International Journal of Mathematics Research.
ISSN 0976-5840 Volume 5, Number 1 (2013), pp. 27-39

http://www.irphouse.com

Study of Recent Advances in Mathematics

Manprit Kaur

Department of Mathematics
Gyan Ganga College of Technology, Jabalpur, India
manpreet2410@gmail.com

Abstract

In this paper the various applications, advances of science in particular and various engineering principles in the field of mathematics are discussed. This paper also highlights the application of mathematics in engineering, medical imaging such as 3D-tomography, EPRI, TEM, and so on.

Keywords: Approximation, Biomedical, parameter, robotic, Transmission, tomography, transform.

Introduction

Mathematics is defined as "the science of quantity", and this definition prevailed until the 18th century. Starting in the 19th century, when the study of mathematics increased in rigor and began to address abstract topics such as group theory and projective geometry, which have no clear-cut relation to quantity and measurement, mathematicians and philosophers began to propose a variety of new definitions. Some of these definitions emphasize the deductive character of much of mathematics, some emphasize its abstractness, some emphasize certain topics within mathematics. Today, no consensus on the definition of mathematics prevails, even among professionals. There is not even consensus on whether mathematics is an art or a science. A great many professional mathematicians take no interest in a definition of mathematics, or consider it undefinable. Some just say, "Mathematics is what mathematicians do."

Three leading types of definition of mathematics are called logicist, intuitionist, and formalist, each reflecting a different philosophical school of thought. All have severe problems, none has widespread acceptance, and no reconciliation seems possible.An early definition of mathematics in terms of logic was "the science that draws necessary conclusions". Intuitionist definitions, developing from the philosophy of mathematician identify mathematics with certain mental phenomena. An example of an intuitionist definition is "Mathematics is the mental activity which consists in carrying out constructs one after the other."Formalist definitions identify mathematics with its symbols and the rules for operating on them. In terms of formal mathematics simply is "the science of formal systems". A formal system is a set of symbols, or tokens, and some rules telling how the tokens may be combined into formulas. In formal systems, the word axiom has a special meaning, different from the ordinary meaning of "a self-evident truth". In formal systems, an axiom is a combination of tokens that is included in a given formal system without needing to be derived using the rules of the system.

2. Modern mathematics

(i) In 19th century:

Throughout the 19th century mathematics became increasingly abstract. In the 19th century lived Carl Friedrich Gauss (1777–1855). Leaving aside his many contributions to science, in pure mathematics he did revolutionary work on functions of complex variables, in geometry, and on the convergence of series. He gave the first satisfactory proofs of the fundamental theorem of algebra and of the quadratic reciprocity law.

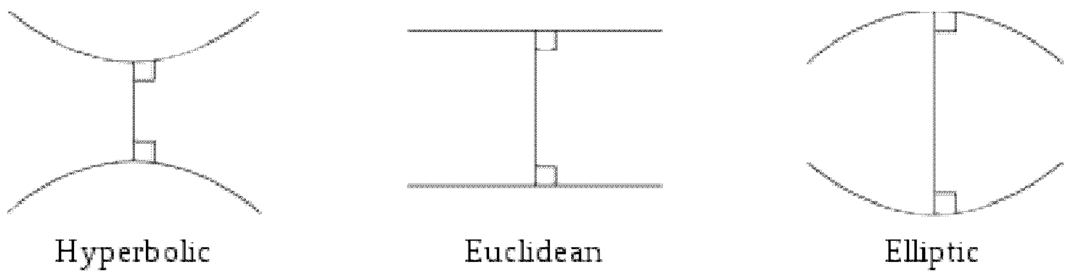

Fig.(i) Behaviour of lines with a common perpendicular in each of the three types of geometry.

This century saw the development of the two forms of non-Euclidean geometry, where the parallel postulate of Euclidean geometry no longer holds. The Russian mathematician Nikolai IvanovichLobachevsky and his rival, the Hungarian mathematician JánosBolyai, independently defined and studied hyperbolic geometry, where uniqueness of parallels no longer holds. In this geometry the sum of angles in a triangle add up to less than 180°. Elliptic geometry was developed later in the 19th century by the German mathematician Bernhard Riemann; here no parallel can be found and the angles in a triangle add up to more than 180°. Riemann also developed Riemannian geometry, which unifies and vastly generalizes the three types of geometry, and he defined the concept of a manifold, which generalizes the ideas of curves and surfaces.

The 19th century saw the beginning of a great deal of abstract algebra. Hermann Grassmann in Germany gave a first version of vector spaces, William Rowan Hamilton in Ireland developed noncommutative algebra. The British mathematician George Boole devised an algebra that soon evolved into what is now called Boolean algebra, in which the only numbers were 0 and 1. Boolean algebra is the starting point of mathematical logic and has important applications in computer science.

Augustin-Louis Cauchy, Bernhard Riemann, and Karl Weierstrass reformulated the calculus in a more rigorous fashion.

Also, for the first time, the limits of mathematics were explored. NielsHenrik Abel, a Norwegian, and Évariste Galois, a Frenchman, proved that there is no general algebraic method for solving polynomial equations of degree greater than four (Abel–Ruffini theorem). Other 19th-century mathematicians utilized this in their proofs that straightedge and compass alone are not sufficient to trisect an arbitrary angle, to construct the side of a cube twice the volume of a given cube, nor to construct a square equal in area to a given circle. Mathematicians had vainly attempted to solve all of these problems since the time of the ancient Greeks. On the other hand, the limitation of three dimensions in geometry was surpassed in the 19th century through considerations of parameter space and hypercomplex numbers.

Abel and Galois's investigations into the solutions of various polynomial equations laid the groundwork for further developments of group theory, and the associated fields of abstract algebra. In the 20th century physicists and other scientists have seen group theory as the ideal way to study symmetry.

In the later 19th century, Georg Cantor established the first foundations of set theory, which enabled the rigorous treatment of the notion of infinity and has become the common language of nearly all mathematics. Cantor's set theory, and the rise of mathematical logic in the hands of Peano, L. E. J. Brouwer, David Hilbert, Bertrand Russell, and A.N. Whitehead, initiated a long running debate on the foundations of mathematics.

The 19th century saw the founding of a number of national mathematical societies: the London Mathematical Society in 1865, the SociétéMathématique de France in 1872, the CircoloMatematico di Palermo in 1884, the Edinburgh Mathematical Society in 1883, and the American Mathematical Society in 1888. The first international, special-interest society, the Quaternion Society, was formed in 1899, in the context of a vector controversy.

In 1897, Hensel introduced p-adic numbers.

(ii) 20th century

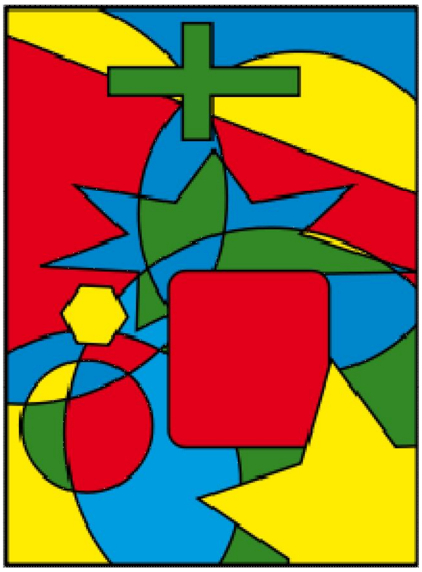

Fig (ii) A map illustrating the Four Color Theorem.

The 20th century saw mathematics become a major profession. Every year, thousands of new Ph.D.s in mathematics are awarded, and jobs are available in both teaching and industry.

In a 1900 speech to the International Congress of Mathematicians, David Hilbert set out a list of 23 unsolved problems in mathematics. These problems, spanning many areas of mathematics, formed a central focus for much of 20th-century mathematics. Today, 10 have been solved, 7 are partially solved, and 2 are still open. The remaining 4 are too loosely formulated to be stated as solved or not.

Notable historical conjectures were finally proven. In 1976, Wolfgang Haken and Kenneth Appel used a computer to prove the four colour theorem. Andrew Wiles, building on the work of others, proved Fermat's Last Theorem in 1995. Paul Cohen and Kurt Gödel proved that the continuum hypothesis is independent of (could neither be proved nor disproved from) the standard axioms of set theory. In 1998 Thomas Callister Hales proved the Kepler conjecture.

Mathematical collaborations of unprecedented size and scope took place. An example is the classification of finite simple groups (also called the "enormous theorem"), whose proof between 1955 and 1983 required 500-odd journal articles by about 100 authors, and filling tens of thousands of pages. A group of French mathematicians, including Jean Dieudonné and André Weil, publishing under the pseudonym "Nicolas Bourbaki", attempted to exposit all of known mathematics as a coherent rigorous whole. The resulting several dozen volumes has had a controversial influence on mathematical education.

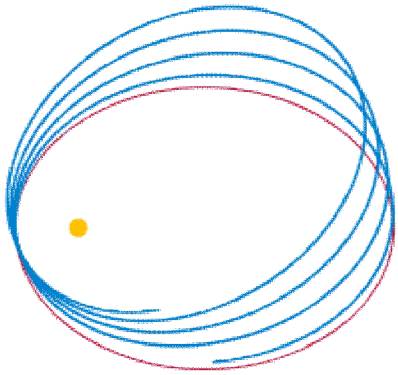

Fig. (iii) Newtonian (red) vs. Einsteinian orbit (blue) of a lone planet orbiting a star, with relativistic precession of apsides

Differential geometry came into its own when Einstein used it in general relativity. Entire new areas of mathematics such as mathematical logic, topology, and John von Neumann's game theory changed the kinds of questions that could be answered by mathematical methods. All kinds of structures were abstracted using axioms and given names like metric spaces, topological spaces etc. As mathematicians do, the concept of an abstract structure was itself abstracted and led to category theory. Grothendieck and Serre recast algebraic geometryusing sheaf theory. Large advances were made in the qualitative study of dynamical systems that Poincaré had begun in the 1890s. Measure theory was developed in the late 19th and early 20th centuries. Applications of measures include the Lebesgue integral, Kolmogorov's axiomatisation of probability theory, and ergodic theory. Knot theory greatly expanded. Quantum mechanics led to the development of functional analysis. Other new areas include, Laurent Schwarz's distribution theory, fixed point theory, singularity theory and René Thom's catastrophe theory, model theory, and Mandelbrot's fractals. Lie theory with its Lie groups and Lie algebras became one of the major areas of study.

Non-standard analysis, introduced by Abraham Robinson, rehabillitated the infinitesimal approach to calculus, which had fallen into disrepute in favour of the theory of limits, by extending the field of real numbers to the Hyperreal numbers which include infinitesimal and infinite quantities.

The development and continual improvement of computers, at first mechanical analog machines and then digital electronic machines, allowed industry to deal with larger and larger amounts of data to facilitate mass production and distribution and communication, and new areas of mathematics were developed to deal with this: Alan Turing's computability theory; complexity theory; Claude Shannon's information theory; signal processing; data analysis; optimization and other areas of operations research. In the preceding centuries much mathematical focus was on calculus and continuous functions, but the rise of computing and communication networks led to an increasing importance of discrete concepts and the expansion of combinatorics including graph theory. The speed and data processing abilities of computers also enabled the handling of mathematical problems that were too time-consuming to deal with by pencil and paper calculations, leading to areas such as numerical analysis and symbolic computation. Some of the most important methods and algorithms of the 20th century are: the simplex algorithm, the Fast Fourier Transform, error-correcting codes, the Kalman filter from control theory and the RSA algorithm of public-key cryptography.

At the same time, deep insights were made about the limitations to mathematics. In 1929 and 1930, it was proved the truth or falsity of all statements formulated about the natural numbers plus one of addition and multiplication, was decidable, i.e. could be determined by some algorithm. In 1931, Kurt Gödel found that this was not the case for the natural numbers plus both addition and multiplication; this system, known as Peano arithmetic, was in fact incompletable. (Peano arithmetic is adequate for a good deal of number theory, including the notion of prime number.) A consequence of Gödel's two incompleteness theorems is that in any mathematical system that includes Peano arithmetic (including all of analysis and geometry), truth necessarily outruns proof, i.e. there are true statements that cannot be proved within the system. Hence mathematics cannot be reduced to mathematical logic, and David Hilbert's dream of making all of mathematics complete and consistent needed to be reformulated.

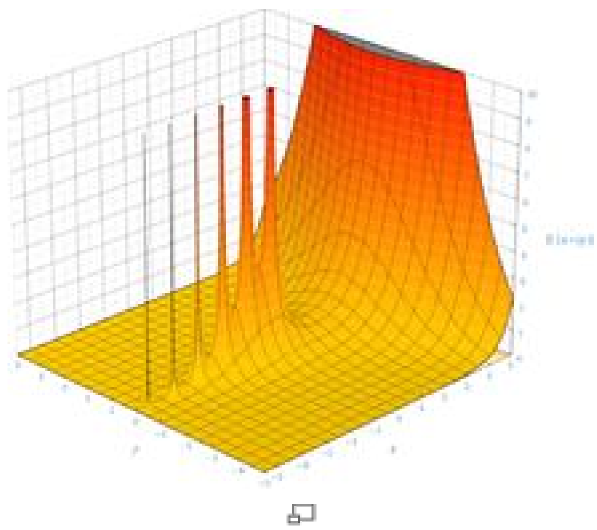

Fig. (iv) The absolute value of the Gamma function on the complex plane.

One of the more colourful figures in 20th-century mathematics was SrinivasaAiyangarRamanujan (1887–1920), an Indian autodidact who conjectured or proved over 3000 theorems, including properties of highly composite numbers, the partition function and its asymptotics, and mock theta functions.

He also made major investigations in the areas of gamma functions, modular forms, divergent series, hypergeometric series and prime number theory.

As in most areas of study, the explosion of knowledge in the scientific age has led to specialization: by the end of the century there were hundreds of specialized areas in mathematics and the Mathematics Subject Classification was dozens of pages long.[133] More and more mathematical journals were published and, by the end of the century, the development of the world wide web led to online publishing.

(iii) In 21st century:

In 2000, the Clay Mathematics Institute announced the seven Millennium Prize Problems, and in 2003 the Poincaré conjecture was solved by Grigori Perelman (who declined to accept an award on this point).

Most mathematical journals now have online versions as well as print versions, and many online-only journals are launched. There is an increasing drive towards open access publishing, first popularized by the arXiv.

There are many observable trends in mathematics, the most notable being that the subject is growing ever larger, computers are ever more important and powerful, the application of mathematics to bioinformatics is rapidly expanding, the volume of data to be analyzed being produced by science and industry, facilitated by computers, is explosively expanding.

3.Application of Mathematics in the field of medical sciences:

The application of mathematics in the field of science has been in place since times immemorial. However, its actual importance and applicability in medical sciences has become more visible in today's world. To understand it medically, one can say that the application of mathematics in medical sciences are sporadic. The study of mathematics and the subject, in general, are contributing to biology and the study of life sciences in many ways.

Today, biologists and professionals in the field of medical sciences use the concepts of mathematics its theorems and formula, among others in obtaining quantitative measures and analysing their studies in quantity along with quality. Many scientific medical equipments and machines have been designed in keeping the applications of mathematics in view. Biologists use mathematics to derive quantitative data from any medical study, such as estimating the number of genes that are involved in inheriting a particular trail from the parents to the child.

Today, the concepts and theories of mathematics are being applied to almost all aspects of medical sciences. However, there are some fields of study under medical sciences where mathematics has found greater scope. These fields are clinical research, biotechnology and genetics. In general, the mathematics and its sub branches play an important role in the study of genetics.

In the study of genetics, the two most important sub sections of mathematics that play a major role are statistics and probability. Through probability, its formula and its applications are used people working in genetics understand the mechanism of meiosis, forming of egg cells and the sperm, and the process of inheritance, among others. They also get an understanding of how phenotypes (observable diseases) and genotypes (the DNA sequences) are related to each other through the study of probability distributions. They also conduct the analysis and study of genetic determinants through the applications of statistics and its methods and principles.

Another advantage that the study of mathematics in medicine and biology provides is to mathematical modeller's. Mathematical modelling refers to looking and studying the biological systems and the medical phenomena through the study of mathematics by applying its principles and theorems. Mathematics has also shown its worth in testing out new ideas in the field of medicine. This has been or high importance in the study of cancerous and tumor cells. Medical practitioners have been successful in understanding the direction in which cancerous cells potentially grow inside the living body with the help of a model strictly based on mathematics.

Many health organizations are now using online systems and software applications to manage their patient records and keep a check on the remedial steps for each patient, coordinating with practitioners and other hospital staff on a regular basis along with many more things. All these online systems and software applications are developed keeping mathematics as the platform.

In biotechnology as well, mathematics is seen to play a major role. The study of mathematics is needed to understand concepts such as concentration/dilution, calibration, molarity, molality, solution preparation, serial dilution, radioactive decay and decomposition, absorption, and cell growth, among others. In clinical research, the understanding of DNA, thumb impressions, and markups, among others is also carried out based on the study of mathematics.

4. Mathematics in medical imaging:

4.1 Tomography refers to imaging by sections or sectioning, through the use of any kind of penetrating wave.

A device used in tomography is called a tomograph, while the image produced is a tomogram. The method is used in radiology, archaeology, biology, geophysics, oceanography, materials science, astrophysics, quantum information, and other sciences. In most cases it is based on the mathematical procedure called tomographic reconstruction. Fig. 4.1 represents the basic principle of tomography superposition free tomographic cross sections S1 and S2 compared with the projected image P.

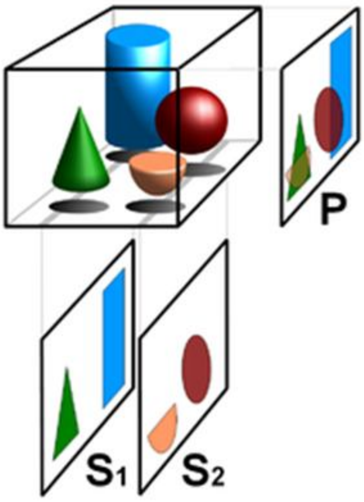

Fig.4.1tomography superposition with projected image P

4.2 Medical Imaging - Perspective

The perspective in medical imaging is dominated by the development of newer measuring technologies:

- 3Dtomography. In non-destructive material testing, 3D X-ray CT is widely used in connection with a circular scanning geometry; i.e., the X-ray tube is moved on a circle in a plane around the examined object with the detector plane positioned at the opposite side. Helical geometry is favoured in medical applications, but there in necessary variable shift of the patient has not been solved yet satisfactorily in existing algorithms. In principle, X-ray tube and detector can be moved along arbitrary trajectories around the patient. The determination of trajectories that are optimal with respect to resolution and stability remains a mathematical challenge. The following figure 4.2(a) represent the X-ray source trajectory.

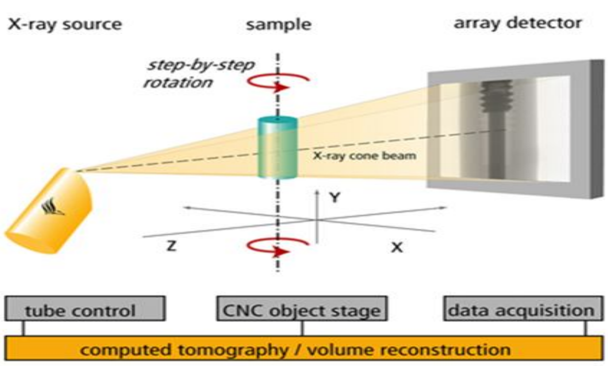

Fig. 4.2(a)X-ray source trajectory

Fig. 4.2(b) and Fig.4.2(c) is the mathematical representation of X-ray source trajectory.

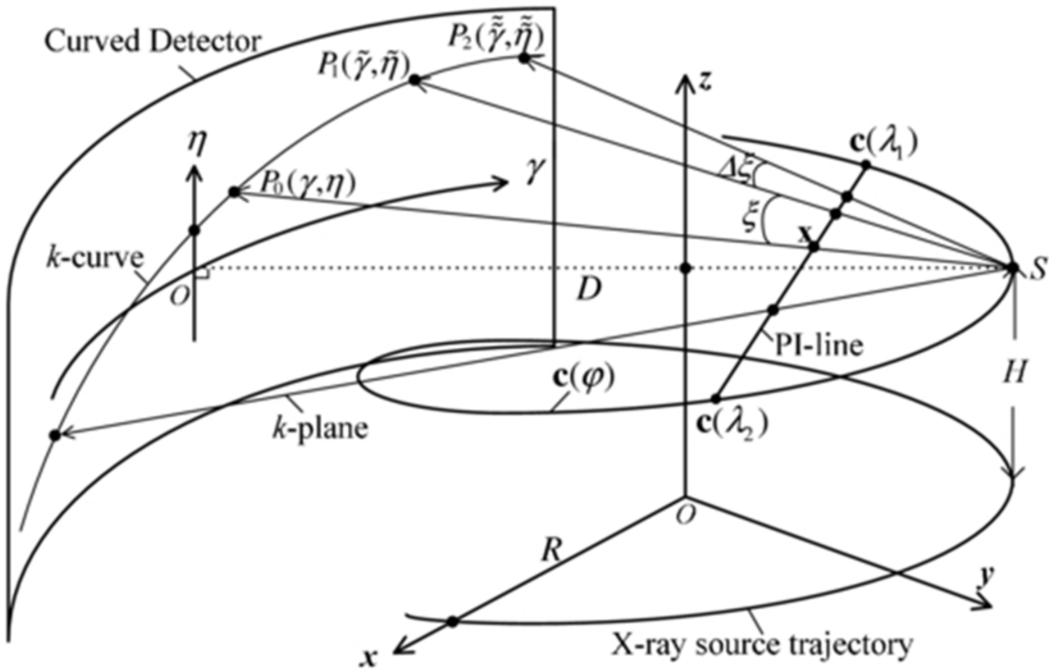

Fig. 4.2(b)Mathematical representation of X-ray source trajectory

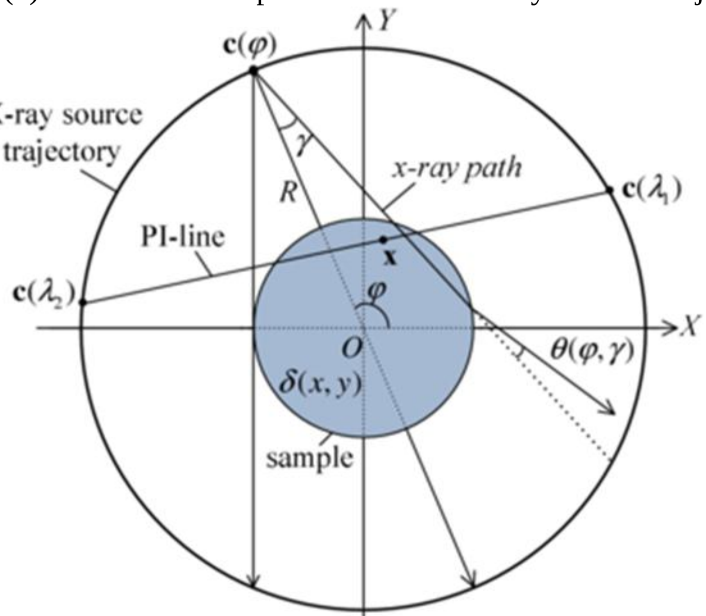

Fig. 4.2(c)Mathematical representation of X-ray source trajectory

- Electron paramagnetic resonance imaging:In this technique a decoupling of the fourth dimension is not possible, since there, besides the three spatial dimensions, a spectral dimension shows up additionally. The corresponding mathematical model is the Radon transform in four dimensions. This technology is presently studied in the stages of pharmaceutical research and animal experiments. However, due to the limitations in field strength, the data in Radon space cannot be sampled completely with the consequence that a limited angle problem has to be solved. Theoretically, the desired distribution would be uniquely determined, if all data in the restricted range were available, but instabilities and strong artefacts complicate the reconstruction problem.

- Ultrasound CT: In this approach sender and receiver are spatially separated, the corresponding mathematical model is an inverse scattering problem for the determination of the spatially varying sound impedance and the scattering properties. The difficulty here is that, in contrast to CT, the paths of the waves depend upon the variable to be computed, which makes the problem highly nonlinear. A linearization of the problem via Born or Rytov approximation neglects the effects of multiple scattering and is therefore not sufficiently accurate. That is why ultrasound tomography today is still a challenge to mathematics and algorithm development.
- Transmission electron microscopy (TEM): For the visualization of biomolecules by TEM, various approaches are pursued. If one does not aim at averaging over many probes of the same kind, again a limited angle problem arises. In addition, wave phenomena enter for small sized objects, leading, as in ultrasound tomography, to nonlinear inverse scattering problems. Fortunately, linearization is feasible here, which facilitates the development of algorithms significantly.
- Phase contrast tomography: In this technology, where complex-valued sizes have to be reconstructed, linearizations are applicable, too. The phase supplies information even when the density differences within the object are extremely small. Due to different scanning geometries, medical application still generates challenging problems for the development of algorithms.
- Diffusion tensor MRI: This method provides a tensor at each reconstruction point, thus bearing information concerning the diffusivity of water molecules in tissue. In this reconstruction and regularization are performed separately. Only after having computed tensors, properties of the tensor like symmetry or positive definiteness are produced point by point.

The above list of imaging techniques under present development is by no means complete. Methods like impedance tomography are studied as well as those applying light to detect objects close to the skin. In all of the mentioned measuring techniques, the technological development is so advanced that the solution of the associated mathematical problems like modelling, determination of achievable resolution and development of efficient algorithms will yield a considerable innovation thrust.

Thus one can conclude that the study of mathematics is an exercise in reasoning. Beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and “mathematicizing” a problem. As college students and employees, high school graduates will need to use mathematics in contexts quite different from the high school classroom. They will need to make judgments about what problem needs to be solved and, therefore, about which operations and procedures to apply. Woven throughout the four domains of mathematics—Number Sense and Numerical Operations; Algebra; Geometry; and Data Interpretation, Statistics and Probability—are the following mathematical reasoning skills:

- Using inductive and deductive reasoning to arrive at valid conclusions.
- Using multiple representations (literal, symbolic, graphic) to represent problems and solutions.
- Understanding the role of definitions, proofs and counter-examples in mathematical reasoning; constructing simple proofs.
- Using the special symbols of mathematics correctly and precisely.
- Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations.
- Distinguishing relevant from irrelevant information, identifying missing information, and either finding what is needed or making appropriate estimates.
- Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem.
- When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results.
- Shifting regularly between the specific and the general, using examples to understand general ideas, and extending specific results to more general cases to gain insight.

References:

[1] Louis AK (1992) Medical imaging: state of the art and future development. Inverse Prob 285 8:709–738.

[2] Louis AK (2008) Combining image reconstruction and image analysis with an application to two-dimensional tomography. SIAM J. Imaging Sciences 1(2):188–208.
[3] Natterer F, Wübbeling F (2001) Mathematical methods in image reconstruction. SIAM.
[4] Jonas P, Louis AK (2004) Phase contrast tomography using holographic measurements. Inverse Prob 20:75–102.
[5] Marr RB, Chen C, Lauterbur PC (1981) On two approaches to 3D reconstruction in NMR zeugmatography. In: Herman GT, Natterer F (eds) Mathematical aspects of computerized tomography. Springer LNMI, Springer, Berlin, pp 225–240.
[6] Zachow S, Zilske M, Hege H-C (2007) 3D reconstruction of individual anatomy from medical image data. In: Proceedings ANSYS conference and CADFEM Users' Meeting, Dresden. Sieheauch: Report 07-41, ZIB