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Dimension Effect on Dynamic Stress Equilibrium in SHPB Tests

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Abstract

The state of dynamic equilibrium of stress waves in a specimen under test by SHPB is not automatically achieved due to the effect of loading rate and the specimen dimensions. Therefore, a guideline for proper SHPB experiments and analytical investigations have been conducted to examine the process of wave equilibrium in different material specimens. Compressive experiments on different materials with SHPB were conducted to determine the effect of length and diameter as well as the density of the material on the time required for the stress equilibrium to be reached within the specimen. The loading rate is kept in all measurements as the loading pulses more like the actual loading stress pulse in SHPB to have a better idea of the real response of the materials under a constant strain rate. Although, the actual incident pulse shaped is not very suitable for a parametric study of the thickness effect.

Keywords: SHPB, stress equilibrium, multiple reflection, dimension effect.

1. Introduction

The split Hopkinson pressure bar (SHPB) technique (Fig. (1)) is a well-established method used for the determination of high strain rate properties of materials. In the SHPB test, the sample (a small solid cylinder of the tested material) is sandwiched between two long-high strength steel bars. The sample can be compressed by a stress pulse generated by impacting the end of one of the steel bars (incident bar) using a steel projectile. The projectile has the same diameter as the steel bars, and a length that is appropriate for providing suitable loading pulse duration.

The stress pulses in the bars are recorded by strain gauges placed in equidistant from the sample. The stress-strain properties can be obtained from the amount of the stress pulse reflected and transmitted by the sample, assuming that stress equilibrium exists throughout the sample.

When the stress wave travels along the incident bar in a positive direction, it hits the first interface between the incident bar and the sample. The difference in impedance between the bars and the sample makes the wave partially reflect back in the negative direction as a reflected pulse σ_R, while the rest of the wave passes through the first interface as a transmitted pulse (from the first interface) at time zero σ_{T0}. The σ_{T0} travels toward the second interface between the sample and the transmitter bar and again due to the impedance mismatch, part of σ_{T0} will reflect back toward the first face and the rest will transmit into the transmitter bar.

The time required for the σ_{T0} pulse to reach the second face of the sample, or in other words, the time required for the pulse to travel between the two faces of the sample is called the traverse time **tt**, which defined as:

$$tt = \frac{\ell}{c_s}$$

where ℓ is the sample length and c_s is the wave speed in the sample.

Therefore, after one period of traverse time, the reflection occurs at the second face of the sample creating σ_{R1} and σ_{T1}. This process continues, so that multiple reflections occur within the sample and a succession of reflected waves become "trapped" inside the sample propagating back and forth between the two interfaces [1].

Theoretically, reflected waves thus "trapped" in this manner will undergo an infinite number of reflections between the interfaces; however, at each reflection the intensity of the reflected stress will decrease since a portion of the wave is transmitted each time. Eventually, the trapped wave will have decayed to negligible amplitude. The effect of multiple reflections within the sample is to cause a dispersion of the incident wave. Thus, if the incident wave has a sharp rise time before reaching a constant maximum stress, the transmitted wave will have a less sharp rise time. These multiple reflections cause a non-uniform stress distribution that may lead to inaccurate estimates of the initial stress/strain properties of the sample [2].

The theory of SHPB analysis is based on the equation $\varepsilon_T = \varepsilon_I + \varepsilon_R$. This equation is only true if the forces and therefore the stresses are equal on both sides of the sample. This equilibrium condition will not arise immediately as the stress wave is incident on a SHPB sample, but occurs after several reflections have taken place inside the sample.

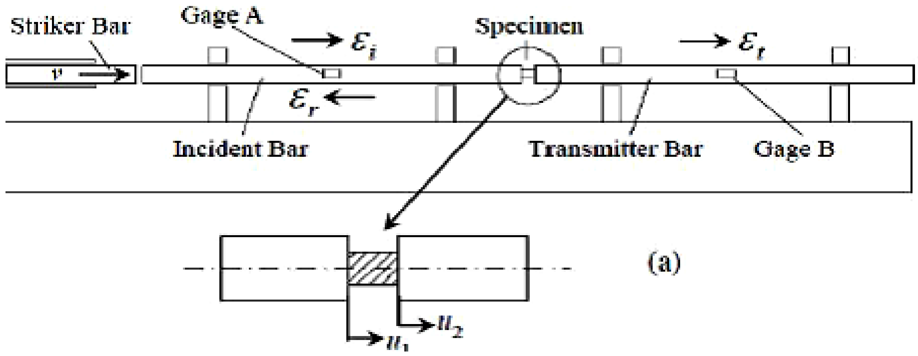

Fig. 1: Schematic diagram for SHPB system.

2. Theory

Wave propagation behaviour for elastic bars is well-understood and mathematically predictable [8], and from the elementary wave theory, the wave equation can be shown:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{C_o^2}\frac{\partial^2 u}{\partial t^2} \tag{1}$$

Where $C_0 = \sqrt{E/\rho}$ is the fundamental longitudinal wave velocity, and u is the displacement. From the wave equation, the stress in the bar can be shown to be

$\sigma = -\rho C_0 v$, where ρ is the density of the bar and v is the velocity of the particles of the bar subjected to the pulse.

Consider an incident elastic wave of compressive stress σ_I moving to the right as in Fig. (2), through the bar s_1 of cross-sectional area A_1. This wave is partially reflected and partially transmitted at the surface of discontinuity AB, where another bar s_2 of cross-sectional area A_2 is perfectly attached to s_1. If A_2 were zero, the wave would be reflected completely, whilst if s_1 and s_2 were of identical area and material, then the incident wave would be totally transmitted. However, since s_1 and s_2 have different areas and are of different materials, then at AB the incident wave must be reflected and transmitted.

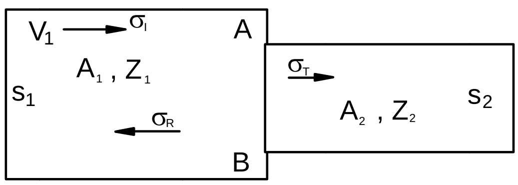

Fig. 2: Interface between two bars.

The stress wave transmitted through s_2 is σ_T, and reflected back through s_1 is σ_R. Where the initial stress in s_1 is $\sigma_I = Z_1\upsilon_1$ at the plane AB the following conditions are satisfied:

i. the forces acting on the plane AB acting from s_1 and s_2 are equal at all times, and,
ii. the particle velocity in plane AB, in the material for s_1 and s_2 are equal.
iii. According to (i) we have, assuming σ_I , σ_R and σ_T are taken to be compressive, then

$$A_1(\sigma_I + \sigma_R) = A_2\sigma_T, \tag{2}$$

σ_I, and σ_R are associated with waves travelling in opposite directions, therefore, (ii) gives

$$\upsilon_I - \upsilon_R = \upsilon_T \tag{3}$$

where υ denotes particle speed and subscripts I, R, and T refer to incident, reflection, and transmission. In general the stress (σ) is related to density (ρ), sound speed (c), and particle speed (υ) by:

$$\upsilon = \frac{\sigma}{\rho c} = \frac{\sigma}{Z} .$$

The transmitted and reflected stresses can be derived to be;

$$\therefore \sigma_T = \frac{2A_1Z_2}{A_1Z_1 + A_2Z_2}\sigma_I = T\,\sigma_I \tag{4}$$

The reflected stress is obtained as:

$$\therefore \sigma_R = \frac{A_2Z_2 - A_1Z_1}{A_1Z_1 + A_2Z_2}\sigma_I = R\,\sigma_I \tag{5}$$

Where T and R are the transmission and reflection coefficients respectively.

3. Analysis and Computation

The principles of SHPB technique are well-documented [6]. The theory of SHPB shows that the nominal strain ε_s, strain rate $\dot{\varepsilon}$, and nominal stress σ_s are given by the following equations:

$$\varepsilon_s = \frac{-2c_b}{\ell}\int_0^t \varepsilon_R dt \tag{6}$$

$$\dot{\varepsilon} = \frac{-2c_b}{\ell}\varepsilon_R \tag{7}$$

$$\sigma_s = \frac{A_b}{A_s}E_b\varepsilon_T \tag{8}$$

where ℓ , and A_s are the length and the cross-sectional area of the sample, while c_b , A_b , and E_b are the wave speed, cross-sectional area and Young's modulus for the bar respectively. The above equations have been derived assuming that stress equilibrium exists in the sample .

The stress at the incident bar/specimen interface (σ_{B1}) is

$$\sigma_{B1} = \frac{(\sigma_I - \sigma_R)A_b}{A_s}$$

And the stress at specimen/transmitter (σ_{B2}) bar interface is

$$\sigma_{B2} = \frac{\sigma_T A_b}{A_s}$$

Where σ_I, σ_R and σ_T are the incident, reflected and transmitted stress in the bars respectively. Assuming incompressible plasticity, then $A_o \ell_o = A_S \ell_S$ (where A_o is the original cross-sectional area of the specimen and A_S is the instantaneous cross-sectional area of the specimen, and A_b is the cross-sectional area of the bar.

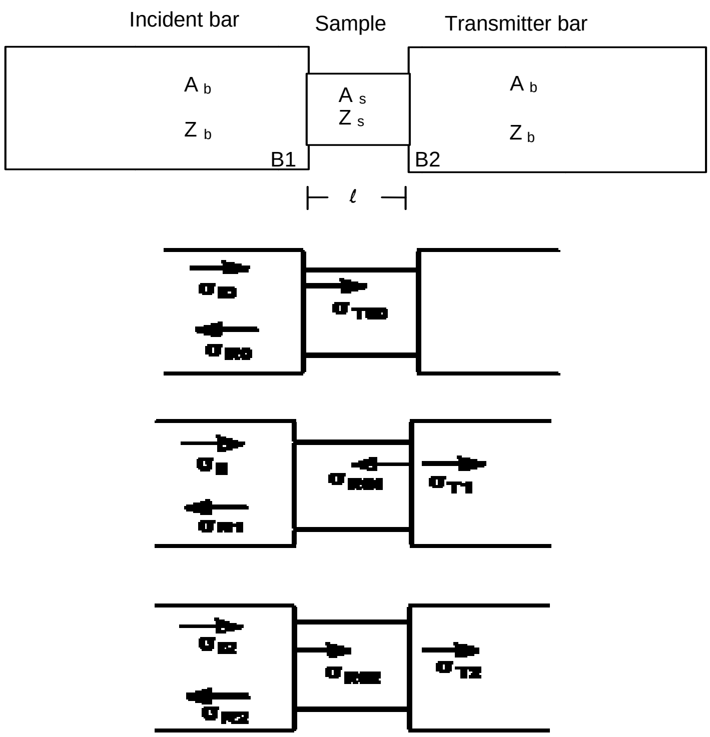

Fig. 3: Pressure bars and sample.

Consider an elastic stress wave incident on the first interface (B1). The first reflection from this interface occurs at time t=0 as in Fig. (3), where:

σ_{I0} = incident stress,

σ_{R0} = reflected stress at B1,

σ_{T1} = transmitted stress at the interface B2 at time t=ℓ / c_s which is called the traverse time (tt), and,

σ_{RS1} = reflected stress at B2 at time tt.

If the incident stress wave has a finite duration, then the stress σ_I may be time dependent.

At t = 1 traverse time, the stress is transmitted into the sample. It is important to note at this stage that if σ_{I0} is compressive (+ve), then according to equation (4) σ_{T1} will also be compressive (+ve), while from equation (5) σ_{R0} may be compressive (+ve) or tensile (-ve) depending on the mechanical impedance Z_s of the sample and its cross-sectional area A_s.

Equations 4 and 5 can be re-written for the SHPB as;

$$\sigma_T = \frac{2A_b Z_s}{A_s Z_s + A_b Z_b}\sigma_I \qquad (8)$$

$$\sigma_R = \frac{A_s Z_s - A_b Z_b}{A_s Z_s + A_b Z_b}\sigma_I \qquad (9)$$

and the transmission and reflection coefficients can be written at the interface B1 as:

$$T_1 = \frac{2A_b Z_s}{A_s Z_s + A_b Z_b} \qquad (10)$$

$$R_1 = \frac{A_s Z_s - A_b Z_b}{A_s Z_s + A_b Z_b} \qquad (11)$$

At the interface B2 the reflection will occur inside the sample, so the coefficient is denoted as R_2 and the transmission coefficient as T_2 (where the stress wave has transmitted partially into the transmitter bar).

$$R_2 = \frac{A_b Z_b - A_s Z_s}{A_s Z_s + A_b Z_b} = -R_1 \qquad (12)$$

$$T_2 = \frac{2A_s Z_b}{A_s Z_s + A_b Z_b} \qquad (13)$$

For a compressive incident stress, the transmitted stress will always be compressive; while the reflected stress can be tensile or compressive. Usually $A_b Z_b \rangle A_s Z_s$ making R_1 negative and R_2 positive [2].

The build-up of the reflected and the transmitted pulses in the pressure bars caused by the multiple reflections between the interfaces, and the build up of the transmitted and reflected stress pulses in the SHPB sample can be equated as;

At 0 traverse time $\sigma_{TS0} = T_1 \sigma_{I0}$ and $\sigma_{R0} = R_1 \sigma_{I0}$, and at 1 traverse time $\sigma_{T1} = T_2 \sigma_{TS0} = T_1 T_2 \sigma_{I0}$, $\sigma_{RS1} = R_2 \sigma_{RS0} = -R_1 T_1 \sigma_{I0}$ and $\sigma_{R1} = R_1 \sigma_{I1}$. Where at the second traverse time the stress are; $\sigma_{T2} = T_1 T_2 \sigma_{I1}$, $\sigma_{RS2} = R_2 \sigma_{RS1} + T_1 \sigma_{I2} = R_1^2 T_1 \sigma_{I1} + T_1 \sigma_{I2}$ and $\sigma_{R2} = R_1 \sigma_{I2} - R_1 T_1 T_2 \sigma_{I0}$ and so on. After N of traverse times the transmitted and reflected stresses equal ;

$\sigma_{TN} = R_1^2 \, \sigma_{T(N-2)} + T_1 T_2 \sigma_{I(N-1)}$, *for* $N > 2$ and $\sigma_{RN} = R_1(\sigma_{IN} - \sigma_{T(N-1)})$, *for* $N > 1$ respectively.

In standard SHPB theory, the transmitted stress σ_T is proportional to the actual stress of the sample. This cannot be correct unless stress equilibrium has been achieved. Stress equilibrium occurs when the equation $\sigma_I + \sigma_R = \sigma_T$ is satisfied. So, the equilibrium condition can be achieved when the ratio $\frac{\sigma_T}{\sigma_I + \sigma_R} \cong 1$ is satisfied.

4. Results and Discussion

Understanding the way the SHPB system operates helps to make some predictions about the nature of the computed results, such as:

1) The transmitted pulse always has the same sign as the incident pulse as can be seen from equation (8) while from equation (9), the reflected pulse does not always have the same sign, but rather depends on the value of the cross-sectional area and the impedance of the sample compared with those of the bars. If $A_s Z_s \langle A_b Z_b$, the reflected pulse will be of opposite sign to the incident pulse.
2) After the stress pulse passes through the sample the multiple reflections inside the sample take a long time to decay, hence equilibrium no longer exists between the bar and the sample until the time tends to infinity. So the equilibrium ratio $\sigma_T / (\sigma_I + \sigma_R)$ will oscillate with a period of 2tt -the time required for the pulse to return to the interface. The above pulses are shown in Fig. (4) for HDPE, Nylatron, CFC and Aluminium samples with length/diameter ratio of 4/8. It can easily be noticed that the more dense material and higher sound speed (higher impedance samples), the shorter time required for the equilibrium to be achieved.
3) Because of the time required for the pulse to propagate through the sample, the transmitted pulse starts one traverse time (tt) after the reflected pulse. This delay

can be compensated experimentally by appropriate positioning of the strain gauges, but would be inconvenient for samples of different thicknesses.

4) In general, the smaller the cross-sectional area of the sample, the greater the reflected pulse and the smaller the transmitted pulse and longer time is required for the equilibrium as shown in Fig. (5).
5) The trend for all the computed results is that the normalised transmitted pulse tends to 1 and the reflected pulse vanishes to zero [1]. The time this process takes depends on the cross-sectional area and the mechanical impedance of the sample and bars.
6) The results in Fig. (6) shows that when the length of the sample increase the time required for the equilibrium increases as well. while when the diameter increase the time decreases.
7) From the previous researches, the ideal length to diameter ratio of the sample is half. This ratio was found to avoid the barrelling effect when the length is big and to avoid the friction effect at thin samples.
8) Due to Poisson's ratio, the combinations of L/D variation have been examined and shown in Fig. (5). This means when the length is changed the diameter has to be changed as well to keep the same L/D ratio with same value.

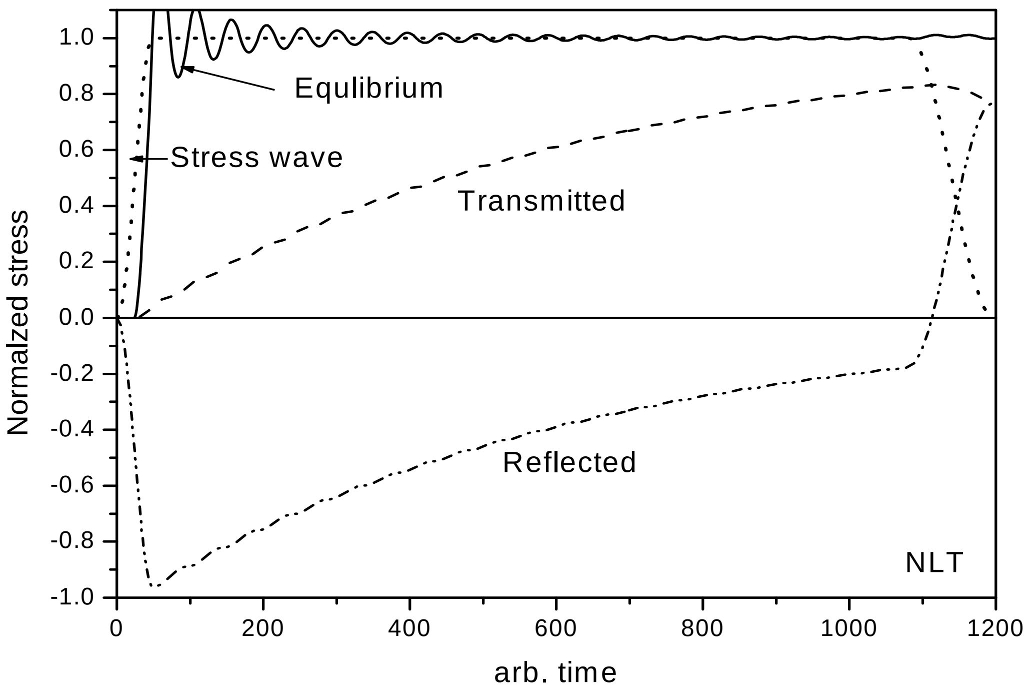

Fig. 4: Incident, transmitted, Reflected waves and the equilibrium for nylatron sample with L/D ratio 4/8.

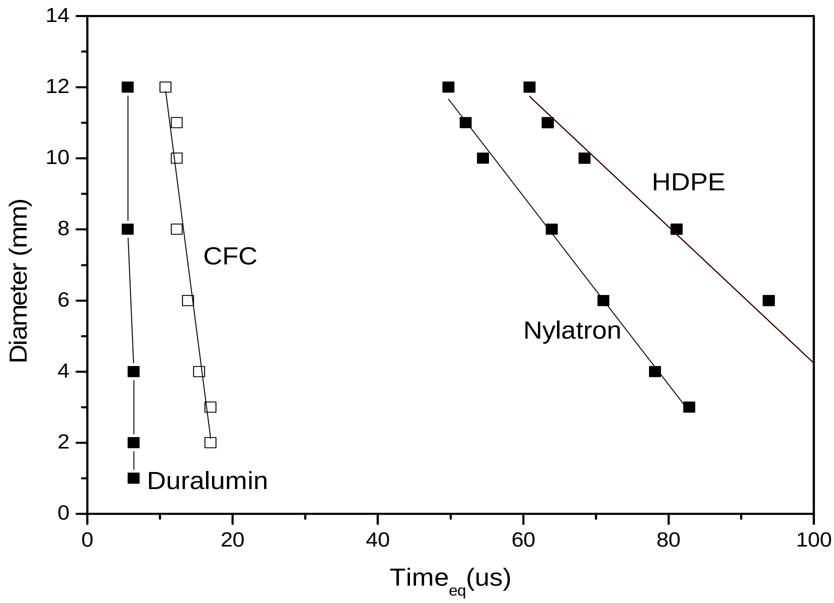

Fig. 5: Diameter versus time for complete equilibrium for HDPE, Nylatron, carbon Fibreglass, and Aluminium.

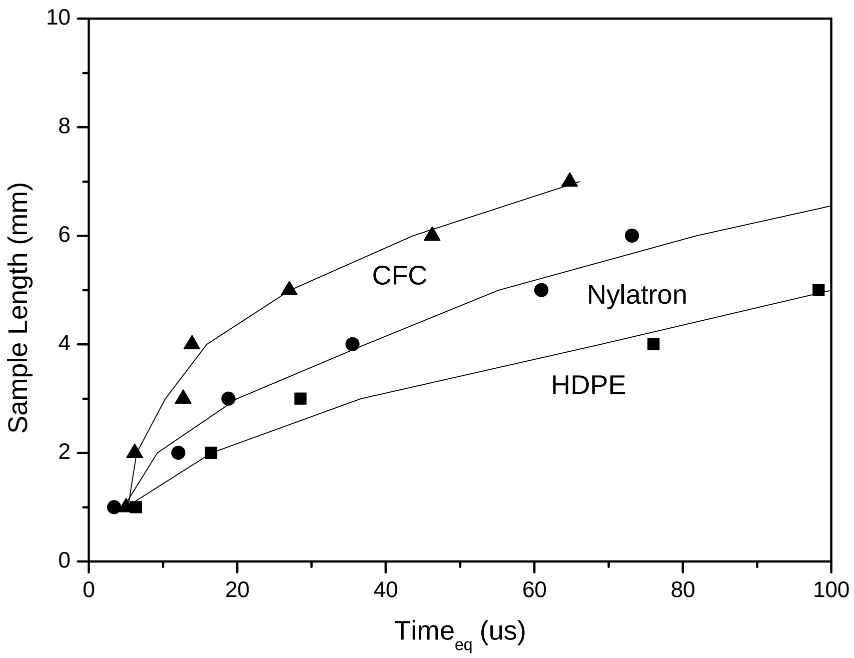

Fig. 6: Length-time for HDPE, Naylatron and CFC.

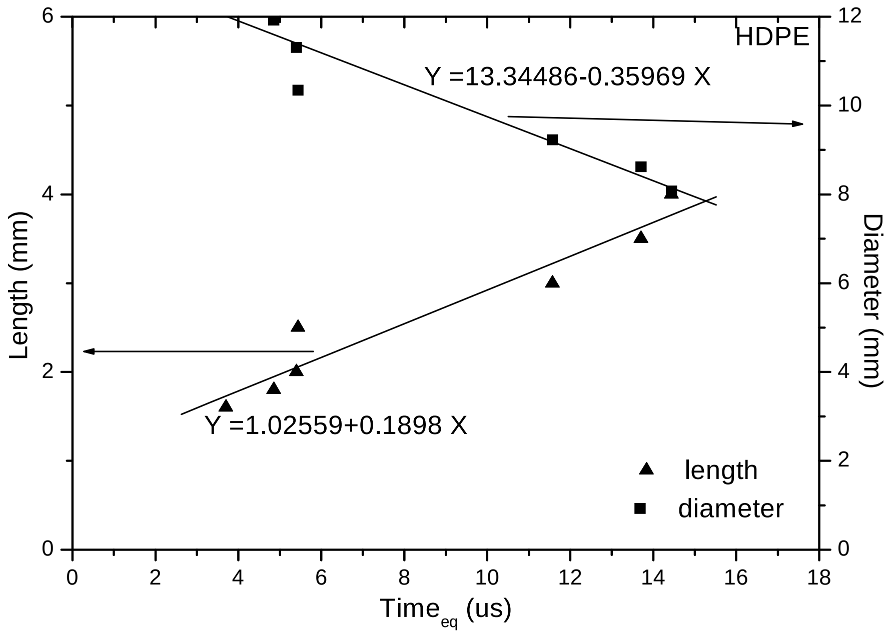

Fig. 7: Length and Diameter versus time for HDPE.

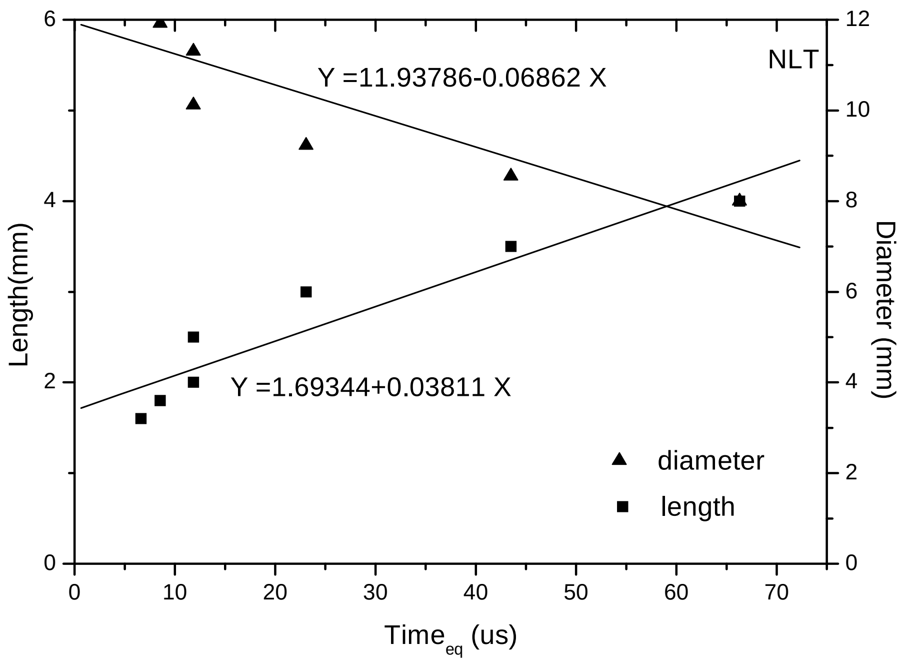

Fig. 8: Length and Diameter versus time for nylatron.

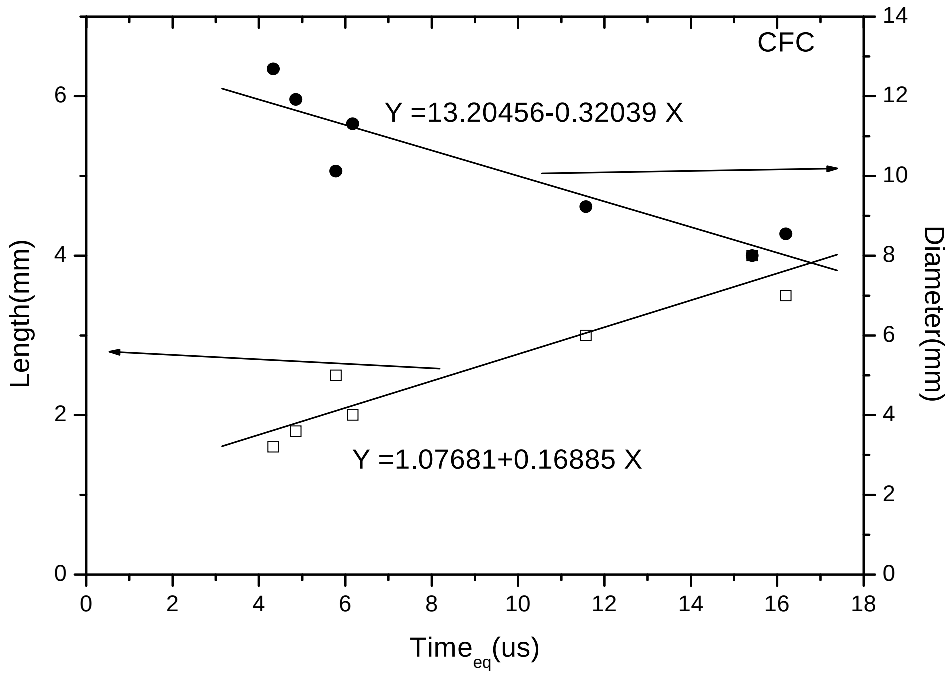

Fig. 9: Length and Diameter for CFC.

5. Conclusions

The dynamic stress equilibrium in materials under SHPB testing has been investigated using the multiple reflections of the stress pulse inside the specimen within the elastic limit of the materials. It was illustrated that the equilibrium condition, which is one of the fundamental requirements in materials dynamic property testing, is not satisfied automatically when a SHPB is used to determine the dynamic response of the material under test.

To ensure studying the effect of dimensions on the dynamic equilibrium, the loading rate must be examined and kept the same [7]. Also, very high loading rate may cause localized failure in the specimen near the front face when impacted by the stress pulse from the incident bar, therefore in the present work the loading rate, which related to the incident pulse is kept the same in all measurements.

A reduction in the specimen thickness may lead to achieving early dynamic stress equilibrium especially in soft materials. However, the thickness cannot be reduced indefinitely, and the friction effects will be more pronounced in thin specimens.

The large difference between the initial front and back-surface stresses due to the large thickness causes a severe non-equilibrium in the specimen. It is thus, necessary to quantitatively understand the effect of the specimen dimensions on the dynamic stress equilibrium in order to properly design SHPB experiments, so valid results for the tested material can be obtained.

In an experiment, the peak stress of the specimen at a certain constant strain rate is part of the experimental goal and is not variable if the experiment is properly designed; therefore, it is important to decrease the amplitude of the initial stress in the front-surface stress pulse to facilitate early equilibrium in the specimen. Thus, in this work, the incident pulse generated with a suitable rise time to achieve the equilibrium in a shorter time.

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