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A Hybrid Approach on Shortest Path in Fuzzy Network

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Abstract

This paper introduces a new type of fuzzy shortest path network problem using triangular fuzzy number. To find the smallest path from each possible path in a network by using fuzzy distance function. Thus the optimum shortest path for the given problem is obtained.

Keywords: Fuzzy Number, Fuzzy Distance, order relation.

Introduction

The shortest path problem was one of the first network problems studied in terms of operations research. In some applications, the numbers associated with the edges of networks may represent characteristics other than lengths, and we may want the optimum paths, where optimum can be defined by different criteria. The shortest-path problem is the most common problem in the whole class of optimum path problems. Consider the edge weight of the network as uncertain; which means that it is either imprecise or unknown. In 1965, Zadeh [8] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line.

Okada and Gen (1994) first tried to resolve the issue of incomparability of intervals using a ranking strategy of their own but their method seems to work in an *ad hoc* manner and so the result is not always unique and self-explanatory when used in an algorithm for solving the shortest path problem. In 1997, Heilpern [9] proposed three definitions of the distance between two fuzzy numbers. These include that mean distance method is generated by expect values of fuzzy numbers, distance method is combined by a Minkowski distance and the h-levels of the closed intervals of fuzzy numbers, and geometrical distance method is based on the geometrical operation of trapezoidal fuzzy numbers. All of them use real number to calculate the distance. Yao and Wu[13], used from signed distance to define ordering. Chen[12], first normalized fuzzy numbers and then used from maximizing set and minimizing set to define ordering.

For ranking of fuzzy numbers, a fuzzy number needs to be evaluated and compared with the others, but this may not be easy. Fuzzy set ranking has been studied by many researchers. Some of these ranking methods have been compared and reviewed by Bortolan and Degain[1]. More recently by Chen and Hwang[2], and it still receives much attention in recent years[3,4,5,6]. Many methods for ranking fuzzy numbers have been proposed, such as representing them with real numbers or using fuzzy relations. Wang and Kerre [6,7] proposed some axioms as reasonable properties to determine the rationality of a fuzzy ranking method and systematically compared a wide array of existing fuzzy ranking methods.

Here, first determine the number of paths in a given network and calculate fuzzy distance for each path. Then find the minimum distance value among the paths. That minimum path is fuzzy shortest path for a network.

Triangular Fuzzy Number

The fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is a triangular number, denoted by (a_1, a_2, a_3), its membership function $\mu_{\tilde{a}}$ is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & , if\ 0 \le x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , if\ a_1 \le x \le a_2 \\ 1 & , if\ x = a_2 \\ \dfrac{x - a_3}{a_2 - a_3} & , if\ a_2 \le x \le a_3 \\ 0 & , if\ x \ge a_3 \end{cases}$$

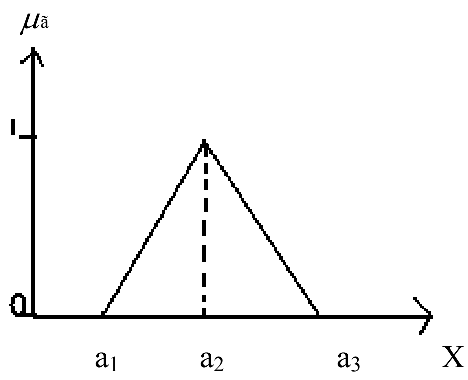


Figure 1: Membership function of a fuzzy number ã .

Positive Fuzzy Number

A fuzzy number ã is called a positive fuzzy number if its membership function is such that $\mu_{\tilde{a}}(x) = 0 \ \forall \ x < 0$.

Addition of Two Fuzzy Numbers

Let ã and $\tilde{b}$ two triangular fuzzy numbers. An addition of fuzzy numbers is $\tilde{c} = \tilde{a} \oplus \tilde{b}$ defined by the membership function.

$$\mu_{\tilde{a}}(t) = \underset{t=u+v}{Sup} \min\{\mu_{\tilde{a}}(u), \mu_{\tilde{b}}(v)\}$$

Addition of ã and $\tilde{b}$ is represented as $(a_1, a_2, a_3) \oplus (b1, b_2, b_3)$. Therefore, the function principle is

$$\tilde{c} = \tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)$$
$$= (a_1+b_1, a_2+b_2, a_3+b_3).$$

Ranking Of Fuzzy Number with Distance Method

Let all of fuzzy numbers be either positive or negative. Without less of generality, assume that all of them are positive. The membership function of a R is $u_a(x)=1$, if $x = a$; and $u_a(x) = 0$, if $x \neq a$. Hence if $a = 0$ we have the following

$$u_0(x) = \begin{cases} 1 & x = 0, \\ 0 & x \neq 0. \end{cases}$$

Definition: For u and v $\in$ E, define the ranking of u and v by saying

$d(u, u_0) > d(v, u_0)$ iff $u \succ v$,
$d(u, u_0) < d(v, u_0)$ iff $u \prec v$,
$d(u, u_0) = d(v, u_0)$ iff $u \approx v$.

Property 1. Suppose u and v $\in$ E are arbitrary then,

(i) if u = v then u = v,

(ii) if $v \subseteq u$ and $\underline{u}(r)^2 + \overline{u}(r)^2 > \underline{v}(r)^2 + \overline{v}(r)^2$ for all $r \in [0,1]$ then $v \prec u$.

Remark 1. The distance triangular fuzzy number u = (a, b, c) of u_0 is defined the following

$$d(u, u_0) = [2a^2 + b^2/3 + c^2/3 + a(c-b)]^{1/2}. \quad \text{.....Eqn(1)}.$$

Algorithm

Step-1: Find the number of paths.

$$P_m = \sum_{i=1, j=2}^{n} l_{ij} \quad \text{where } i \neq j \quad \text{and} \quad m = 1,2.....k$$

Step-2: Determine the fuzzy distance for each path (P_{ij}).

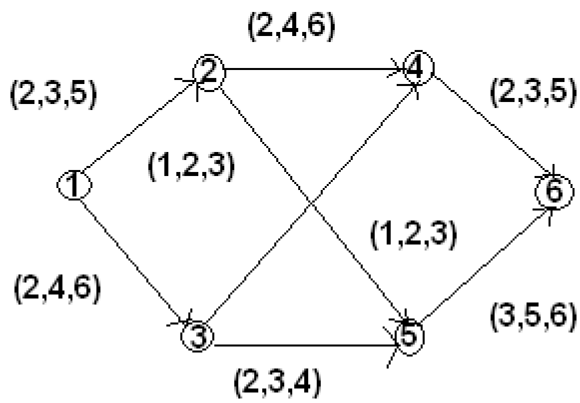
$$d(P_m) = [\, 2a^2 + b^2 / 3 + c^2 / 3 + a\,(c-b)\,]^{1/2}$$

where P_m = (a, b, c), m= 1 , 2, ….k.

Step-3: Select the minimum path by using the function Pij = min [$d_1(P_{ij})$, $d_2(P_{ij})$,…$d_k(P_{ij})$]

where i , j = 1,2,……..n for all $i \neq j$. Thus, the shortest path is obtained.

Numerical Example



In this example, the graph consists of 6 nodes and 8 edges. There are four paths available such as P1, P2, P3 and P4. Determine the fuzzy distance of each paths using Eqn(1). Now compare the paths which is smallest than the other paths. Thus, the fuzzy shortest path is found.

PATHS		**PATH LENGTH**	**FUZZY DISTANCE**	**RANK**
P1	(1-2-4-6)	(6,10,16)	15.0552	3
P2	(1-2-5-6)	(6,10,14)	13.9521	1
P3	(1-3-4-6)	(5,10,16)	14.0947	2
P4	(1-3-5-6)	(7,12,16)	16.1037	4

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