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A Comparative Study of Different Techniques for P-Median Problem: A review

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Abstract

P-median problem refers to decrease the average distances in facility location problem. It is the burning issue in facility location problem, the prominent area of research in present scenario. Through this paper, efforts are made to highlight the different aspects of p-median solving techniques. Different techniques are analyzed and compared to reveal the best results.

Introduction

There is a set of facilities such as setting up of a supply chain of a business, locating necessary services such as health care and education, and construction of transportation networks etc. which are provided to public as well as private sector. In many locations service to customers from the facilities that are being located depends on the distance of the customer and the facility to which the customer is assigned. Customers are generally, though not always, assigned to the nearest facility. So set covering problem is to allocate different facilities at different locations so that different sectors got benefits. Associated with each demand node i is subset of N_i of the candidate facility j that can cover demand nodes. Often, demand nodes are said to be covered if the shortest path distance between the demand nodes and the facility is less than or equal to a coverage distance. For this we have to adjust these facilities so that

1. We can cover maximum area.
2. Average distance of facilities should be as minimum as possible.
3. The distance of each node to the facility should be minimum.

The set covering problem is to find a minimum cost set of facilities from among a finite set of candidate facilities so that every demand node is covered by at least one facility, or we can say that Facility location problem is a resource allocation problem that mainly deals with adequate placement of various types of facilities. a) To serve a distributed set of demands satisfying the nature of interaction between the demands. b) Optimizing the cost of placing the facilities and the quality of service [1]. Several facility location problems can be formulated depending on the constraints on the nature of the facilities, and objective functions, e.g. the cost reduction, demand capture, fast response time etc. For locating the emergency facilities, such as hospitals, fire-fighting stations etc., covering a region using minimum radius circles is a natural mapping of the corresponding facility location problem where the objective is to minimize the worst-case response time. A comprehensive treatment of facility location problems and their formulations can be found in [2, 3, 4].

Literature Review

The *p*-median Problem (*p*-M) is to locate *p* new facilities, called medians, on the network *G* in order to minimize the sum of the weighted distances from each node to its nearest new facility (Francis *et al*., 1992). If $p \geq 2$, then this problem can be viewed as a location-allocation problem (LAP). This is because the location of the new facilities will determine the allocation of their service in order to best satisfy the nodal demands. In 1964, Hakimi proved that in networks, a set of optimal locations will always correspond with the vertices. He also proposed an enumerative-graph theoretic approach for the problem. In 1970, Revelle and Swain proposed other procedures to solve this problem after reformulating it as an integer programming (IP) problem. After that in 1972, Jarvinen *et al*. also used this IP formulation and proposed a branch-and-bound algorithm for this problem. Several heuristic procedures have been developed, such as in 1964, Maranzana and in 1968, Teitz and Bart due to NP-hardness. In 1993, Beasly has also developed Lagrangian heuristics for this *p*-median location problem, based on Lagrangian relaxation and subgradient optimization concepts. Several variants and extensions of the *p*-median problem have been addressed in the literature. In 1991, one type of variant studied by Pesamosca considers the interaction weights between the new facilities as well as the connection scheme as a tree. This case was treated as a problem EMFLP (Euclidean distance multifacility location problem) on a tree because here p=2 and its optimality conditions were then obtained using the optimality conditions of *p* problems of the type ESFL (Euclidean single facility location problem) a problem is called ESFL when n =1. Accordingly, for solving the problem EMFLP, a fixed point algorithm was developed to iteratively solve Problem ESFL using the Weiszfeld algorithm if differentiability is met, and otherwise, the algorithm switches over to Miehle's algorithm. Another type of variant involves placing the capacity restrictions on the facilities to be located. When the capacity is finite, the resulting problem is called a capacitated problem; otherwise the problem is uncapacitated. In 1986b, Cavalier and Sherali presented exact algorithms to solve the *p*-median problem on a chain graph and the 2-median problem on a tree graph, where the demand density functions are assumed to be piecewise uniform. In 1987, for the uncapacitated p-median problem, Chiu addressed the 1- median problem on a general network as well as on a tree network. Dynamic location considerations on networks are addressed by Sherali in 1991. In 1993, Francis *et al*. developed a median-row-column aggregation algorithm to solve large-scale rectilinear distance *p*-median problems. On the other hand, Sherali and Nordai in 1988 gave certain localization results and algorithms for solving the capacitated *p*-median problem on a chain graph and the 2- median problem on a tree graph. Another variant considered the treatment of a continuous demand over the network, which arises in some situations such as the location of public service facilities or in probabilistic distributions of demand. In 1978 Minieka, in 1979 Handler and Mirchandani, in 1987 Chiu and in 1982 Derardo *et al*. contribute in this varient. Combining the last 2 variants, in 1991 Sherali and Rizzo solved an unbalanced, capacitated *p-median* problem on a chain graph with a range of link demands. For solving this problem, they considered two unbalanced cases, the shortage and over-capacitated cases, provided a first-order characterization of optimality for these two problems and developed an enumerative algorithm based on a partitioning of the dual space. There are still further variants that include capacity restrictions on links, probabilistic travel times on links, and maximum distance constraints. A lot more work is done in this p-median stream.

Techniques used to solve p-median problem

Greedy Heuristics

A number of algorithms are there for solving this P-median problem of set covering. There is a heuristic algorithm for solving set covering problem this is known as Greedy Algorithm since it does what is best at each step without looking ahead the impact of decision taken by algorithm on future decisions. Greedy heuristics were the first to be proposed for facility location problems by Hochbaum [11]. In this firstly we find a candidate site that covers the most uncovered demand. Secondly locate next facility at that site. Thirdly remove all demands covered by the most recently sited facility from the problem. If P sites been located or all demands covered then we can stop the process otherwise we repeat the process again and again. In this we simply evaluate that how many demand nodes are covered by a facility and we select a facility which covers most demands. Hochbaum [11] considered greedy heuristics for facility location problems in which the distances need not satisfy metric properties.

LP Rounding Techniques

The idea behind LP Rounding is to write the problem as an integer linear program, relax its integrality restraints to efficiently solve the general linear program, and then move the LP solution to a nearby integral point in the feasible solution space. The difficulty of this process lies in the rounding step, which need bound. It's a 2-steps process.

Step 1 (Filtering): Consider all the facilities to which a client j is fractionally assigned. Among these, let N_j denote the subset of "near" facilities located within a distance $2D_j$ of client j. Here we have to choose the clients with fractional assignment value given for facility.

Step 2 (Rounding the Filtered Solution): Among all clients, select the client j with minimum D_j . Among the facilities in N_j, select the facility with the minimum f_i, say $k \in 2N_j$. Form a new solution x'', y'' in which the client j is connected to the cheapest facility k in N_j and to no other facility . Facility k is selected into the solution and all others facilities in N_j are dropped. The cost of selecting facility k and dropping all other facilities in N_j is not greater. Another round of allotments is started by selecting the un-allotted client j with minimum D_j and allotting j and all un-allotted clients in the extended neighborhood of j using the same procedure. The algorithm continues till all clients are allotted to some facility. This algorithm based on rounding the fractional optimal solution to the LP relaxation of the original integer programs were proposed by Shmoys et al. [12]. They used the filtering idea proposed by Lin and Vitter [13] to round the optimal fractional solution of a linear program relaxation of the integer program formulation to obtain a half-integral solution. They also proposed a heuristic to convert the half-integral solution to an integral solution and proved an approximation factor of 6.66. This idea was also combined with randomization by Chudak and Shmoys [14].

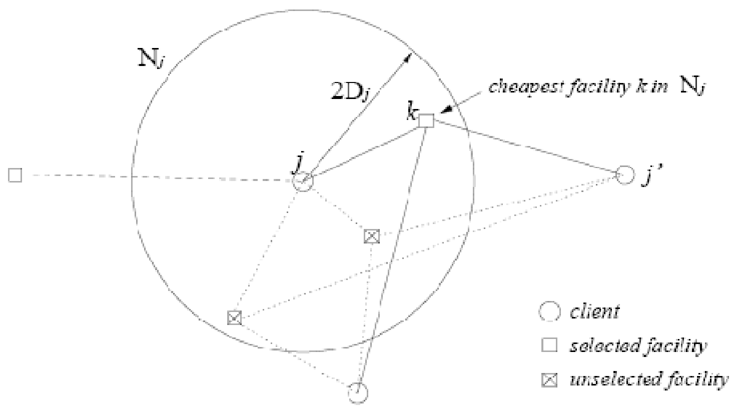

Figure3: LP Rounding

Primal-Dual Techniques

This primal dual technique perform its work in following way (i) starts with the feasible dual solution $y = 0$ (ii) increases y continuously until a dual constraint(s) becomes tight (iii) in this case the corresponding primal variable(s) enter the solution (iv) continues as long as the primal solution generated so far is not feasible (v) goes over the primal variables in the solution in a reverse order and removes them whenever feasibility is maintained. Algorithms for facility location based on primal-dual techniques were proposed by Jain and Vazirani [16]. Two-phase primal-dual scheme is used to solve the uncapacitated facility location problem. Their technique's originality was in relaxing the primal conditions while satisfying all the complimentary slackness conditions. This allowed them to prove a stronger approximation theorem for uncapacitated facility location. This also allowed them to obtain approximation algorithms for a variety of facility location problems including the k-median problem using the Lagrangian relaxation technique.

Local Search Techniques

Algorithms for facility location based on local search are perhaps the most adaptable. Local search heuristics have been used for many years and one such heuristic was proposed by Kuehn and Hamburger [17]. However, Korupolu et al. [18] showed for the first time that a worst case analysis of the local minimas computed by these heuristics was possible and they showed constant factor approximations to many facility location problems which were comparable to those obtained by other techniques. For certain variants of facility location problems, local search is the only technique known to give constant factor approximations. The first analysis of a local search heuristic for the k-median problem was given by Korupolu et al. [18, 19]. They considered local operations of adding a facility, dropping a facility and swapping a pair of facilities at each step. Their result gives a pseudo-approximation to the k-median problem. Here [21] analyze local search heuristic for the k-median problem proposed by Kuehn and Hamburger [17]. And [20] prove that its locality gap is 5. [21] show that this guarantee can be improved by considering stronger local operation. He considers p-swap operation which deletes p facilities from the current solution and adds p facilities to the current solution. We show that the local search heuristic with p-swaps has a locality gap of $3+2/p$. This is the first analysis of a local search heuristic which opens at most k facilities.

Comparative Study of p-median Techniques

In the previous section we have gone through the number of approaches to solve the p-median problem. This section presents a comparative study of all these techniques as well as their pros and cons. Greedy heuristic was the first to be proposed for facility location problems by Hochbaum [11]. This technique describe that how many demand nodes are covered by a facility and we select a facility which covers most demands. But, this technique did not yield any approximation guarantee for the k-median problem. LP rounding technique based on rounding the fractional optimal solution to the LP relaxation of the original integer programs were proposed by Shmoys et al. [12]. They used the filtering idea proposed by Lin and Vitter [13] to round the optimal fractional solution of a linear program relaxation of the integer program formulation to obtain a half-integral solution. They also proposed a heuristic to convert the half-integral solution to an integral solution and proved an approximation factor of 6.66. Another proposed primal-dual algorithm for the UFL problem which has an approximation factor of 3. Their solution satisfied a stronger property that the algorithm computes a solution whose sum of facility cost and three times the service cost is at most three times the total cost of the optimal solution. This algorithm with the Lagrangian relaxation obtains an approximation ratio of 6 for the k-median problem. The modified primal-dual algorithm for the UFL problem given by Jain and Vazirani is satisfy a “continuity” property and used it to demonstrate the integrality gap. Kuehn and Hamburger [6] proposed a local search heuristic which considered a simple swap operation at each step. The algorithm starts with an arbitrary subset of k facilities. At each step, it tries to improve the solution by removing one of the facilities from current solution and adding a new facility. The algorithm terminates when the solution cannot be improved in this manner. This is one of the most popular heuristic used in practice.

Results and Conclusion

In this paper, we present the set covering problem. This deals with the allocation of facilities with an objective to reduce the total or an average distance between facilities and demand nodes. We have discussed the different available techniques to solve the P-median problem, which addresses the set covering problem. During this discussion we have taken Greedy Heuristic, LP-Rounding, Primal Dual and Local Search techniques into account. On the basis of this comparative study we conclude that each technique has its own way to solve the problem but the technique which has shown high impact on final solution that is Local Search. In other words, we can say that Local search is the one algorithm which has provided best results for P-median problem so far.

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