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Harmonic Mean Labeling for Some Special Graphs

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ABSTRACT

A graph G = (V, E) with p vertices and q edges is said to be a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1, 2, …, q+1 in such a way that when each edge e =uv is labeled with f(e = uv) $= \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic mean labeling of G.

In this paper we prove the Harmonic mean labeling behavior for some special graphs.

Keywords: Graph, Harmonic mean graph, path, comb, kite, Ladder, Crown.

1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges.

The vertex set is denoted by V(G) and the edge set is denoted by E(G). A cycle of length n is C_n and a path of length n is denoted by P_n. For all other standard terminology and notations we follow Harary [1].

S. Somasundaram and R.Ponraj introduced mean labeling of graphs in [2]. We introduce Harmonic mean labeling of graphs in [3] and studied their behavior in [4], [5] and [6]. In this paper, we investigate Harmonic mean labeling for some special graphs.

The definitions and other informations which are useful for the present investigation are given below:

Definition 1.1: A Graph G with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…q+1 in such a way that when each edge e = uv is labeled with f (e = uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are distinct. In this case f is called a Harmonic mean labeling of G.

Definition 1.2: An (m, n) kite graph consists of cycle of length m with n edges path attached to one vertex of a cycle.

Definition 1.3: Comb is a graph obtained by joining a single pendant edge to each vertex of a path.

Definition 1.4: The product graph $P_m \times P_n$ is called a planar grid and $P_2 \times P_n$ is a called a ladder.

Definition 1.5: The corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed by one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.6: The graph P_m A$K_{1,n}$ is obtained by attaching $K_{1,n}$ to each vertex of P_m.

We shall make frequent reference to the following results

Theorem 1.7 [3]: Any path is a Harmonic mean graph.

Theorem 1.8 [3]: Any cycle is a Harmonic mean graph.

Theorem 1.9 [3]: Combs are Harmonic mean graphs.

Theorem 1.10 [3]: Ladders are Harmonic mean graphs.

2. Main Results

Theorem 2.1: An (m, n) kite graph G is a Harmonic mean graph.

Proof: Let $u_1 u_2 u_3 \ldots\ldots u_m u_1$ be the given cycle of length m and $v_1 v_2 \ldots\ldots v_n$ be the given path of length n. Define a function $f : V(G) \to \{1,2\ldots.q+1\}$ by

$f(u_i) = i,\ 1 \le i \le m,\ f(v_i) = n+i,\ 1 \le i \le n.$

Then the edge labels of the cycle are $f(u_1u_m) = 1$, $f(u_iu_{i+1}) = i+1$, $1 \leq i \leq m$ and the edge labels of the path are $\{m+1, m+2\ldots.m+n\}$

Hence the kite graph is a Harmonic mean graph

Example 2.2: A Harmonic mean labeling of (5, 6) kite graph is show below.

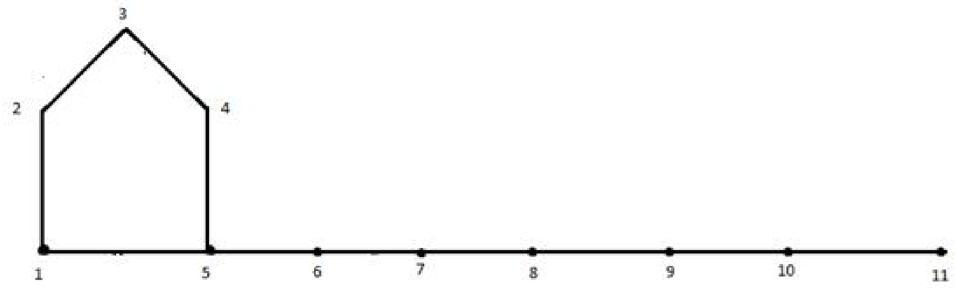

Figure: 1

Next we investigate special type of trees generated from a path

Theorem 2.3: Let G be a graph obtained by joining a pendent vertex with a vertex of degree two of a comb graph. Then G is a Harmonic mean graph.

Proof: Comb is a graph with $V(G) = \{v_1\ v_2\ldots\ldots\ldots\ldots..v_n,\ u_1,\ u_2\ldots\ldots\ldots.u_n\}$.

Let us take P_n as $v_1\ v_2\ldots\ldots v_n$ and join a vertex u_i to v_i $1 \leq i \leq n$.

Let G be a graph obtained by joining a pendant vertex w to v_n.

Define a function $f : V(G) \rightarrow \{1,2\ldots q+1\}$by

$f(v_i) = 2i$, $1\leq i \leq n$, $f(u_i) = 2i-1$, $1\leq i \leq n$, $f(w) = 2n+1$

The label of the edge u_iv_i is $2i$, $1\leq i \leq n$.

The label of the edge v_iv_{i+1} is $2i-1$, $1\leq i \leq n$.

The label of the edge $v_n\,w$ is $2n$.

This gives a Harmonic mean labeling for G.

Example 2.4: Harmonic mean labeling of G with 11vertices and 10 edges is given below:

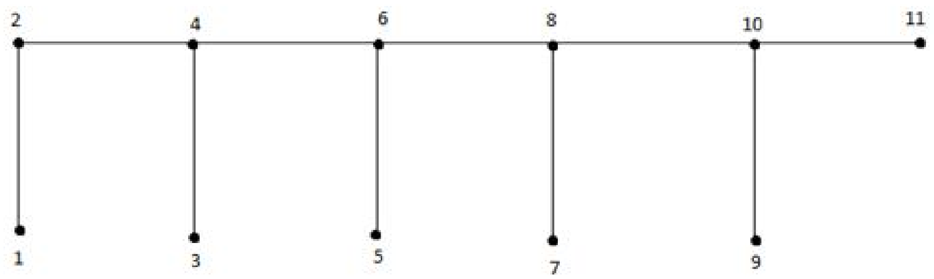

Figure: 2

In the similar manner, we can see the Harmonic mean labeling of G obtained by joining a pendant vertex with a vertex of degree two on both ends of a comb graph. Harmonic mean labeling G with 10 vertices 9 edges is shown below:

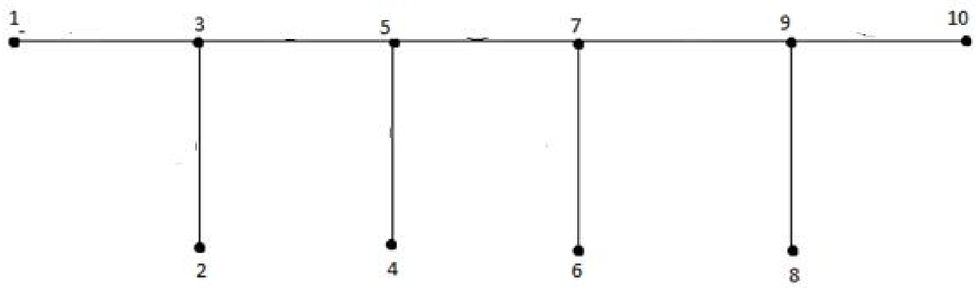

Figure: 3

Now we have

Theorem 2.5: Let A be the collection of Paths P_n^i where n is odd. $P_n^i = u_1^i u_2^i \ldots\ldots u_n^i$, $(1 \le i \le m)$. Let G be the graph obtained from A with

$V(G) = \bigcup_{i=1}^{n} V(P_n^i)$ and $E(G) = \bigcup_{i=1}^{n} E(P_n^i) \cup (u^i_{\frac{n+1}{2}} u^{i+1}_{\frac{n+1}{2}}, 1 \le i \le m-1\}$

Then G is a Harmonic Mean graph

Proof: Define $f : V(G) \to \{1,2\ldots.q+1\}$ by

$f(u_i^1) = i,\ 1 \le i \le n$ and $f(u_j^k) = f(u_n^{k-1}) + (j-1),\ 2 \le k \le m,\ 1 \le j \le n.$

Obviously f is a Harmonic mean labeling of G.

Examples 2.6: A Harmonic mean labeling of G with m = 5 and n = 5 is given below:

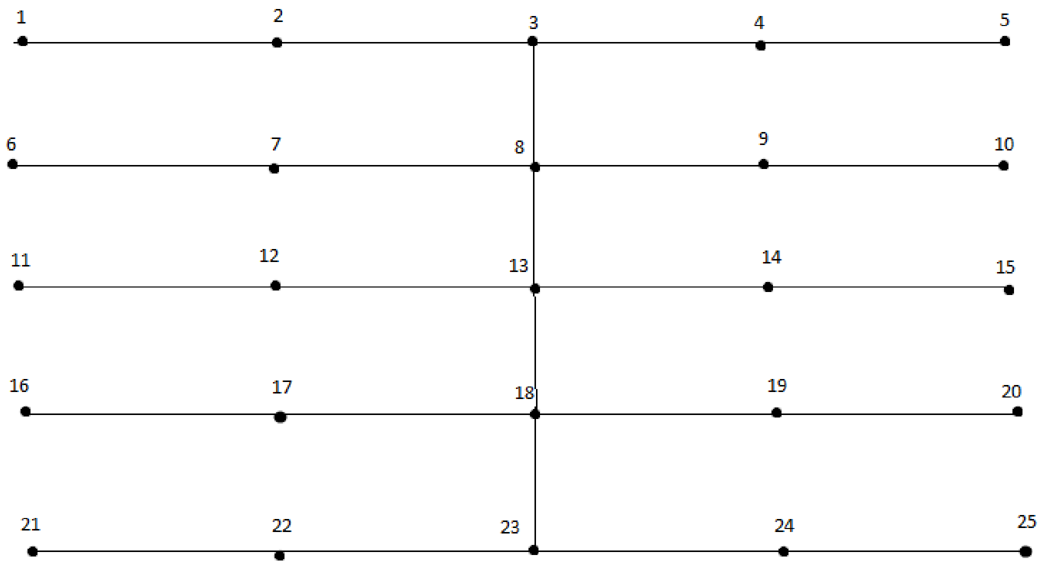

Figure: 4

Now we prove the following

Theorem 2.7: Let P_n be the path and G be the graph obtained from P_n by attaching C_3 in both the end edges of P_n. Then G is a Harmonic mean graph.

Proof: Let P_n the path $u_1u_2\ldots.u_n$ and $v_1u_1u_2$, $v_2u_{n-1}u_n$ be the triangles at the end.

Define a function $f : V(G) \to \{1,2\ldots.q+1\}$ by

$f(u_i) = i+1$, $1 \le i \le n-1$, $f(u_n) = n+3$, $f(v_1) = 1$, $f(v_2) = n+2$

The edge labels are given below:

$f(u_1v_1) = 1$, $f(u_2v_1) = 2$.

$f(u_{n-1}\,v_2) = n+1$, $f(u_nv_2) = n+3$

$f(u_{n-1}u_n) = n+2$

Obviously G is a Harmonic mean graph.

Example 2.8: A Harmonic mean labeling of G obtained from P_8 is given below

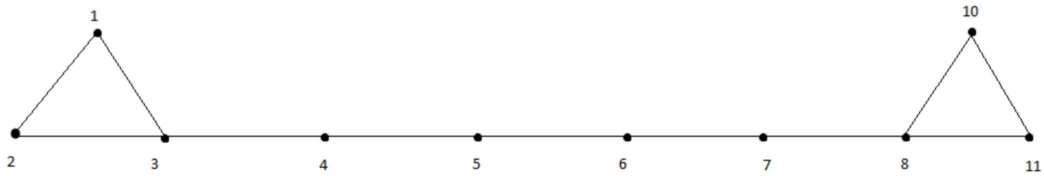

Figure: 5

Next we investigate

Theorem 2.9: Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$. Then G admits a Harmonic mean labeling.

Proof: Let P_n be the path $u_1u_2......u_n$ and a_i, b_i the vertices of $K_{1,2}$ which are attached to vertex u_i of P_n. The graph contains 3n vertices and 3n-1 edges.

Define a function $f : V(G) \to \{1,2.....q+1\}$ by

$f(u_i) = 3i-1, \ 1 \leq i \leq n, \ f(a_i) = 3i-2, \ 1 \leq i \leq n,$

$f(b_i) = 3i, \ 1 \leq i \leq n, \ f(u_i \ u_{i+1}) = 3i, \ 1 \leq i \leq n.$

$f(a_i \ u_i) = 3i-2, \ 1 \leq i \leq n, \ f(b_i \ u_i) = 3i, \ 1 \leq i \leq n.$

Hence G admits a Harmonic mean labeling.

Example 2.10: A Harmonic mean labeling of P_4 A$K_{1,2}$ is shown in figure 6.

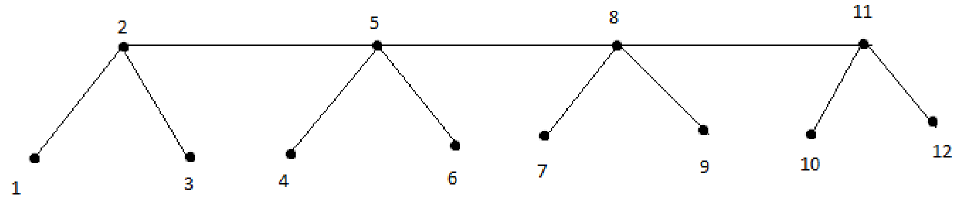

Figure: 6

The same argument as in Theorem 2.9 gives the following

Theorem 2.11: Let G be a graph obtained by attaching P_n to the central vertex of $K_{1,3}$. Then G admits a Harmonic mean labeling

Proof: Let P_n be the path $u_1u_2.....u_n$ and a_i, b_i, c_i be the vertices of $K_{1,\ 3}$ which are attached to the vertex u_i of P_n.

The graph G has 4n vertices ad 4n-1 edges.

Define $f : V(G) \to \{1,2....q+1\}$ by

$f(u_i) = 4i-1, \ 1 \leq i \leq n.$

$f(a_i) = 4i-3, \ 1 \leq i \leq n, \ f(b_i) = 4i-2, \ 1 \leq i \leq n, \ f(c_i) = 4i-1, \ 1 \leq i \leq n.$

Edges are labeled with

$f(u_iu_{i+1}) = 4i, \ 1 \leq i \leq n.$

$f(u_ia_i) = 4i-3, \ 1 \leq i \leq n.$

$f(u_ib_i) = 4i-2, \ 1 \leq i \leq n, \ f(u_i \ c_i) = 4i-1, \ 1 \leq i \leq n.$

Hence G admits a Harmonic mean labeling

Example 2.12: A Harmonic mean labeling $P_3 AK_{1,3}$ is displayed below.

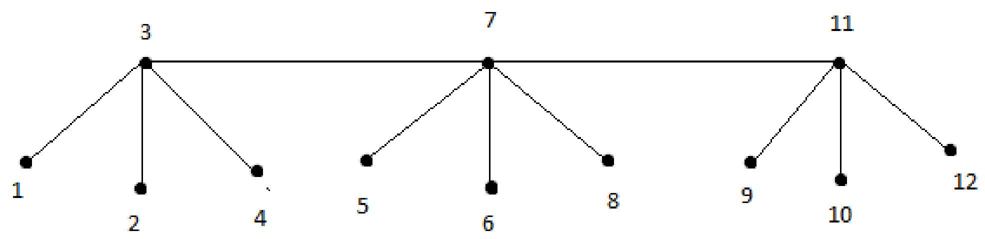

Figure: 7

Now we prove the Harmonic mean labeling of Corona with Ladder

Theorem 2.12 : $L_n AK_1$ is a Harmonic mean graph.

Proof: Let L_n be a Ladder with $V(L_n) = \{ a_i, b_i \ 1 \leq i \leq n\}$ and
$E(L_n) = \{a_ib_i: 1 \leq i \leq n\} \cup \{a_i a_{i+1}: 1 \leq i \leq n-1\} \cup \{b_i b_{i+1}: 1 \leq i \leq n-1\}$

Let c_i be the pendant vertex adjacent to a_i and d_i be the pendent vertex adjacent to b_i.

Define a function $f : V(L_n AK_1) \to \{1,2\ldots\ldots q+1\}$ by

$f(a_i) = 5i-3, 1\leq i \leq n,$

$f(b_i) = 5i-2, 1\leq i \leq n,$

$f(c_i) = 5i-4, 1\leq i \leq n,$

$f(d_i) = 5i-1, 1\leq i \leq n-1.$

Edges labels are given below

$f(a_i b_i) = 5i-3, 1\leq i \leq n$

$f(b_i d_i) = 5i-1, 1\leq i \leq n.$

$f(a_i c_i) = 5i-4, 1\leq i \leq n.$

$f(a_i a_{i+1}) = 5i-1, 1\leq i \leq n-1$

$f(b_i b_{i+1}) = 5i, 1\leq i \leq n-1$

Hence $L_n AK_1$ is a harmonic mean graph.

Example 2.13: The Harmonic Mean labeling of $L_4 AK_1$, is given below

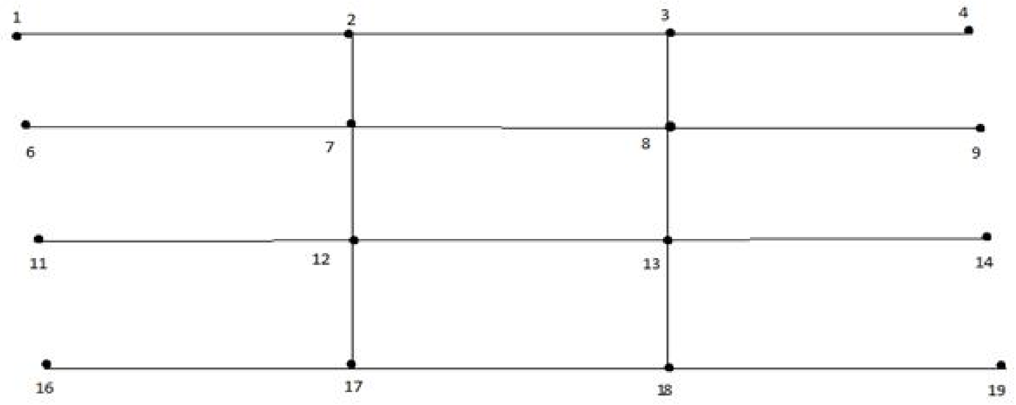

Figure: 8

Remark 2.14[4]: The crown $C_n\mathrm{A}K_1$ is a Harmonic mean graph for all n ≥ 3.
Now we prove

Theorem 2.15: $C_n\mathrm{A}\overline{K_2}$ is a Harmonic mean graph for all n ≥ 3.

Proof: Let C_n be the cycle u_1 u_2…..$u_n u_1$ and v_i, w_i be the pendant vertices adjacent to u_i $1 \le i \le n$.

Define a function f : $V(C_n \mathrm{A}\overline{\mathbf{K}_2}) \to \{1,2\ldots q+1\}$ by

$f(u_i) = 3i-1$, $1 \le i \le n$, $f(v_i) = 3i-2$, $1 \le i \le n$, $f(w_i) = 3i$, $1 \le i \le n$.

Then the edge labels are all distinct. Hence $C_n \mathrm{A}\overline{\mathbf{K}_2}$ is a Harmonic mean graph.

Example 2.15: Here we display the Harmonic mean labeling of $C_5\mathbf{A}\overline{K}_2$

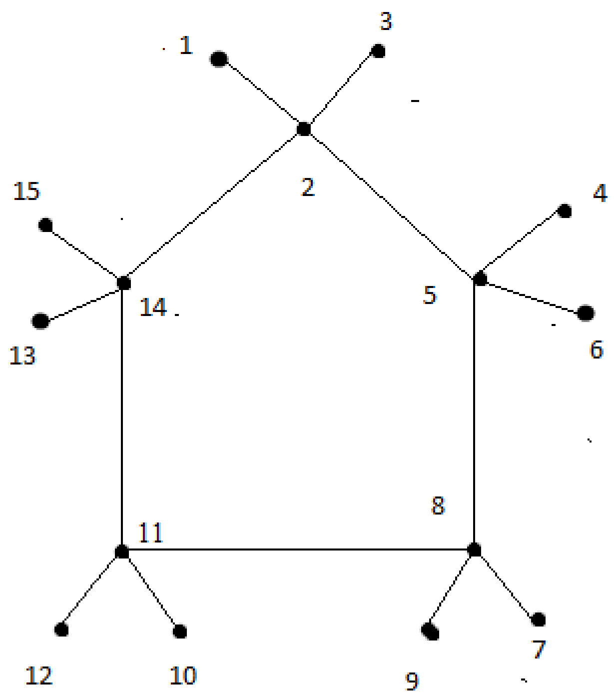

Figure: 9

Remark 2.16: Harmonic Mean labeling of $C_4A\overline{K}_3$ is shown in the following figure.

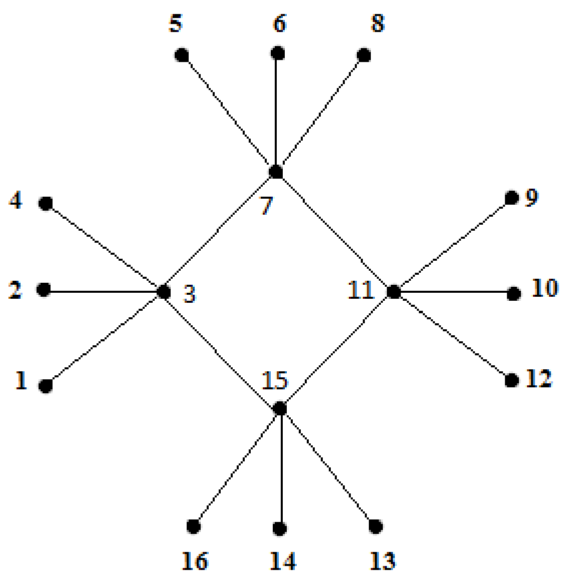

Figure: 10

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