International Journal of Materials Physics.
ISSN 0974-309X Volume 5, Number 1 (2014), pp. 27-34

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An Unsteady MHD Flow Past a Porous Medium with Oscillatory Suction Velocity and Newtonian Heating

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Abstract

An analytical study of MHD flow past a porous medium with oscillatory suction velocity and Newtonian heating has been carried out. Closed form solution to the governing equations and its analysis showed decrease in temperature profile as a result of increase in prandtl number and angular frequency. Increase in magnetic Hartmann number and Newtonian heating result in a decrease in velocity profile of the fluid while increase in angular frequency and porosity result in an increase in velocity profile of the fluid.

Keywords: Magnetohydrodynamics, Porous medium, Angular frequency, Newtonian heating, Boussinesq approximation.

1. Introduction

The study of MHD flows through porous medium has gained the attention of researchers in fluid dynamics due to its importance in many real life applications. According to Branover(1978), since magnetic field exist everywhere, it follows that magnetohydrodynamics (MHD) phenomena must occur wherever conducting fluids are available. The specific fields of application of MHD theory and technology are many and varied. For the particular case of liquid metal MHD, magnetic fields are used to levitate samples of liquid metal, to control their shape and to induce internal stirring for the purpose of homogenization of the final product (Mebine 2007). Permeability which is a measure of the ease with which a formation permits a fluid to flow through it is applicable in bio-circulatory systems, geophysics and engineering to mention a few. Several studies in related areas are abounding. Okedoye and Bello (2008), examined the MHD flow of a uniformly stretched vertical permeable surface under oscillatory suction velocity and deduced that increase in magnetic Hartmann number reduces the velocity as a result of opposing Lorentz force and the oscillatory suction velocity affects all parameters under consideration shortly away from the boundary. Orukari et al (2011) considered the influence of viscous dissipation and radiation on MHD Couette flow in a porous medium and the result is in agreement with the work of Okedoye and Bello (2008) in the simultaneous effect of magnetic field and permeability. Flow generated by Newtonian heating is used in convective heat transfer. Our aim in this study is to incorporate Newtonian heating to the earlier study of Okedoye and Bello (2008) with a view to complementing it as well as its extension. This practice widens the applicability of studies of this nature.

2. Nomenclature

$u' =$ fluid axial velocity
A= constant
$\omega' =$ angular velocity
$y' =$ transverse coordinate
g= acceleration due to gravity
$\beta =$ coefficient of volume expansion
T= temperature
$T_e =$ equilibrium temperature
$\sigma_c =$ electrical conductivity
$B_o^2 =$ magnetic field term
$\rho =$ fluid density
$\upsilon =$ kinematic viscosity
$\kappa =$ permeability of the medium
E= activation energy
R= universal gas constant
$\kappa_T =$ thermal conductivity
$c_p =$ heat capacity at constant pressure
$t' = time$
d= characteristic distance
$\varphi =$ porosity term
$P_r =$ prandtl number
H= magnetic Hartmann number
G= Newtonian heating term
$\theta =$ dimensionless temperature
y= dimensionless coordinate

t= dimensionless time
$\omega =$ dimensionless angular velocity
u= dimensionless fluid velocity

3. Mathematical Formulation of the Physical Problem

An unsteady MHD flow of viscous, incompressible, electrical conducting fluid past an infinite plate in a porous medium with oscillatory suction velocity and Newtonian heating is considered. In Cartesian coordinate system, x-axis is assumed to be along the plate in the direction of the flow and y-axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. In the analysis, it is assumed that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic Hartmann number. Further, all fluid properties are assumed to be constant except that of the influence of density variation with temperature which shall be taken into account by the introduction of the usual Boussinesg approximation. The basic hydrodynamic equations governing the physics of the problem following the argument of Mebine (2007) are

$$\frac{\partial u'}{\partial t'} - \left(1 + Ae^{i\omega' t'}\right)\frac{\partial u'}{\partial y'} = \mu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T - T_e\right) - \frac{\sigma_c B_o^2 u'}{\rho} - \frac{\upsilon}{\kappa}u' \quad (2.1a)$$

$$\frac{\partial T}{\partial t'} - (1 + ae^{i\omega' t'})\frac{\partial T}{\partial y'} = \frac{\kappa_T}{\rho c_p}\frac{\partial^2 T}{\partial y'^2} \quad (2.1b)$$

The boundary conditions are

$$u' = 0,\ T = 1 + Ae^{i\omega' t'} \text{ as } y' = 0 \quad (2.2a)$$

$$u' \to 1 + Ae^{i\omega' t'},\ T \to 0 \text{ as } y' \to \infty \quad (2.2b)$$

where the parameters are defined in nomenclature.

In order to analyze equations (2.1a) and (2.1b) subject to equation (2.2), we introduce the following dimensionless quantities

$$y = \frac{y't}{d}, \quad \varphi = \frac{\upsilon\mu d^2}{\kappa\rho}, \quad H = \frac{\sigma_c \mu B_o^2 \upsilon}{\rho u'^2}, \quad \theta = \frac{E\left(T - T_e\right)}{RT_o^2},$$

$$G = \frac{\mu g\beta RT_e^2}{Eu^3},\ P_r = \frac{\mu c_p}{\kappa_T}$$

$$t = \frac{ud}{t'},\ u = \frac{u't}{d},\ \omega = \frac{\omega' y'}{u'}$$

Equations (2.1a), (2.1b), (2.2a) and (2.2b) can now be written as

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + Ae^{i\omega t}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G\theta - Hu - \varphi u \quad (2.3a)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \left(1 + Ae^{i\omega t}\right)\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \quad (2.3b)$$

The boundary conditions are

$$u = 0,\ \theta = 1 + Ae^{i\omega t} \text{ on } y = 0 \quad (2.4a)$$

$$u \to 1 + Ae^{i\omega t},\ \theta \to 0 \text{ as } y \to \infty \quad (2.4b)$$

3.1 Method of solution

For sufficiently large values of the time, that is after the initial transients have disappeared, we seek solution to equations (2.3) in the forms

$$u(y,t) = h(y)e^{-nt} \text{ and } \theta(y,t) = \phi(y)e^{-nt} \quad (3.1)$$

where n is a decay constant.

Therefore, equations (2.3) and (2.4) give rise to

$$h''(y) + \left(1 + Ae^{i\omega t}\right)h'(y) - \left(H + \varphi - \frac{n}{4}\right)h(y) = G\phi(y) \quad (3.2a)$$

$$\phi''(y) + P_r\left(1 + Ae^{i\omega t}\right)\phi'(y) + \frac{P_r}{4}n\phi(y) = 0 \quad (3.2b)$$

with the boundary conditions

$$h = 0,\ \phi = \left(1 + Ae^{i\omega t}\right)e^{nt} \text{ at } y = 0 \quad (3.3a)$$

$$h \to \left(1 + Ae^{i\omega t}\right)e^{nt},\ \phi \to 0 \text{ as } y \to \infty \quad (3.3b)$$

The solution of equation (3.2b) with the imposition of the boundary condition (3.3) and substituting in equation (3.1), we obtain

$$\theta(y,t) = A_1e^{\alpha_1 y} + A_2e^{\alpha_2 y} \quad (3.4a)$$

where $A_2 = \dfrac{2\left(1 + Ae^{i\omega t}\right)}{-\left(-b + \sqrt{b^2 - P_r n}\right) + 1}$, $\alpha_1 = \dfrac{-b - \sqrt{b^2 - P_r n}}{2}$, $A_1 \approx -b$

$b = 1 + Ae^{i\omega t}$ $\quad \alpha_2 = \dfrac{-b + \sqrt{b^2 - P_r n}}{2}$

Also we assume that $\alpha_2 >> 1$

Similarly, substituting equation (3.4a) into equation (3.2a) and solving together with the boundary conditions (3.3) and substituting into equation (3.1), we obtain

$$u(y,t) = B_1e^{\beta_1 y} + B_2e^{\beta_2 y} + \psi_1e^{\alpha_1 y} + \psi_2e^{\alpha_2 y} \quad (3.4b)$$

$$\beta_1 = \frac{-b - \sqrt{b^2 + 4\left(H + \varphi - \frac{n}{4}\right)}}{2}$$

$$\beta_2 = \frac{-b + \sqrt{b^2 + 4\left(H + \varphi - \frac{n}{4}\right)}}{2}$$

$$B_2 = \frac{2\left(1 + Ae^{iwt}\right)}{-\left(-b + \sqrt{b^2 + 4\left(H + \varphi - \frac{n}{4}\right)}\right)}$$

$$B_1 \approx b$$

$$\psi_1 = \frac{GA_1}{\alpha_1^2 + \left(1 + Ae^{i\omega t}\right)\alpha_1 - \left(H + \varphi - \frac{n}{4}\right)}$$

where
$$\psi_2 = \frac{GA_2}{\alpha_2^2 + \left(1 + Ae^{i\omega t}\right)\alpha_1 - \left(H + \varphi - \frac{n}{4}\right)}$$

The solution is now complete.

4. Results and Discussion

The problem of MHD flow past a porous medium, with oscillatory suction velocity and Newtonian heating has been investigated based on fairly realistic assumptions. In order to get physical insight and numerical validation of the problem, values are chosen as shown on the tables

Table 1: Increasing P_r on the temperature profile of the fluid with $\omega = 0,\ A = 0.5\ \ t = 10,\ n = 0.0035$

y	$P_r = 0.71$	$P_r = 1.00$	$P_r = 3.00$	$P_r = 5.00$	$P_r = 7.00$
0	1.497516372	1.49650272	1.489524429	1.482567726	1.475632486
1	2.661438974	2.659865171	2.649009089	2.638189542	2.627405176
2	2.920290412	2.918240198	2.904124448	2.890067929	2.876069799
3	2.97710874	2.974568875	2.957160988	2.939723376	2.922435345
4	2.988828849	2.985789552	2.964902254	2.951668007	2.923514855
5	2.990482357	2.986940226	2.962612951	2.938462056	2.914486263

Increase in Prandtl number as shown on table 1 brings about a corresponding decrease in the temperature profile of the flow and this observation is consistent with that of Okedoye and Bello (2008).

Table 2: Increasing ω on the temperature profile of the fluid with $p_r = 0.71$, $A = 0.5$, $t = 10$, $n = 0.0035$

y	$\omega = 0.00$	$\omega = 5.00$	$\omega = 10.00$	$\omega = 15.00$	$\omega = 20.00$
0	1.497516372	1.296766603	0.775122615	0.57026165	0.527669007
1	2.661438974	2.264264237	1.238962787	0.811829801	0.744211302

2	2.920290412	2.521578204	1.396190424	0.947122372	0.871290752
3	2.977108740	2.589348744	1.426724567	1.022166125	0.945658745
4	2.988828849	2.606535302	1.405711291	1.063046181	0.988969424
5	2.990482357	2.61022154	1.364504662	1.084561594	1.013981541

From table 2, an increase in the angular frequency of the flow caused a decrease in temperature profile of the flow. This observation is in agreement with the work of Israel-Cookey *et al* (2003) that an increase in the free stream frequency is associated with a decrease in the temperature distribution.

Values recorded on table 3, shows that increase in prandtl number increases the velocity profile of the fluid flow and this observation is in accord with an earlier study of Okedoye and Bello (2008).

Table 3: Increasing P_r on the velocity profile of the fluid with $A = 0.5$, $t = 10$, $n = 0.0035$, $\omega = 5$ $\varphi = 0.3$ $H = 2$, $G = 3$.

y	$P_r = 0.71$	$P_r = 1.00$	$P_r = 3.00$	$P_r = 5.00$	$P_r = 7.00$
0	-4.937321549	-4.370720777	-4.360320281	-4.361276813	-13.49924554
1	-4.792602185	-6.702740026	-6.686229423	-6.681066471	-8.543800432
2	-16.42839005	-9.965207215	-13.37245885	-13.36213294	92.86423626
3	-24.64258508	-20.10822008	-20.05868827	-20.04319941	252.431166
4	-32.85678011	-26.81096010	-26.74491769	-26.73663682	686.1790513
5	-41.07097514	-33.51370013	-33.43114711	-33.40533236	1865.228046

Table 4: Increasing angular frequency(ω) on the velocity profile of the fluid with P_r= 0.71 $A = 0.5$, $t = 10$, $n = 0.0035$, $\varphi = 0.3$, $H = 2$, $G = 3$.

y	$\omega = 0.00$	$\omega = 5.00$	$\omega = 10.00$	$\omega = 15.00$	$\omega = 20.00$
0	-4.437365326	-6.864359766	-2.779909534	-1.84277962	-1.715933004
1	-5.065814145	-8.632628879	-4.663991325	-2.709995659	-2.522387845
2	-3.837350319	-10.30651834	-9.327982650	-7.097094478	-5.044775689
3	-10.43099964	-28.01602151	-13.99197438	-8.129986977	-7.567163534
4	-28.35439678	-75.36138901	-18.65596532	-10.83998264	-10.08955138
5	-77.07524151	-207.0119546	-23.31995662	-13-54997829	-12.61193922

We discovered that increase in angular frequency of the fluid as depicted on table 4, shows an increase in the velocity profile of the fluid.

Effect of Hartmann number on the velocity profile of the fluid is displayed on table 5. We observed that increase in magnetic Hartmann number results in a decrease in the velocity profile of the fluid and this observation is in accord with an earlier results of

Israel-Cookey (2003), Mebine(2007), Okedoye and Bello (2008) and Orukari et al (2011).

Table 5: Increasing Hartmann number (H) on the velocity profile 0f the fluid with Pr= 0.71 $A = 0.5$, $t = 10$, $n = 0.0035$, $\omega = 5$ $\varphi = 0.3$, $G = 3$.

y	H =0.00	H =2.00	H =4.00	H =6.00	H =8.00
0	-2659.45978	-3.219849041	-3.11356222	-2.634298256	-2.37391482
1	-4592.712952	-5.553298458	-5.641356091	-5.867221171	-6.575882926
2	-3894.292231	-11.10659692	-11.28271218	-11.72382562	-13.15176585
3	-13773.75724	-16.65989537	-16.92406827	-17.60166351	-19.72764878
4	-18370.85181	-22.21319383	-22.56542437	-23.46888468	-29.33705849
5	-22963.56476	-27.76649229	-28.20678046	-29.33610585	-32.87941463

Table 6: Increasing permeability(φ) on the velocity profile of the fluid with Pr= 0.71 $A = 0.5$, $t = 10$, $n = 0.0035$, $\omega = 5$ $H = 2$, $G = 3$.

y	$\varphi = 0.00$	$\varphi = 0.30$	$\varphi = 0.60$	$\varphi = 0.90$	$\varphi = 1.20$
0	-4.76829321	-6.864359766	-4.065049469	-3.819486982	-3.618444902
1	-7.18459651	-8.632628879	-6.361210335	-6.111971104	-5.931852551
2	-14.36919302	-10.30651834	-12.72242067	-12.22394221	-11.8637051
3	-21.55378953	-28.0160215	-19.08363097	-18.33591331	-17.79555765
4	-28.73838604	-75.36138901	-25.44484134	-24.44788442	-23.72741021
5	-35.92298255	-207.0119546	-31.80605168	-30.55985552	-29.65926276

Increase in porosity term as shown on table 6 result in an increase in the velocity profile of the fluid and the result is in agreement with the work of Orukari et al (2011).

Table 7: Increasing Newtonian heating (G) on the velocity profile of the fluid with Pr= 0.71 $A = 0.5$, $t = 10$, $n = 0.0035$, $\omega = 5$ $H = 2$, $\varphi = 0.3$

y	G=0.00	G=3.00	G=6.00	G=9.00	G=12.00
0	-2.651725103	-4.372232645	-6.092740187	-7.81324773	-9.533755269
1	-0.53340425	-3.515990888	-6.498577525	-9.481164164	-12.4637508
2	-7.4450107136	-13.41018399	-19.38183528	-25.34053054	-31.30570381
3	-11.1676607	-20.11542061	-29.06318053	-38.01094044	-46.95870035
4	-14.89021427	-26.82056082	-38.75090737	-50.68125393	-62.61160046
5	-18.61276744	-33.52570063	-48.43863382	-63.35156701	-78.26450018

Increase in Newtonian heating term as displayed on table 7, brings about a corresponding decrease in the velocity profile of the fluid caused by increase in temperature and occasional evaporation.

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