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Fractal Dimension for New Fractional Derivative

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1.Interoduction

Many paper and books on fractional derivative have appeared recently. Most of them are developed to the solvability of numerical method.(see,e.g.[]). In this report we will survey new definition of fractional derivative in [1]. According old definition form different ions [3] for $\alpha > 0$ of a function $f\colon (0,) \to \mathbb{R}$ we have:

$$D_0^\alpha f(x) = \frac{1}{\Gamma(x-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-\xi)^{n-\alpha} \big(f(\xi) - f(0)\big) d\xi \,, \tag{1.1}$$

For $x \in [0,1]$, $n - 1 \leq \alpha < n$ and n>1, but so many problem are in physics and engineering that we couldn't use this definition because prime condition apply (1.1) is continues f(x), so we need new definition depended from continues f(x). Recently Prof. Chen *et al* [1] suggested new definition where discontinues place described by fractal dimension.

Definition 1.2.

$$\frac{Df(t)}{D^\alpha t} = \lim_{t_2 \to t_1} \quad \frac{f(t_1) - f(t_2)}{t_1^\alpha - t_2^\alpha}, \tag{1.3}$$

That $t_1^\alpha - t_2^\alpha$ means distance between t_1 *and* t_2 .

Using fractal geometrical has been expressed in [4] we can rewrite (1.3)

$$\frac{Df(t)}{D^\alpha t} = \lim_{\Delta x \to L_0} \quad \frac{f(t_1) - f(t_2)}{\text{distance between two point}} = \frac{df}{ds} = \lim_{\Delta x \to L_0} \frac{f(t_1) - f(t_2)}{k{L_0}^\alpha}, \tag{1.4}$$

Where L_0smallest measure is size from the space and α is fractal dimension and k is a constant, so looking for find fractal dimension is more important.

2.Fractal Dimension

How many disks does it take to cover the Koch coastline? Well, it depends on their size of course. 1 disk with diameter 1 is sufficient to cover the whole thing, 4 disks with diameter ⅓, 16 disks with diameter 1/9, 64 disks with diameter 1/27, and so on. In general, it takes 4n disks of radius (⅓)n to cover the Koch coastline. If we apply this procedure to any entity in any metric space we can define a quantity that is the equivalent of a dimension. TheHausdorff-Besicovitch dimension of an object in a metric space is given by the formula :

$$D = \lim_{h\to 0} \frac{\log N(h)}{\log(1/h)}, \tag{2.1}$$

where N(h) is the number of disks of size h needed to cover the object. Thus the Koch coastline has a Hausdorff-Besicovitch dimension which is the limit of the sequence.

$$\frac{\log 1}{\log 1}, \frac{\log 4}{\log 3}, \frac{\log 16}{\log 9}, \ldots, \frac{\log 4^n}{\log 3^n} = \frac{n\log 4}{n\log 3} = 1.261859507\ \ldots, \tag{2.2}$$

Is this really a dimension? Apply the procedure to the unit line segment. It takes 1 disk of diameter 1, 2 disks of diameter ½, 4 disks of diameter ¼, and so on to cover the unit line segment. In the limit we find a dimension of

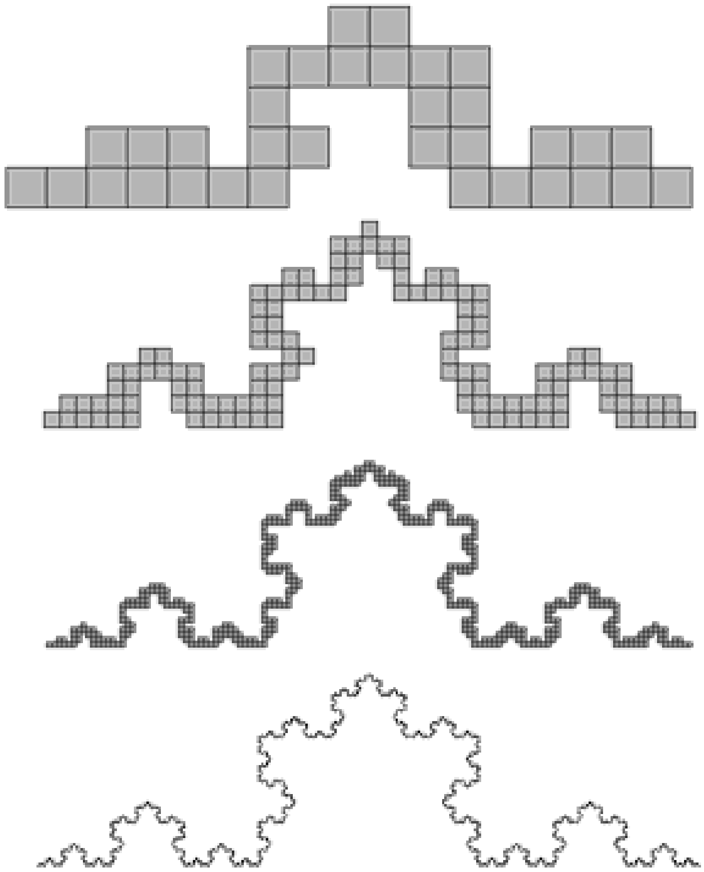
$$\frac{\log 2^n}{\log 2^n} = \frac{n\log 2}{n\log 2} = 1 \tag{2.3}$$

This agrees with the topological dimension of the space.

The problem now is, how do we interpret a result like 1.261859507…? This does not agree with the topological dimension of 1 but neither is it 2. The Koch coastline is somewhere between a line and a plane. Its dimension is not a whole number but a fraction. It is a fractal. Actually fractals can have whole number dimensions so this is a bit of a misnomer. A better definition is that a fractal is any entity whose Hausdorff-Besicovitch dimension strictly exceeds its topological dimension ($D > D_T$). Thus, the Peano space-filling curve is also a fractal as we would expect it to be. Even though its Hausdorff-Besicovitch dimension is a whole number ($D = 2$) its topological dimension ($D_T = 1$) is strictly less than this. The monster has been tamed.

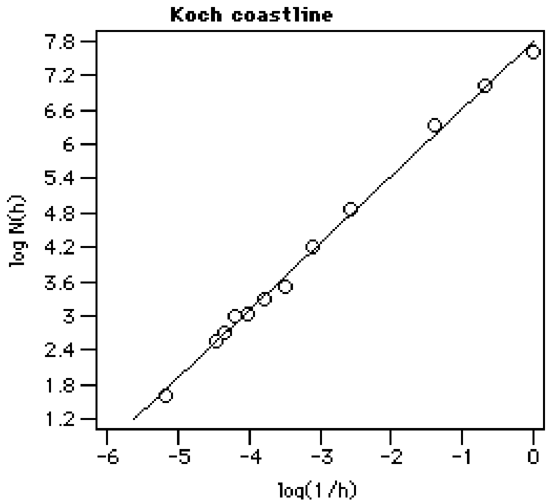
It should be possible to use analytic methods like those described above on all sorts of fractal objects. Whether this is convenient or simple is another matter. Fractals produced by simple iterative scaling procedures like the Koch coastline are very easy to handle analytically. Julia and Mandelbrot sets, fractals produced by the iterated mapping of continuous complex functions, are another matter. There's no obvious fractal structure to the quadratic mapping, no hint that a "monster" curve lurks inside, and no simple way to extract an exact fractal dimension. If there are analytic techniques for calculating the fractal dimension of an arbitrary Julia set they are well hidden. A narrow and quick search of the popular literature reveals nothing on the ease or impossibility of this task. There are, however, experimental techniques.

Surrounding the Koch Coastline with Boxes (a way to determine its dimension)



Take any plane geometric object of finite extent (fractal or otherwise) and cover it with a single closed disk. Any type of disk will do, so to make life easy we will use a square; the disk of the Manhattan metric in the plane. Record its dimension and call it "h". Repeat the procedure with a smaller box. Record its dimension and the number of boxes "N(h)" required to cover the object. Repeat with ever smaller boxes until you have reached the limit of your resolving power as shown in the figure to the right. Plot the results on a graph with "log N(h)" on the vertical axis and "log (1/h)" on the horizontal axis. The slope of the best fit line of the data will be an approximation of the Hausdorff-Besicovitch dimension of the object. The following are the results of a few sample experiments using thisbox-counting method. I think with a bit of refinement, the deviations could all be brought below 5%.
dimension (experimental) = 1.18
dimension (analytical) = 1.26
deviation = 6%

log (1/h)	log N(h)
0	7.60837
−0.69315	7.04054
−1.38629	6.32972
−2.56495	4.85981
−3.09104	4.21951
−3.49651	3.52636
−3.78419	3.29584
−4.00733	3.04452
−4.18965	2.99573
−4.34381	2.70805
−4.47734	2.56495
−5.17615	1.60944



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