483.7 \times 10^3 & -153.15 \times 10^4 \\ -153.15 \times 10^4 & -952.2 \times 10^3 \end{bmatrix} \begin{Bmatrix} X_3 \\ X_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$-2483.7 \times 10^3 X_3 - 153.15 \times 10^4 X_4 = 0$$

$$-153.15 \times 10^4 X_3 - 952.2 \times 10^3 X_4 = 0$$

❖ $\boldsymbol{If\ X_4 = 1\ then\ X_3 = -0.62}$

Substitute $\omega_2^2 = \lambda_2 = 1.04 \times 10^{10}$ in $[K_t - J_w^2]\{X\} = 0$ then

$$\begin{bmatrix} 153.15 \times 10^4 - (0.56 \times 10^{-4} \times 1.04 \times 10^{10}) & -153.15 \times 10^4 \\ -153.15 \times 10^4 & 306.3 \times 10^4 - (0.56 \times 10^{-4} \times 1.04 \times 10^{10}) \end{bmatrix} \begin{Bmatrix} X_3^I \\ X_4^I \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

➢ $\begin{bmatrix} 94.91 & -153.15 \\ -153.15 & 248.06 \end{bmatrix} \times 10^4 \begin{Bmatrix} X_3^I \\ X_4^I \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$94.91 \times 10^4 X_3^I - 153.15 \times 10^4 X_4^I = 0$$

$$-153.15 \times 10^4 X_3^I + 248.06 \times 10^4 X_4^I = 0$$

❖ $\boldsymbol{If\ X_3^I = 1\ then\ X_4^I = 0.612}$

3. **Analysis of blade :-**

➢ $Torsional\ Stiffnes\ K_t = \begin{bmatrix} K_{t5} & -K_{t5} \\ -5 & K_{t5} + K_{t6} \end{bmatrix}$

$$= \begin{bmatrix} 112.6 \times 10^3 & -112.6 \times 10^3 \\ -112.6 \times 10^3 & 225.1 \times 10^3 \end{bmatrix} N.mm$$

➢ $Mass\ Moment\ of\ Inertia\ J = \begin{bmatrix} J_5 & 0 \\ 0 & J_6 \end{bmatrix}$

$$= \begin{bmatrix} 36.92 \times 10^{-3} & 0 \\ 0 & 36.92 \times 10^{-3} \end{bmatrix} Kg.mm^2$$

The Characteristic Equation has $[K_t - J_w^2]\{X\} = 0$

Then $|K_t - J_w^2| = 0$

➢ $\begin{bmatrix} 112.6\times10^3 - 36.92\times10^{-3}\omega^2 & -112.6\times10^3 \\ -112.6\times10^3 & 225.1\times10^3 - 36.92\times10^{-3}\omega^2 \end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$$1353.0864\times10^{-6}\omega^4 - 12467.884\omega^2 + 12667.5\times10^6 = 0$$

$Let\ \omega^2 = \lambda$, then

➢ $1353.0864\times10^{-6}\lambda^2 - 12467.884\lambda + 12667.5\times10^6 = 0$

By Solving above quadratic equation the roots are

❖ $\omega_1^2 = \lambda_1 = 7.98\times10^6$

❖ $\omega_2^2 = \lambda_2 = 1.16\times10^6$

Substitute $\omega_1^2 = \lambda_1 = 7.98\times10^6$ in $[K_t - J_w^2]\{X\} = 0$ then

$$\begin{bmatrix} 112.6\times10^3 - (36.92\times10^{-3}\times7.98\times10^6) & -112.6\times10^3 \\ -112.6\times10^3 & 225.1\times10^3 - (36.92\times10^{-3}\times7.98\times10^6) \end{bmatrix}\begin{Bmatrix}X_5\\X_6\end{Bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

➢ $\begin{bmatrix} -182.02 & -112.6\times10^3 \\ -112.6\times10^3 & -69.42 \end{bmatrix}\begin{Bmatrix}X_5\\X_6\end{Bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$$-182.02X_5 - 112.6\times10^3X_6 = 0$$

$$-112.6\times10^3X_5 - 69.42X_6 = 0$$

❖ $\boldsymbol{If\ X_6 = 1\ then\ X_5 = 0.618}$

Substitute $\omega_2^2 = \lambda_2 = 1.16\times10^6$ in $[K_t - J_w^2]\{X\} = 0$ then

$$\begin{bmatrix} 112.6\times10^3 - (36.92\times10^{-3}\times1.16\times10^6) & -112.6\times10^3 \\ -112.6\times10^3 & 225.1\times10^3 - (36.92\times10^{-3}\times1.16\times10^6) \end{bmatrix}\begin{Bmatrix}X_5^I\\X_6^I\end{Bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

➢ $\begin{bmatrix} 69.70 & -112.6\times10^3 \\ -112.6\times10^3 & 182.17 \end{bmatrix}\begin{Bmatrix}X_5^I\\X_6^I\end{Bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$$69.70X_5^I - 112.6\times10^3X_6^I = 0$$

$$-112.6\times10^3X_5^I + 182.17X_6^I = 0$$

❖ $\boldsymbol{If\ X_5^I = 1\ then\ X_6^I = 0.619}$

Table: 2

Natural Frequency	HUB		ROTOR		BLADE	
1. ω_1	$X_1 = 0.618$	$X_2 = 1$	$X_3 = -0.62$	$X_4 = 1$	$X_5 = 0.618$	$X_6 = 1$
2. ω_2	$X_1^I = 1$	$X_2^I = 0.618$	$X_3^I = 1$	$X_4^I = 0.612$	$X_5^I = 1$	$X_6^I = 0.619$

MODE SYNTHESIS OF SUPER ALLOY:-

From the Ritz approach for the first two vectors of the isolated system when junction is fixed and assuming third vector. The junction is having unit displacement and the rest are zero.

$$\psi = \begin{bmatrix} X_1^I & X_1 & 0 \\ X_2^I & X_2 & 0 \\ X_3^I & X_3 & 0 \\ X_4^I & X_4 & 0 \\ 0 & 0 & 1 \\ X_5^I & X_5 & 0 \\ X_6^I & X_6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0.618 & 0 \\ 0.618 & 1 & 0 \\ 1 & -0.62 & 0 \\ 0.612 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0.619 & 0 \\ 0.618 & 1 & 0 \end{bmatrix}_{7X3}$$

$$\psi^T = \begin{bmatrix} 1 & 0.618 & 1 & 0.612 & 0 & 1 & 0.618 \\ 0.618 & 1 & -0.62 & 1 & 0 & 0.619 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}_{3X7}$$

$$K = \begin{bmatrix} K_{t1} & -K_{t1} & 0 & 0 & 0 & 0 & 0 \\ -K_{t1} & K_{t1}+K_{t2} & 0 & 0 & -K_{t2} & 0 & 0 \\ 0 & 0 & K_{t3} & -K_{t3} & 0 & 0 & 0 \\ 0 & 0 & -K_{t3} & K_{t3}+K_{t4} & -K_{t4} & 0 & 0 \\ 0 & -K_{t1} & 0 & -K_{t4} & K_{t2}+K_{t4}+K_{t5} & -K_{t5} & 0 \\ 0 & 0 & 0 & 0 & -K_{t6} & K_{t6}+K_{t7} & -K_{t7} \\ 0 & 0 & 0 & 0 & 0 & -K_{t7} & K_{t7} \end{bmatrix}$$

$$K = \begin{bmatrix} 16.35 & -16.35 & 0 & 0 & 0 & 0 & 0 \\ -16.35 & 32.70 & 0 & 0 & --16.35 & 0 & 0 \\ 0 & 0 & 1.53 & -1.53 & 0 & 0 & 0 \\ 0 & 0 & -1.53 & 2.99 & -1.53 & 0 & 0 \\ 0 & -16.35 & 0 & -1.53 & 17.63 & -0.1 & 0 \\ 0 & 0 & 0 & 0 & -0.1 & 0.2 & -0.1 \\ 0 & 0 & 0 & 0 & 0 & -0.1 & 0.1 \end{bmatrix}_{7X7}$$

$$\psi^T K = 10^6 \begin{bmatrix} 6.25 & 3.89 & 0.59 & 0.3 & -11.14 & 0.14 & -0.04 \\ -6.25 & 22.60 & -2.48 & 3.94 & -17.94 & 0.02 & 0.04 \\ 0 & -16.35 & 0 & -1.53 & 17.63 & -0.1 & 0 \end{bmatrix}_{3X7}$$

$$\psi^T K \psi = \begin{bmatrix} 9.54 & 7.73 & -11.14 \\ 7.70 & 24.27 & -17.94 \\ 11.14 & -17.94 & -17.63 \end{bmatrix}_{3X3}$$

$$J = \begin{bmatrix} J_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & & J_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_7 \end{bmatrix}_{7X7}$$

$$J = 10^{-6} \begin{bmatrix} 1.92 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.92 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.56 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.56 & 0 & 0 & 0 \\ 0 & 0 & 0 & & 0.36 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.36 \end{bmatrix}_{7X7}$$

$$\psi^T J = 10^{-6} \begin{bmatrix} 1.92 & 1.19 & 0.56 & 0.34 & 0 & 0.36 & 0.22 \\ 1.19 & 1.92 & -0.35 & 0.56 & 0 & 0.22 & 0.36 \\ 0 & 0 & 0 & -0 & 0.36 & 0 & 0 \end{bmatrix}_{3X7}$$

$$\psi^T J\psi = \begin{bmatrix} 3.92 & 2.82 & 0 \\ 2.81 & 3.92 & 0 \\ 0 & 0 & -0.36 \end{bmatrix}_{3X3}$$

$$\bar{K} = \psi^T K\psi = \begin{bmatrix} 9.54 & 7.73 & -11.14 \\ 7.70 & 24.27 & -17.94 \\ 11.14 & -17.94 & -17.63 \end{bmatrix}_{3X3}$$

$$\bar{J} = \psi^T J\psi = \begin{bmatrix} 3.92 & 2.82 & 0 \\ 2.81 & 3.92 & 0 \\ 0 & 0 & -0.36 \end{bmatrix}_{3X3}$$

Where

$$[\bar{K} - \bar{J}\omega^2]\{X\} = [0]$$

$$|\bar{K} - \bar{J}\omega^2| = 0$$

$$\begin{vmatrix} 9.54 \times 10^6 - 3.92 \times 10^{-6}\omega^2 & 7.73 \times 10^6 - 2.82 \times 10^{-6}\omega^2 & -11.14 \times 10^6 \\ 7.70 \times 10^6 - 2.81 \times 10^{-6}\omega^2 & 24.27 \times 10^6 - 3.92 \times 10^{-6}\omega^2 & -17.94 \times 10^6 \\ 11.14 \times 10^6 & -17.94 \times 10^6 & -17.63 \times 10^6 + 0.36 \times 10^{-6}\omega^2 \end{vmatrix} = 0$$

$$9.54 \times 10^6 - 3.92 \times 10^{-6}\omega^2\big[(24.27 \times 10^6 - 3.92 \times 10^{-6}\omega^2)(-17.63 \times 10^6 + 0.36 \times 10^{-6}\omega^2) - \big(17.94 \times 10^6\big)(-17.94 \times 10^6) - (7.73 \times 10^6 - 2.82 \times 10^{-6}\omega^2)[(7.70 \times 10^6 - 2.81 \times 10^{-6}\omega^2)(-17.63 \times 10^6 + 0.36 \times 10^{-6}\omega^2) - (11.14 \times 10^6)(-17.94 \times 10^6)] - 11.14 \times 10^6[(7.70 \times 10^6 - 2.81 \times 10^{-6}\omega^2)(-17.94 \times 10^6) - (11.14 \times 10^6)(24.27 \times 10^6 - 3.92 \times 10^{-6}\omega^2)] = 0$$

$\boldsymbol{Let\ \omega^2 = \lambda}$

After simplification

$$4\lambda^3 - 402.31 \times 10^6\lambda^2 + 44.84 \times 10^{10}\lambda - 1120.618 \times 10^{10} = 0$$

$Therefore\ the\ roots\ are$

$$\lambda_1 = \omega_1^2 = 453.38 \times 10^6$$

$$\lambda_2 = \omega_2^2 = -147.56 \times 10^6$$

$$\lambda_3 = \omega_3^2 = -173.8 \times 10^6$$

$$\omega_1 = 21.28 \times 10^3 \ {rad}/{s^2}$$

$$\omega_2 = 12.12 \times 10^3 \ {rad}/{s^2}$$

$$\omega_3 = 13.15 \times 10^3 \ {rad}/{s^2}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$f_1 = 3388 \ Hz$$

$$f_2 = 2029 \ Hz$$

$$f_3 = 2143 \ Hz$$

Consider super alloy we obtain the natural frequencies f_{N1}, f_{N2}, f_{N3}. They are

$$f_{N1} = 3388 \ Hz$$

$$f_{N2} = 2029 \ Hz$$

$$f_{N3} = 2143 \ Hz$$

The fundamental frequency among the three frequencies is generally the lowest frequencies. Hence f_N is obtained as $2029 \ Hz$

Now we can obtain the Damping factor and logarithm decrement value (δ), As we know,

$$f_N = \frac{4.987}{\sqrt{\delta}}$$

- $2029 = \frac{4.987}{\sqrt{\delta}}$
- $\sqrt{\delta} = \frac{4.987}{2029}$

$$\sqrt{\delta} = 2.457 \times 10^{-3}$$

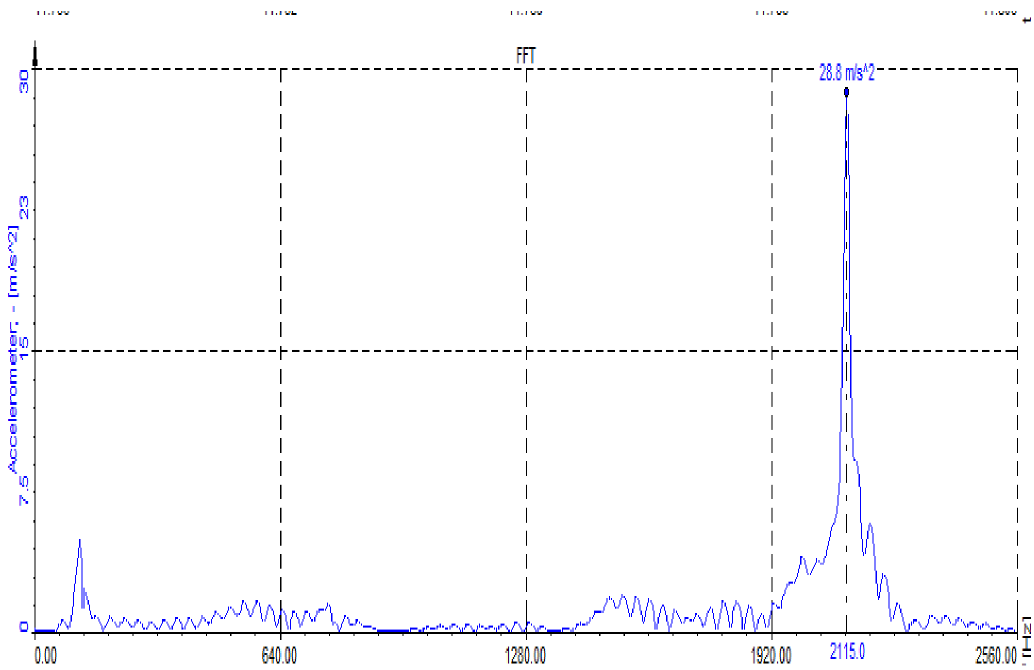
- $\boldsymbol{\delta = 6.041 \times 10^{-6}}$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

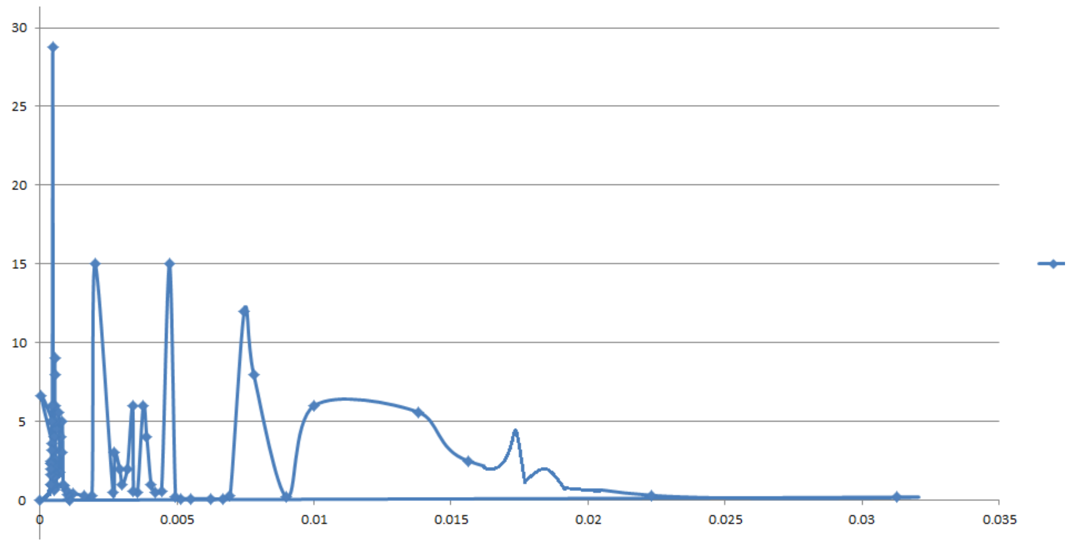
$$\xi = 9.614 \times 10^{-7}$$

$$\xi^2 = 9.244 \times 10^{-13}$$

EXPERIMENTAL ANALYSIS WORK:



Graph-1



Graph-2

Here δ − logarithmic decrement.

X_a and X_b are the peaks of two consecutive waves.

ζ – damping factor

from the above graph

$$X_a = 28.71 m/s^2$$

$$X_b = 28.82\ m/s^2$$

$$\delta = \log\frac{28.82}{28.71} = 0.00000346 = 3.46\times10^{-6}$$

$$3.46\times10^{-6} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Damping factor **ζ = 5.507×10^{-7}**

EXPERIMENTAL PROCEDURE USING FFT ANALYZER:

1. Arrange the cantilever by fixing blade at the end of FFT equipment.

2. Calculate the length of the fixture used for holding the blade of the turbine and hence leave its mark on the blade assembly.

3. At the centre of the blade affix an accelerometer on the face of the bar opposite to the face of the blade.

4. Fix the bar exactly into the lot of the fixture such that a cantilever will be formed.

5. Now connections of various wires , cables of vibration analysers ,pc or laptop, accelerometer and the impact hammer are done with the aid of manuals or under the guidance of experts.

6. Switch on the AC power supply.

7. Run vibration analysis and experimental modal analysis software that are previously installed in pc/laptop. Provide necessary inputs and correspondingly check the settings in the software

8. Always ensure for proper power supply and check for the communications between the devices that are connected

9. The four channel analyser used here is a DEWE software. Adjust the x and y axis before starting the experiment.

10. Acceleration is taken on x-axis while frequency is considered to be on Y-axis.

11. Now with the help of a impact hammer, provide impacts at the tip of specimen on the cantilever, one by one.

12. Vibration analyser receives the corresponding signals from the impact hammer and accelerometer, which it compares and analyses using the software.

13. We can obtain the natural frequencies of the cantilever from the frequency function response curve .Read the curves corresponding to the peaks of these curves.

14. The obtained graphs will demonstrate the first natural frequency.

15. The damping factor can be obtained from the logarithmic decrement formula.

RESULTS AND DISCUSSION

Performance evaluation of Super alloy designed gas turbine.

1. A prototype physical model gas turbine made of Super alloy has been examined experimentally.
2. In their experiment it is found that it is effective in its functioning on the basis of output/results.
3. The theoretical and experimental values are compared and analyzed in this output and results of the functioning of aluminium alloy gas turbine.
4. In the comparative analysis of both the theoretical values and experimental values of natural frequency damping factor logarithmic decrement are examined.
5. The range between theoretical values and experimental values have been calculated and it is found that the values of range between them is within the permitted levels the range between theoretical values and experimental value of natural frequency damping factor logarithmic decrement are calculated and got the error values.

I. In the case of natural frequency the error value it was found 4.066%.

II. In the case of damping factor the error value between theoretical and experimental values is found if 10.79.

III. In the case of the logarithmic decrement between the theoretical and experimental values is found at 8.11.

On the base of the above error values it shows that the performance of the gas turbine is effective and getting the operational output on the expected lines.

Table: 3

FACTORS AFFECTING	THEORETICAL VALUE	EXPERIMENTAL VALUE	RANGE	% ERROR
Natural frequency(ω_n)	2029	2115	86	4.066
Damping factor (ζ)	$9.91X10^{-7}$	$8.84 X 10^{-7}$	$1.07X10^{-7}$	10.79
Logarithmic decrement value(δ)	$6.04X10^{-6}$	$5.55 X 10^{-6}$	$0.49X10^{-6}$	8.11

CONCLUSIONS:

From the above study of performance evaluation of super alloy designed gas turbine it is observed that all the critical parameters namely blade(Nickel Chromium), disc (nickel Chromium), rotor(mild steel), performance is critically examined and investigated on the different operational parameters using the component mode synthesis.

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