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An Appropriate Method for Real Life Fuzzy Transportation Problems

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Abstract

A new method is proposed to find the fuzzy optimal solution to a fuzzy transportation problem (FTP) where all parameters are fuzzy numbers. The proposed method is based on the crisp transportation algorithm, the zero point method and also, provides that the optimal fuzzy solution and the optimal fuzzy objective value of the FTP do not contain any negative part. For illustrating, a FTP is solved by using the proposed method. The proposed method is an appropriate method to apply for finding the fuzzy optimal solution of FTPs occurring in real life situations.

2000 Mathematics Subject Classifications: 90D06, 90C08, 90C90

Keywords: Positive fuzzy number; Fuzzy transportation Problem; Zero point method; Positive fuzzy optimal solution.

Introduction

In today's highly competitive market, many organizations trying to find better ways to create and deliver value to customers become stronger. How and when to send the products safely to the customers in the quantities with minimum cost become more challenging. To meet this challenging, transportation models provide a powerful framework. Transportation models have wide applications in logistics and supply chain for reducing the transportation cost. Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. Fuzzy numbers introduced by Zadeh [11] may represent impressive data. Zimmermann [12] obtained an optimal solution to a FTP by fuzzy linear programming. In the literature, there are several methods [1-9] for finding the fuzzy optimal solution of FTPs where some or all parameters are represented by trapezoidal fuzzy numbers. But there are shortcomings in some existing methods which are as follows:

i. In the existing methods [7,1,6], FTPs are first converted into equivalent crisp transportation problem (CTP) using $\alpha-$cut method, which are then solved by standard methods. Thus, the final results of a FTP are real numbers, which represents a compromise in terms of fuzzy numbers.

ii. In the existing methods [3,8], the optimal solution of some of the fuzzy decision variables and the optimal objective fuzzy value of a FTP have negative part which depicts that quantity of the product and transportation cost may be negative. But the negative quantity of the product and negative transportation cost has no physical meaning.

In this paper, we develop a new method for finding a fuzzy optimal solution of a FTP where all parameters are fuzzy numbers. To overcome the shortcomings of the existing methods [7,1,6, 3, 8], the proposed method provides non-negative fuzzy optimal solution and non-negative optimal fuzzy objective value of FTPs. The proposed method is based on the zero point method which is an algorithm for solving of crisp transportation problems. So, unbalanced transportation problem can be also solved by the new method. By means of a numerical example, the proposed method of solving a fuzzy transportation problem is illustrated. For finding the fuzzy optimal solution of fuzzy transportation problems occurring in real life situations, the proposed method is an appropriate method for solving a real fuzzy life transportation problems and also, provides an applicable optimal solution.

Fuzzy number and Fuzzy transportation problem

We need the following mathematical orientated definitions of fuzzy number and membership function which can be found in Zadeh [12] .

Definition 2.1: A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number denoted by (a_1,a_2,a_3,a_4) where a_1,a_2,a_3 and a_4 are real numbers and its member ship function $\mu_{\tilde{a}}(x)$ is given below.

$$\mu_{\tilde{a}}(x)=\begin{cases} 0 & \text{for } x\le a_1 \\ (x-a_1)/(a_2-a_1) & \text{for } a_1\le x\le a_2 \\ 1 & \text{for } a_2\le x\le a_3 \\ (a_4-x)/(a_4-a_3) & \text{for } a_3\le x\le a_4 \\ 0 & \text{for } x\ge a_4 \end{cases}$$

Let $F(R)$ be a set of all trapezoidal fuzzy numbers over R, a set of real numbers.

Definition 2.2: Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be in $F(R)$. Then,

a. $\tilde{A}$ and $\tilde{B}$ are said to be equal if $a_i = b_i$, i =1,2,3,4 and

b. $\tilde{A}$ is said to be less than or equal $\tilde{B}$ if $a_i \le b_i$, i =1,2,3,4.

Definition 2.3: Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be in $F(R)$. $\tilde{A}$ is said to be positive if $a_i \ge 0$, i =1,2,3,4.

Definition 2.4: Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be in $F(R)$. $\tilde{A}$ is said to be integer if $a_i \ge 0$, i =1,2,3,4 are integers.

Consider the following fuzzy transportation problem (FTP) having fuzzy costs , fuzzy sources and fuzzy demands,

$$\text{(P)}\quad \text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \text{ , for i = 1,2,…,m} \qquad (2.1)$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j \text{ , for j = 1,2,…,n} \qquad (2.2)$$

$$\tilde{x}_{ij} \succeq 0 \text{, for i = 1,2,…,m and j = 1,2,…,n,} \qquad (2.3)$$

where m = the number of supply points; n = the number of demand points; $\tilde{x}_{ij} \approx (x_{ij}^{\ 1}, x_{ij}^{\ 2}, x_{ij}^{\ 3}, x_{ij}^{\ 4})$ is the uncertain number of units shipped from supply point i to demand point j; $\tilde{c}_{ij} \approx (c_{ij}^{\ 1}, c_{ij}^{\ 2}, c_{ij}^{\ 3}, c_{ij}^{\ 4})$ is the uncertain cost of shipping one unit from supply point i to the demand point j ; $\tilde{a}_i \approx (a_i^{\ 1}, a_i^{\ 2}, a_i^{\ 3}, a_i^{\ 4})$ is the uncertain supply at supply point i and $\tilde{b}_j \approx (b_j^{\ 1}, b_j^{\ 2}, b_j^{\ 3}, b_j^{\ 4})$ is the uncertain demand at demand point j.

Dynamic Backward Method

We need the following theorem to prove the proposed method solution of TP is optimal.

Theorem 3.1: Let $[x_{ij}^{\circ 4}] = \{x_{ij}^{\circ 4}, i = 1,2,...,m \text{ and } j = 1,2,...,n\}$ be an optimal solution of (P_4), $[x_{ij}^{\circ 3}] = \{x_{ij}^{\circ 3}, i = 1,2,...,m \text{ and } j = 1,2,...,n\}$ be an optimal solution of (P_3) ,

$[x_{ij}^{\circ 2}] = \{x_{ij}^{\circ 2}, i = 1,2,...,m \text{ and } j = 1,2,...,n\}$ be an optimal solution of (P_2) and $[x_{ij}^{\circ 1}] = \{x_{ij}^{\circ 1}, i = 1,2,...,m \text{ and } j = 1,2,...,n\}$ be an optimal solution (P_1) where

$$(P_4) \text{ Minimize } Z_4 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^4 x_{ij}^4$$

subject to

$$\sum_{j=1}^{n} x_{ij}^4 = a_i^4 \text{ ,for i = 1,2,…,m}$$

$$\sum_{i=1}^{m} x_{ij}^4 = b_j^4 \text{ , for j = 1,2,…,n}$$

$$x_{ij}^4 \geq 0 \text{, for i = 1,2,…,m and j = 1,2,…,n and integers.}$$

and for k = 4, 3, 2,

$$(P_{k-1}) \text{ Minimize } Z_{k-1} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k-1} x_{ij}^{k-1}$$

subject to

$$\sum_{j=1}^{n} x_{ij}^{k-1} = a_i^{k-1} \text{ , for i = 1,2,…,m}$$

$$\sum_{i=1}^{m} x_{ij}^{k-1} = b_j^{k-1} \text{, for j = 1,2,…,n}$$

$$x_{ij}^{k-1} \leq x_{ij}^{\circ k} \text{ , for i = 1,2,…,m and j = 1,2,…,n}$$

$$x_{ij}^{k-1} \geq 0 \text{, for i = 1,2,…,m and j = 1,2,…,n and integers.}$$

Then, $[\tilde{x}_{ij}^{\circ}] = \{\tilde{x}_{ij}^{\circ} = (x_{ij}^{\circ 1}, x_{ij}^{\circ 2}, x_{ij}^{\circ 3}, x_{ij}^{\circ 4}), i = 1,2,...,m \text{ and } j = 1,2,...,m\}$ is an optimal solution of the given problem (P).

Proof: Let $[\tilde{y}_{ij}] = \{\tilde{y}_{ij} = (y_{ij}^1, y_{ij}^2, y_{ij}^3, y_{ij}^4), i = 1,2,...,m \text{ and } j = 1,2,...,n\}$ be a feasible solution of (P). Clearly, $[y_{ij}^1],[y_{ij}^2],[y_{ij}^3]$ and $[y_{ij}^4]$ are feasible solutions of $(P_1),(P_2),(P_3)$ and (P_4) respectively.

Now, since $[x_{ij}^{\circ 1}],[x_{ij}^{\circ 2}],[x_{ij}^{\circ 3}]$ and $[x_{ij}^{\circ 4}]$ are optimal solutions of $(P_1),(P_2),(P_3)$ and (P_4) respectively, we have

$$Z_1([x_{ij}^{\circ 1}]) \le Z_1([y_{ij}^1]);\ \ Z_2([x_{ij}^{\circ 2}]) \le Z_2([y_{ij}^2]);$$

$$Z_3([x_{ij}^{\circ 3}]) \le Z_3([y_{ij}^3]) \ \text{ and }\ Z_4([x_{ij}^{\circ 4}]) \le Z_4([y_{ij}^4])$$

That is, $Z([\tilde{x}_{ij}^{\circ}]) \le Z([\tilde{y}_{ij}])$, for all feasible solution of the problem (P).

Therefore, $[\tilde{x}_{ij}^{\circ}] = \{\tilde{x}_{ij}^{\circ} = (x_{ij}^{\circ 1}, x_{ij}^{\circ 2}, x_{ij}^{\circ 3}, x_{ij}^{\circ 4}), i = 1,2,...,m$ and $j = 1,2,...,m\}$ is an optimal solution to the given problem (P).

Hence the theorem.

Remark 3.1: The optimal fuzzy solution

$$[\tilde{x}_{ij}^{\circ}] = \{\tilde{x}_{ij}^{\circ} = (x_{ij}^{\circ 1}, x_{ij}^{\circ 2}, x_{ij}^{\circ 3}, x_{ij}^{\circ 4}), i = 1,2,...,m \text{ and } j = 1,2,...,m\}$$

and the total minimum fuzzy transportation cost

$$Z([\tilde{x}_{ij}^{\circ}]) = (Z_1([x_{ij}^{\circ 1}]), Z_2([x_{ij}^{\circ 2}]), Z_3([x_{ij}^{\circ 3}]), Z_4([x_{ij}^{\circ 4}]))$$

are positive because fuzzy supply at each origin, fuzzy demand at each destination, fuzzy transportation costs and fuzzy decision variables are positive.

Dynamic backward Method

We, now introduce a new method for solving fuzzy transportation problem which is based on the crisp transportation algorithm namely, the zero point method introduced by Pandian and Natarajan [9].

The proposed method is as follows

Algorithm

Step 1: Construct the problem (P_4) from the given FTP (P) and solve it by the zero point method. Let $\{x_{ij}^{\circ 4}, i = 1,2,...,m$ and $j = 1,2,...,m\}$ be an optimal solution of (P_4) .

Step 2: Construct the problem (P_3) from the given FTP and solve it by the zero point method. Let $\{x_{ij}^{\circ 3}, i = 1,2,...,m$ and $j = 1,2,...,n\}$ be an optimal solution of (P_3) .

Step 3: Construct the problem (P_2) from the given FTP and solve it by the zero point method. Let $\{x_{ij}^{\circ 2}, i = 1,2,...,m$ and $j = 1,2,...,n\}$ be an optimal solution of (P_2) .

Step 4: Construct the problem (P_1) from the given FTP and solve it by the zero point method. Let $\{x_{ij}^{\circ 1}, i = 1,2,...,m$ and $j = 12,..,n\}$ be an optimal solution of (P_1) .

Step 5: $\{\tilde{x}_{ij}^{\circ} = (x_{ij}^{\circ 1}, x_{ij}^{\circ 2}, x_{ij}^{\circ 3}, x_{ij}^{\circ 4}), i = 1,2,...,m$ and $j = 1,2,...,n\}$ is an optimal solution to the given FTP, (P) by the Theorem 3.1.

Remark 3.2: An unbalanced fuzzy transportation problem can be also solved by the proposed method because it is based on the zero point method, the crisp transportation algorithm .

Numerical Example

The proposed method is illustrated by the following example.

Example 4.1: Consider the following fully fuzzy transportation problem.

					Supply
	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
	(0,1,2,4)	(0,0,1,1)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand	(4,7,8,11)	(0,5,6,11)	(1,3,4,6)	(1,2,3,4)	(6,17,21,32)

The given FTP is a balanced one since total fuzzy demand = total fuzzy supply = (6,17,21,32).

Now, from the given FTP, the problem (P_4) as follows:

					Supply
	4	6	14	11	12
	4	1	8	3	3
	8	12	19	12	17
Demand	11	11	6	4	32

Now, by the zero point method , the optimal solution of (P_4) is $x_{12}^{\circ 4} = 11$; $x_{13}^{\circ 4} = 1$; $x_{23}^{\circ 4} = 3$; $x_{31}^{\circ 4} = 11$; $x_{33}^{\circ 4} = 2$; $x_{34}^{\circ 4} = 4$ and the minimum transportation cost = 278.

Now, from the given FTP, the problem (P_3) as follows:

					Supply
	3	4	12	8	7
	2	1	7	2	2
	6	9	16	10	12
Demand	8	6	4	3	21

with $x_{1j}^{3} \leq x_{ij}^{\circ 4}$.

Now, by zero point method , the optimal solution of (P_3) is $x_{12}^{\circ 3} = 6$; $x_{13}^{\circ 3} = 1$; $x_{23}^{\circ 3} = 2$; $x_{31}^{\circ 3} = 8$; $x_{33}^{\circ 3} = 1$; $x_{34}^{\circ 3} = 3$ and the minimum transportation cost is 144.

Now, from the given FTP, the problem (P_2) as follows:

					Supply
	2	3	11	7	6
	1	0	6	1	1
	5	8	15	9	10
Demand	7	5	3	2	17

with $x_{ij}^{2} \leq x_{ij}^{\circ 3}$.

Now, by zero point method , the optimal solution of (P_2) is $x_{12}^{\circ 2} = 5$; $x_{13}^{\circ 2} = 1$; $x_{23}^{\circ 2} = 1$; $x_{31}^{\circ 2} = 7$; $x_{33}^{\circ 2} = 1$; $x_{34}^{\circ 2} = 3$ and the minimum transportation cost is 100.

Now, from the given FTP, the problem $(P1)$ as follows:

					Supply
	1	1	9	5	1
	0	0	5	0	0
	3	5	12	7	5
Demand	4	0	1	1	6

with $x_{ij}^{1} \leq x_{ij}^{\circ 2}$

Now, by zero point method, the optimal solution of (P_1) is $x_{12}^{\circ 1} = 0$; $x_{13}^{\circ 1} = 1$; $x_{23}^{\circ 1} = 0$; $x_{31}^{\circ 1} = 4$; $x_{33}^{\circ 1} = 0$; $x_{34}^{\circ 1} = 1$ and the minimum transportation cost is 28.

Thus, the optimal solution to the given FTP is $\tilde{x}_{12} = (0,5,6,11)$; $\tilde{x}_{13} = (1,1,1,1)$; $\tilde{x}_{23} = (0,1,2,3)$; $\tilde{x}_{31} = (4,7,8,11)$; $\tilde{x}_{33} = (0,1,1,2)$; $\tilde{x}_{34} = (1,2,3,4)$ and the total minimum fuzzy transportation cost is (28,100,144, 278).

Conclusion

The main advantage of the proposed method is that the obtained fuzzy optimal solution and fuzzy optimal value both are non-negative fuzzy numbers. Since the proposed method is based on the classical transportation method so it is easy to learn and to apply the proposed method to find the fuzzy optimal solution of fuzzy transportation problems occurring in real life situations. The proposed method provides an applicable optimal solution which helps the decision makers while they are handling real life transportation problems having fuzzy parameters.

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