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Strong Edge Graceful Labeling of Windmill Graphs

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Abstract

A (p, q) graph G is said to have strong edge graceful labeling if there exists an injection f from the edge set to $\left\{1,2,\ldots\left[\frac{3q}{2}\right]\right\}$ so that the induced mapping f^+ defined on the vertex set given by $f^+(x) = \sum\{f(xy)/xy \in E(G)\}\ (\text{mod } 2p)$ are distinct. A graph G is said to be strong edge graceful if it admits a strong edge graceful labeling. In this paper we investigate strong edge graceful labeling of Windmill graph.

Definition: The windmill graphs $K_m^{(n)}$ (n >3) to be the family of graphs consisting of n copies of K_m with a vertex in common.

Theorem: 1. The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when n is even.

Proof: Let {v_1, v_2, v_3, ..., v_{3n}, } be the vertices of $K_4^{(n)}$ and {e_1, e_2, e_3, ...,e_{3n-1}, e_{3n}, , f_1, ,f_2, ,f_3, ...f_{3n-1}, f_{3n}. } be the edges of $K_4^{(n)}$ which are denoted as in the following Fig. 1.

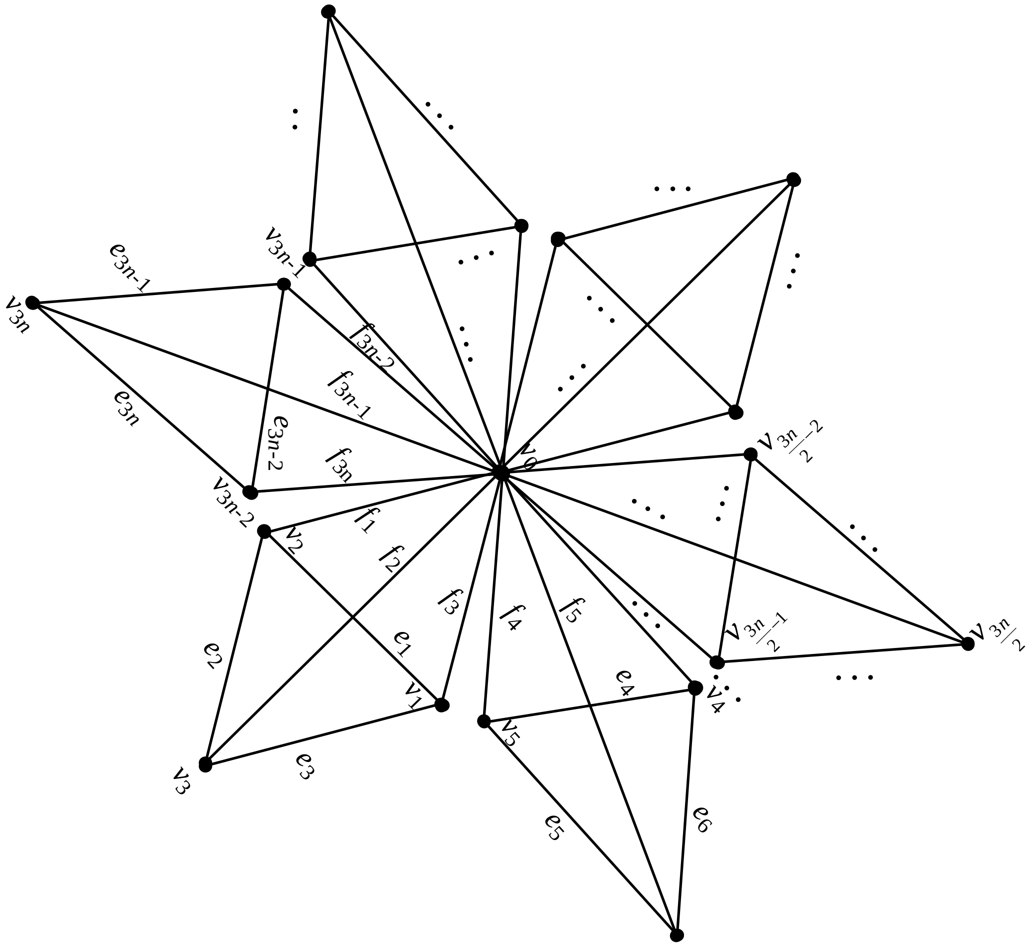

Fig. 1: $K_4^{(n)}$ with ordinary labeling

We first label the edges of $K_4^{(n)}$ as follows:

$f(f_i) = i$ $\quad 1 \le i \le \frac{3n}{2}$

$f(f_i) = 3n+1+i$ $\quad \frac{3n}{2}+1 \le i \le 3n$

$f(e_i) = 3n+1-i$ $\quad 1 \le i \le \frac{3n}{2}$

$f(e_i) = 6n+2-i$ $\quad \frac{3n}{2}+1 \le i \le 3n$

Then the induced vertex labels are:

$f^+(v_0) = 0$

$f^+(v_i) = 6n+2-i \quad 1 \le i \le \frac{3n}{2}$

$f^+(v_i) = 3n+1-i$ $\quad \frac{3n}{2}+1 \le i \le 3n$

Clearly, the vertex labels are all distinct. Hence The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when n is even.

The *SEGL* of $K_4^{(4)}$, $K_4^{(8)}$ are illustrated in Fig.2, Fig.3, respectively.

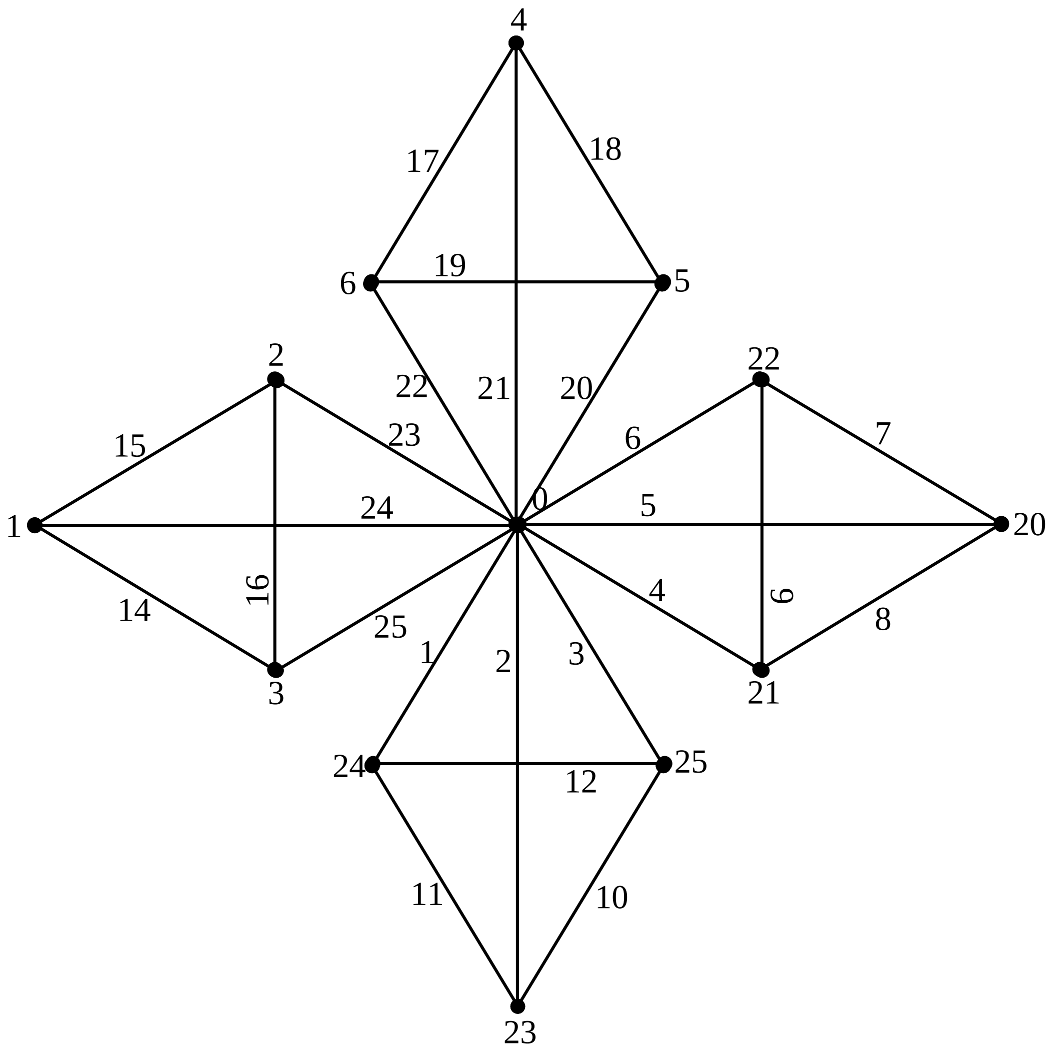

Fig.2. $K_4^{(4)}$ *with SEGL*

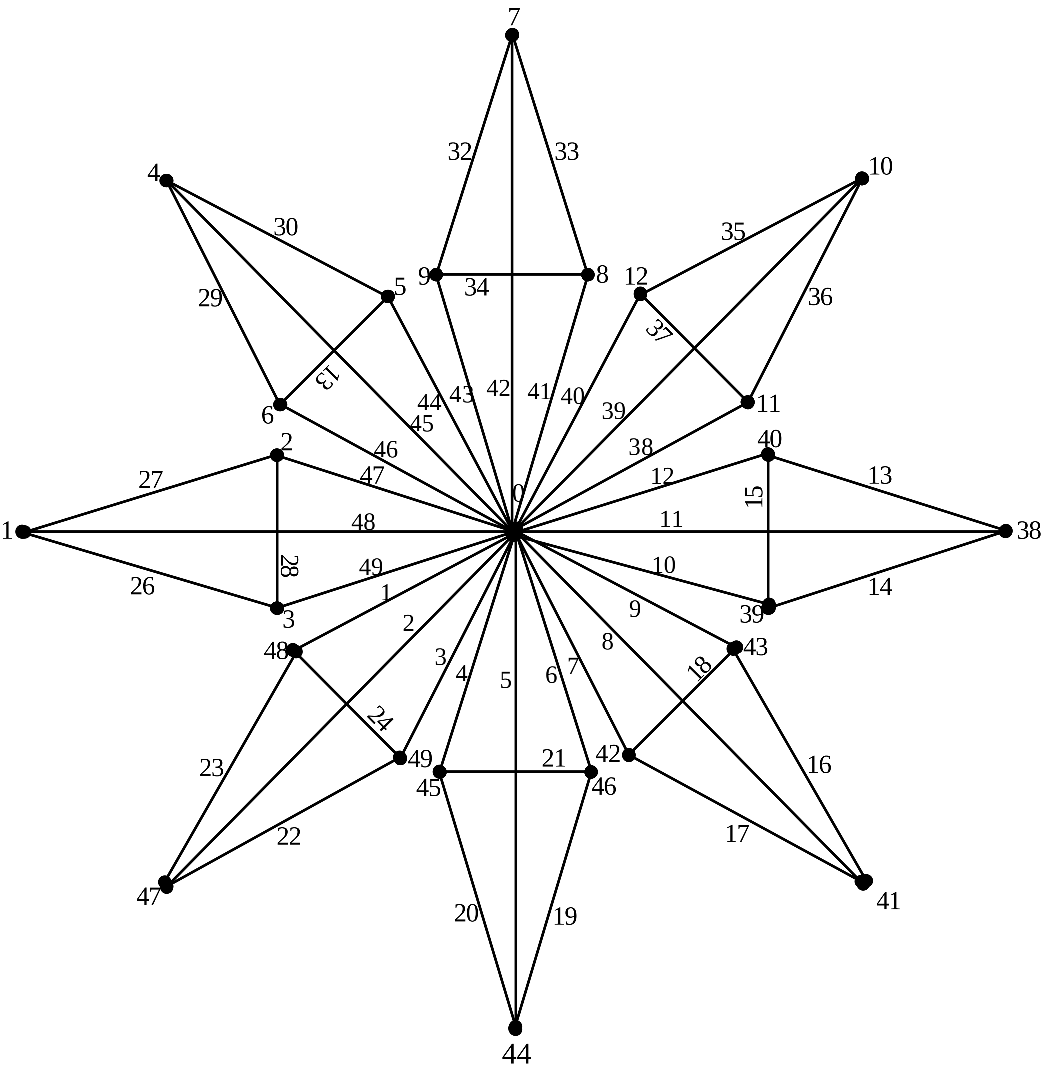

Fig.3. $K_4^{(8)}$ *with SEGL*

Theorem: 2. The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n \equiv 1 (\text{mod } 4)$.

Proof: Let $\{v_1, v_2, v_3, ..., v_{3n}, \}$ be the vertices of $K_4^{(n)}$ and $\{e_1, e_2, e_3, ..., e_{3n-1}, e_{3n}, f_1, f_2, f_3, \ldots f_{3n-1}, f_{3n}. \}$ be the edges of $K_4^{(n)}$ which are denoted as in the above Fig. 1.

We first label the edges of $K_4^{(n)}$ as follows:

$f(f_i) = i \qquad 1 \leq i \leq 3n-1$

$f(f_{3n}) = 6n$

$f(e_i) = 6n - i \quad 1 \leq i \leq 3n$

Then the induced vertex labels are:

$f^{+}(v_0) = \frac{3n-1}{2}$

$f^{+}(v_i) = 6n - 2 - i \quad 1 \leq i \leq 3n\text{-}3$

$f^{+}(v_{3n-2}) = 6n$

$f^{+}(v_i) = 6n - 2 - i \quad 3n - 1 \leq i \leq 3n$

Clearly, the vertex labels are all distinct. Hence The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n \equiv 1 (\text{mod } 4)$.

The *SEGL* of $K_4^{(5)}$, $K_4^{(9)}$ are illustrated in Fig.4, Fig.5. respectively.

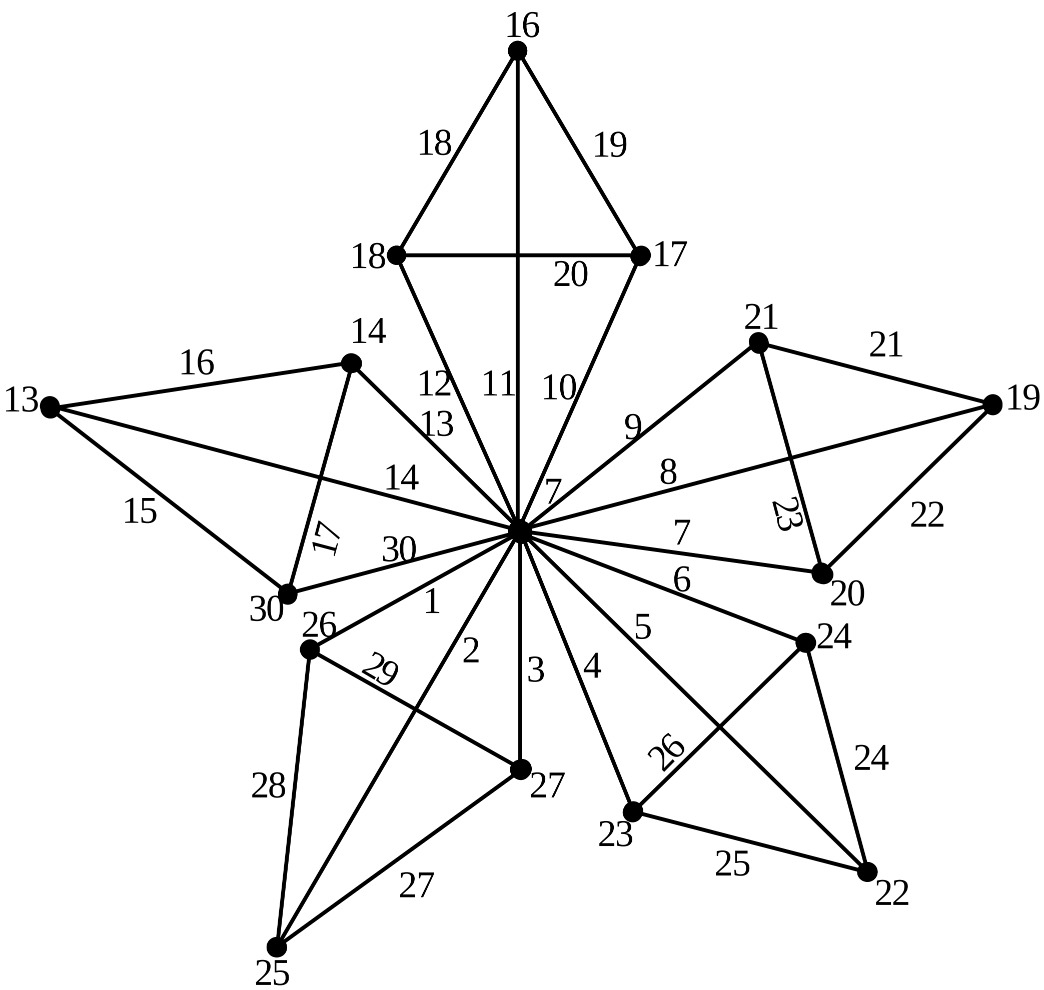

Fig.4. $K_4^{(5)}$ *with SEGL*

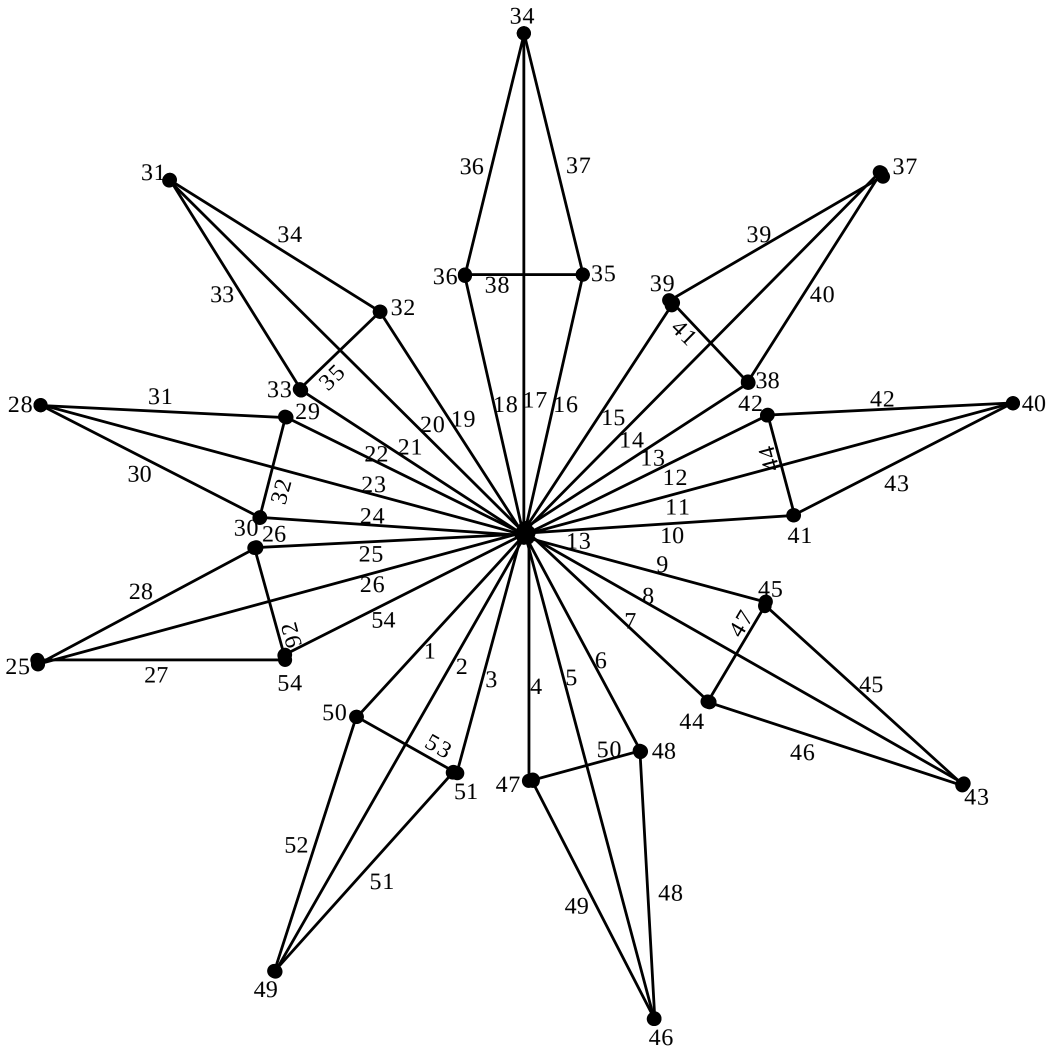

Fig.5. $K_4^{(9)}$ *with SEGL*

Theorem: 3. The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n \equiv 3 (\text{mod } 4)$.

Proof: Let $\{v_1, v_2, v_3, ..., v_{3n}, \}$ be the vertices of $K_4^{(n)}$ and $\{e_1, e_2, e_3, ..., e_{3n-1}, e_{3n}, , f_1, ,f_2, ,f_3, ...f_{3n-1}, f_{3n}. \}$ be the edges of $K_4^{(n)}$ which are denoted as in the above Fig. 1.

We first label the edges of $K_4^{(n)}$ as follows:

$f(f_i) = i \qquad 1 \leq i \leq 3n$

$f(e_i) = 6n+1 - i \qquad 1 \leq i \leq 3n$

Then the induced vertex labels are:

$f^{+}(v_0) = \frac{3n+1}{2}$

$f^{+}(v_i) = 6n - i \quad 1 \leq i \leq 3n$

Clearly, the vertex labels are all distinct. Hence The windmill graph $K_4^{(n)}$ is strong edge graceful for all $n \geq 3$ when $n \equiv 3 \pmod 4$.

The *SEGL* of $K_4^{(3)}$, $K_4^{(7)}$ are illustrated in Fig.6, Fig.7. respectively.

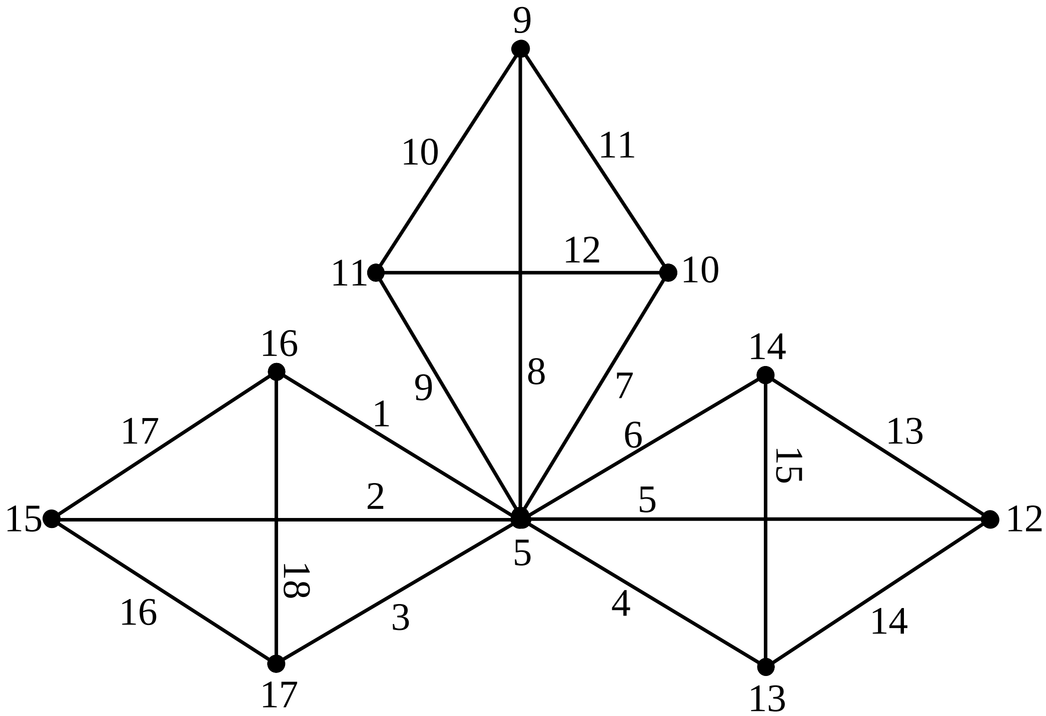

Fig.6. $K_4^{(3)}$ *with SEGL*

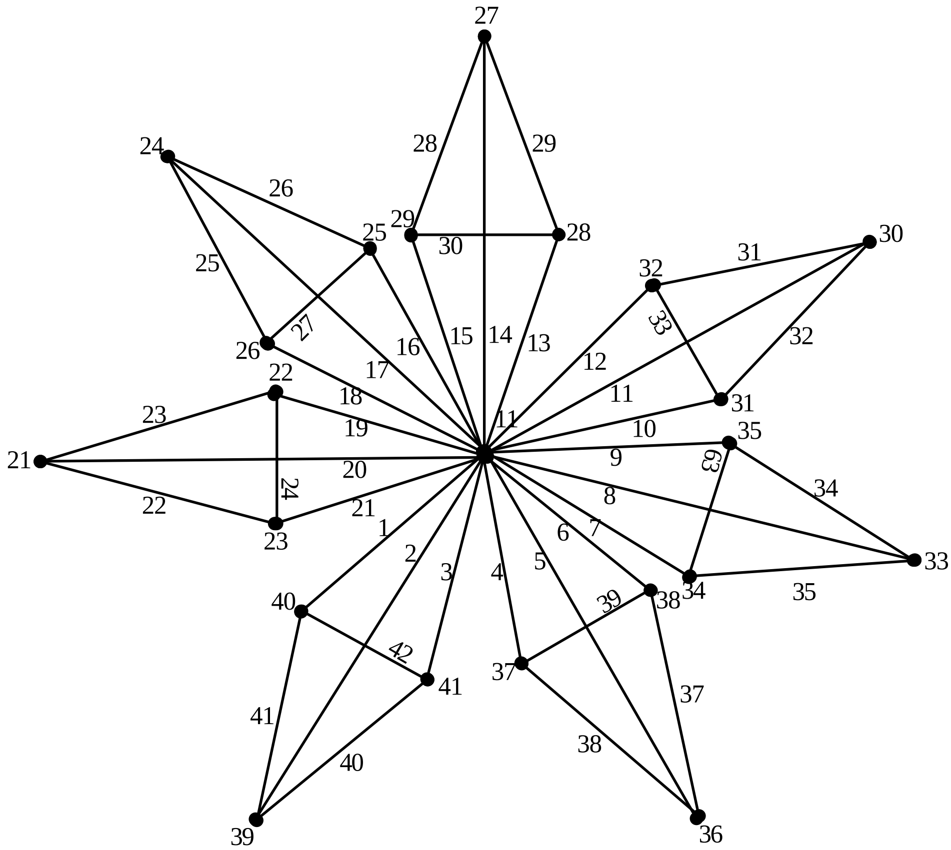

Fig.7. $K_4^{(7)}$ *with SEGL*

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