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Seeking the Best Shape of Pans Heated in an Oven

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Abstract

When baking cakes, we expect that the whole cake is heated evenly.In order to find the best shape of pans to make cakes, webuild two models. Model one is built to explore the distribution of heat across the outer edges of different pans.To be specific, under the condition that the ambient temperature is constant, we simulate the distribution of heat across the outer edges of pans in different shapes. The pan can be rectangle, pentagon, hexagon and so on. By means of calculating the slope of the temperature variation at the edge we can get heat distribution degree of pans in different shapes.We find that circular pans can be heated more evenly than any other shape of pans. Model two is built to explore which shape of pans can make the most use of the space in the rectangular oven. Giving different schemes for different shapes of pans and calculating the ratio of each scheme occupying in the oven we can get the area occupying ratio. We find that the rectangular pans similar to the oven make the most use of the space inside the oven. Considering both sides of heat distribution degree and area occupying ratio we can get the most optimized model of shape of pans to make cakes.

Keywords: shape of pans, heat distribution degree and area occupying ratio

1 Model One

1.1 Principle Introduction

An oven is mainly consisted of the body, electrical heating element, thermolator, timer, power switching regulator and so on. When it works, the electrical heating element will give out heat to attain and maintain the temperature set before. By means of heat conduction, the temperature of the food will rise up.

Food in the oven obeysthe heat conduction equation as follows [1]

$$\nabla^2 T(\boldsymbol{r}, t) + \frac{1}{k} g(\boldsymbol{r}, t) = \frac{1}{\alpha} \frac{\partial T(\boldsymbol{r}, t)}{\partial t} \quad (1\text{-}1)$$

where kdenotes the coefficient of heat conductivity, and αdenotes the thermal diffusion coefficient.

Since food has no heat producer inside, equation (1-1) can be changed into

$$\nabla^2 T(\boldsymbol{r}, t) = \frac{1}{\alpha} \frac{\partial T(\boldsymbol{r}, t)}{\partial t} \quad (1\text{-}2)$$

The boundary conditions are

$$k_i \frac{\partial T}{\partial n_i} + h_i T = h_i T_a \quad (1\text{-}3)$$

$where$ n_i denotes thenormal vector of boundary $S_i(i = 1, 2, 3 \cdots)$, h_i denotes the convective heat transfer coefficient$(i = 1, 2, 3 \cdots)$ and T_a denotes the ambient temperature.

The initial condition is

$$T = F(\boldsymbol{r}) \quad (1\text{-}4)$$

To simplify the model, we just consider the pan in two-dimensional space. That is to say, by decreasing the depth of the pan, only the bottom of the pan is considered.Take arectangular pan as an example. Define that the length of the pan is a and the width of it is b. Define that the initial temperature of the pan is $T_1$and the temperature inside the oven is T_2constantly.

Establish rectangular coordinate system below

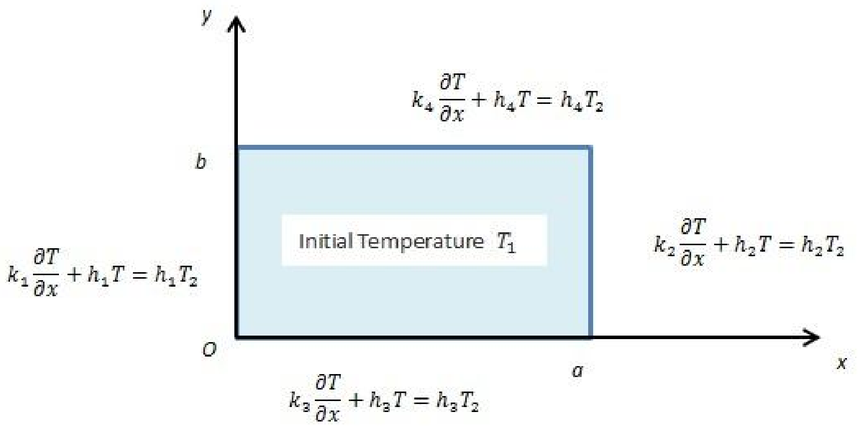

Fig. 1: Rectangular pan in rectangular coordinate system

From Fig. 1 we get

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t},\quad 0<x<a, 0<y<b, t>0 \tag{1-5}$$

$$k_1\frac{\partial T}{\partial x}+h_1 T=h_1 T_2,\quad x=0, t>0 \tag{1-6}$$

$$k_2\frac{\partial T}{\partial x}+h_2 T=h_2 T_2,\quad x=a, t>0 \tag{1-7}$$

$$k_3\frac{\partial T}{\partial x}+h_3 T=h_3 T_2,\quad y=0, t>0 \tag{1-8}$$

$$k_4\frac{\partial T}{\partial x}+h_4 T=h_4 T_2,\quad y=b, t>0 \tag{1-9}$$

$$T=T_1,\quad t=0 \tag{1-10}$$

From (1-5) to (1-10), we get the model of a rectangular pan in an oven.

1.2 Assumptions

a) The pan is considered in two-dimensional space, which means that the depth of the pan is not considered.
b) The ambient temperature in the oven keeps150 Celsiusdegrees constantly.
c) Coefficient of heat conductivity$k$ is a constant and do not change when the temperature changes.
d) Thermal diffusion coefficientα is a constant.
e) Convective heat transfer coefficient$h$ is a constant.
f) The initial temperature of the pan is 25 Celsiusdegrees.
g) No matter in which kind of shape, each pan has an area of A.

1.3 Parameters

k: coefficient of heat conductivity
α: Thermal diffusion coefficient
n_i: Normal vector of boundary $S_i (i=1,2,3\cdots)$
h_i: Convective heat transfer coefficient$(i=1,2,3\cdots)$
T_a: Ambient temperature in the oven
T_b: Room temperature
A: area of pans
a_i: The length of an edge of polygons $(i=4,5,6\cdots)$
Q: Homogeneitydegree of heat

1.4 Model Building

From (1-2), (1-3), and (1-4),we get all the models of different shapes as follows:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t},\quad 0<x<a, 0<y<b, t>0 \tag{1-11}$$

$$\frac{\partial T}{\partial n_i}+HT=HT_a,\quad at\ the\ boundaries\ S_i, i=1,2,3\cdots,N \tag{1-12}$$

$$T=T_b,\quad \mathrm{t}=0 \tag{1-13}$$

$$H=\frac{h}{k} \tag{1-14}$$

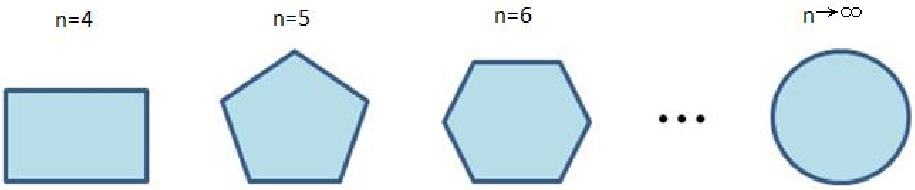

Fig. 2: The relationship between *n* and the shape of the pan

Referring to *Heat Condition* writtenby$M. Necati\ddot{O}zisik$[1], we get values of constants we need in the equation set.

Table 1. Values of constants needed

Material	Temperature/℃	$k/W\cdot(m\cdot ℃)^{-1}$	$\alpha/m^2\cdot s^{-1}$	T_a/℃	T_b/℃
low-carbon steel	0	45	12.4	150	25

By simulating in Matlab we solve the equation set consisted of (1-11) to (1-14).To simplify the model we just consider regular polygon.The value of *A* is 1.
To measure the degree of the heat distribution across the out edge of pans, we define Q as homogeneity degree of heat. Its expression is

$$Q = \frac{\Delta T_{imax}}{a_i} \tag{1-15}$$

WhereΔT_{imax}denotes the range of the temperature of an edge.

By simulating in Matlab we estimate the maximum and minimum of the temperature across the edge. [2]

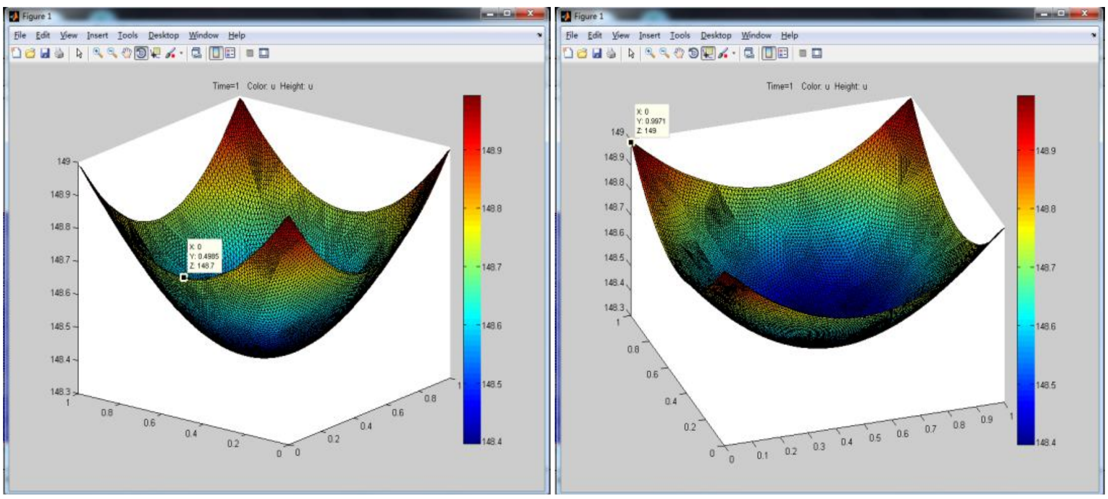

Fig. 3. The heat distribution of a square pan

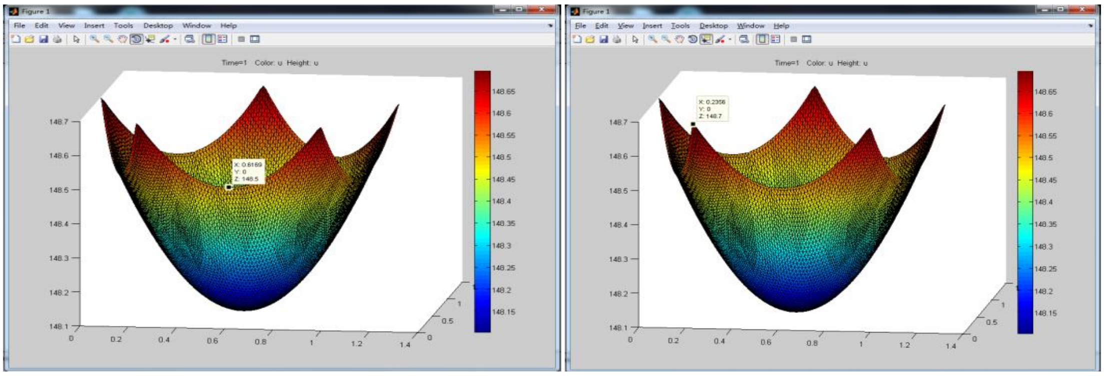

Fig. 4: The heat distribution of a pan shaped in regular pentagon

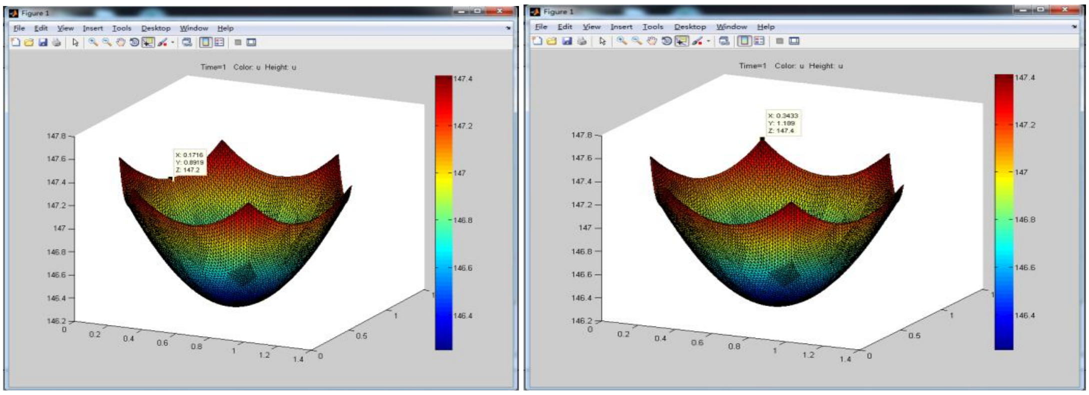

Fig. 5: The heat distribution of a pan shaped in regular hexagon

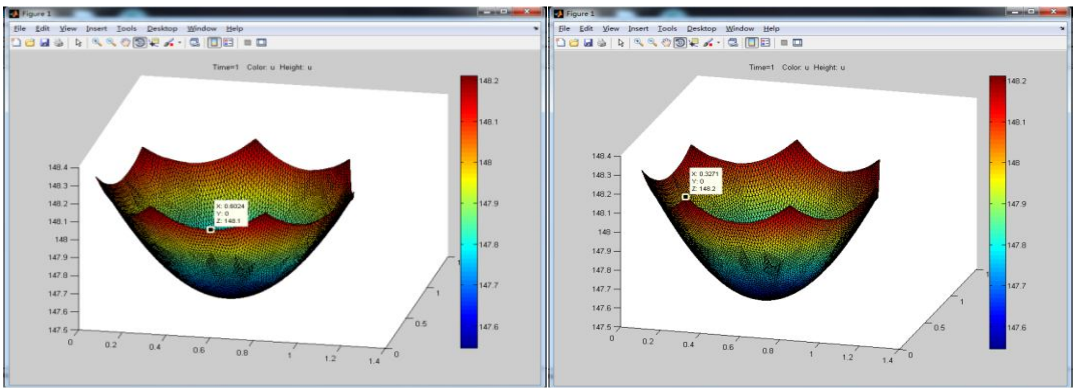

Fig. 6: The heat distribution of a pan shaped in regular heptagon

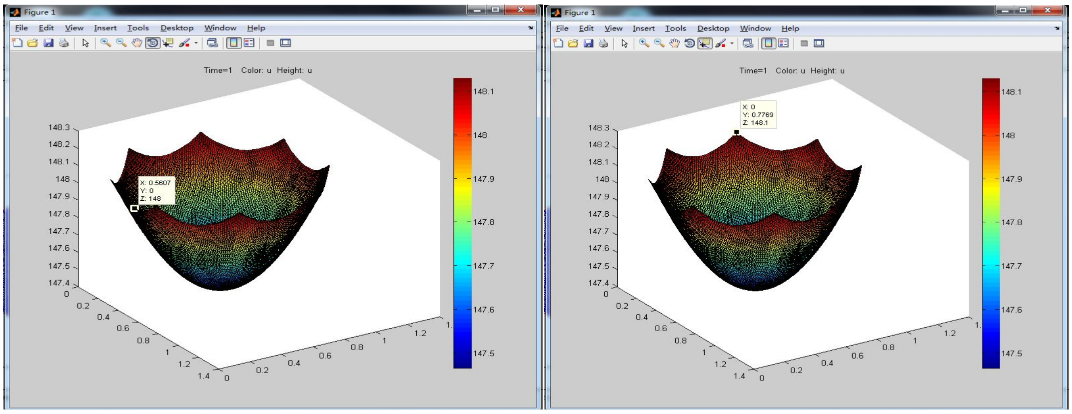

Fig. 7: The heat distribution of a pan shaped in regular octagon

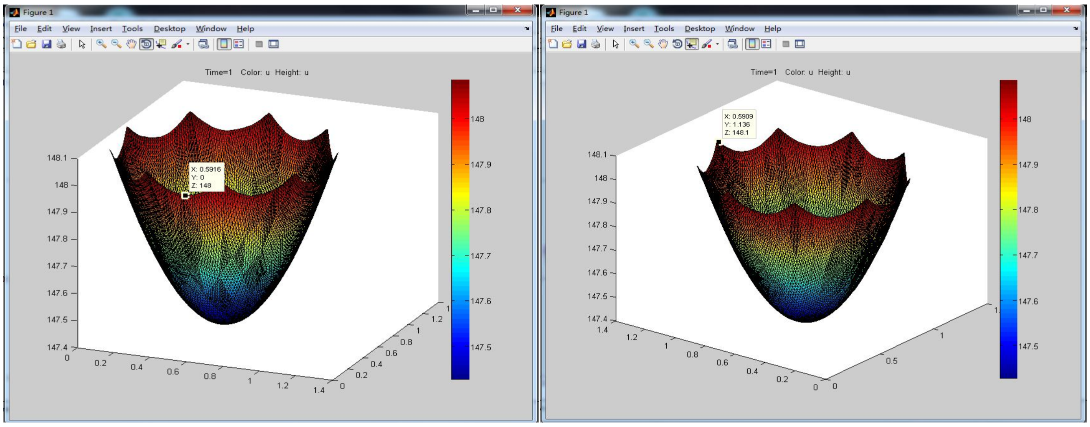

Fig. 8: The heat distribution of a pan shaped in regular enneagon

From the results that we get by simulating in Matlab, we calculate homogeneity degree of heat across the edge and draw the table below.

Table 2. The relationship between shapes of pans and edge homogeneity degree

Number of the edge(n)	4	5	6	7	8	9
Temperature of the vertex(y1)	149.05	148.7	147.4	148.2	148.1	148.105
Temperature of the midpoint(y2)	148.74	148.54	147.21	148.2	148.06	148.08
Range of the temperature(y=y1-y2)	0.31	0.16	0.11	0.06	0.04	0.015
Length of the edge(x)	1	0.7625	0.68659	0.52455	0.45509	0.4022
Ratio between y and x(Q)	0.31	0.2098	0.1602	0.1148	0.0879	0.0597

Establish rectangular coordinate system in Matlab to estimate the relationship between *Q* and *n*.

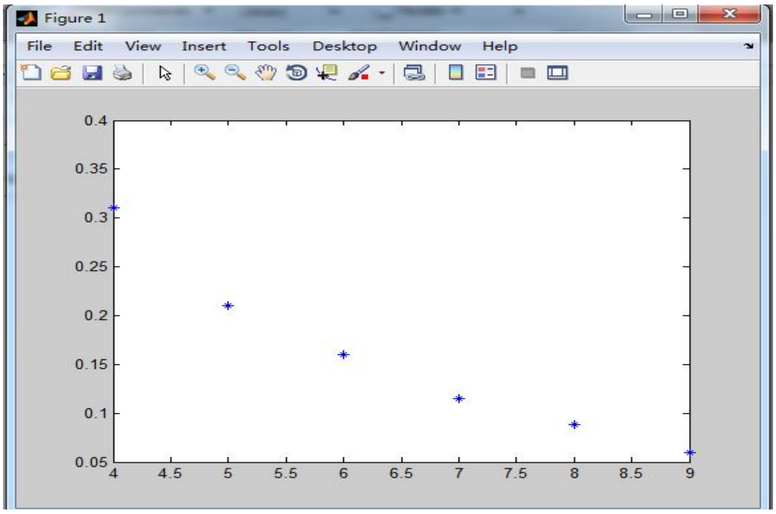

Fig. 9: Distribution of (n, Q) in rectangular coordinate system
Combine the separate points with a smooth line

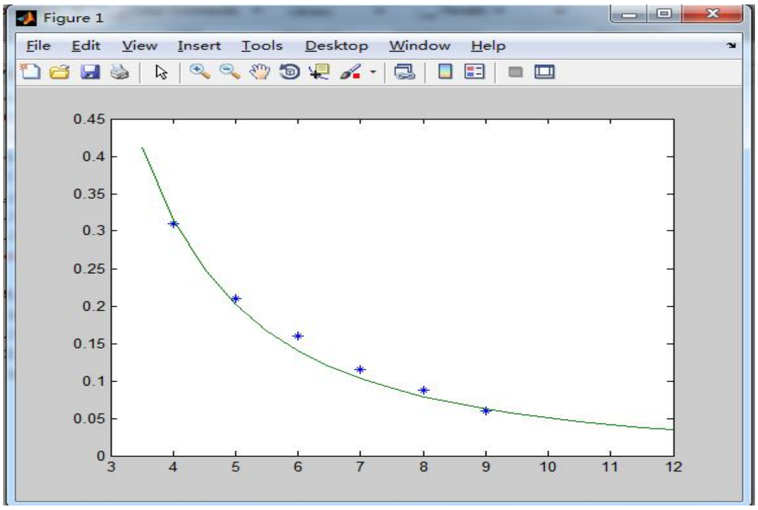

Fig. 10: Line consisted of (n, Q) in rectangular coordinate system

Calculating with Matlab, we get

$$Q=5.043/(n^2) \quad (1\text{-}16)$$

Now we use regular decagon to prove the accuracy of equation (1-16):
Set

$$n=10 \quad (1\text{-}17)$$

We get

$$Q=0.05043 \quad (1\text{-}18)$$

Simulating heat distribution of regular decagon in Matlab

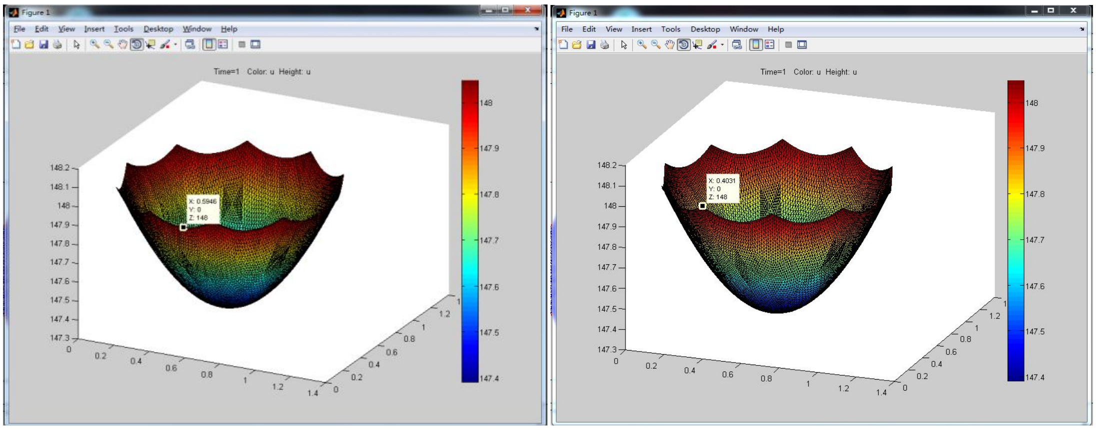

Fig. 11: The heat distribution of a pan shaped in regular decagon

We get

Vertex temperature:148.53℃. Midpointtemperature: 148.35℃

$$y=0.018 \tag{1-19}$$

$$y/x=0.050\approx Q \tag{1-20}$$

Fig.12 shows that the heat distribution of edge of a circular pan is even.

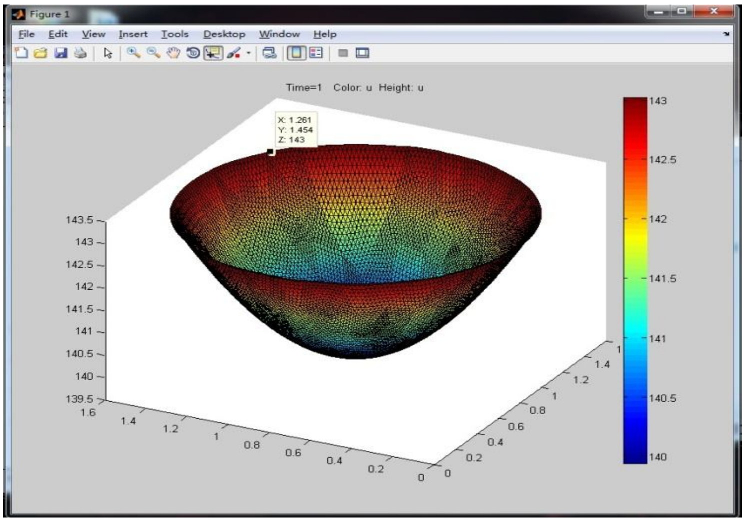

Fig. 12: The heat distribution of a circular pan

1.5 Analyze the Result

From the figures and the data above we see that except circular pans, heat is much more concentrated in the corners than the edges no matter what polygon pans shaped in. The only exception is circle, whose heat distribute evenly across the outer edge. Another phenomenon is that the homogeneity degree of heat decreases progressively to zero with the increasing of *n*(the number of edges). When *n* attend to infinite, we get a circle and its homogeneity degree of heat is zero.

2 Model Two

2.1 Assumptions

a) Each pan hasthe same area of A^2.

b) From rectangular pans to circular ones, it just experiences square, regular pentagon, regular hexagon, regular heptagon, regular octagon and circle.

2.2 Model Building

The problem of polygon arrangement is very complicated. Even we assume the polygon is regular, we hardly find the accurate regulation of it. Therefore, we give several specific examples to uncover part of the regulation and make it understood to some degree. We assume three conditions that

Condition 1: When

$$L/W=1 \tag{2-1}$$

we get

$$L = 6A, W = 6A \tag{2-2}$$

Condition 2: When

$$L/W=2 \tag{2-3}$$

we get

$$L = 12A, W = 6A \tag{2-4}$$

Condition 3: When

$$L/W=3 \tag{2-5}$$

we get

$$L = 18A, W = 6A \tag{2-6}$$

The following is the maximize number of pans that can fit in the oven. We give the explanation in detail under condition one and just give the final results under the other two conditions.

To arrange more uniform polygons on the limited area, we try to make more polygons share the same point to save more space. When the interior of a polygon is bigger than 120°, only two polygons can share the same point. In this case, we try to make several polygons surround one regular or irregular polygon. In this way, we can arrange them tightly. [3]

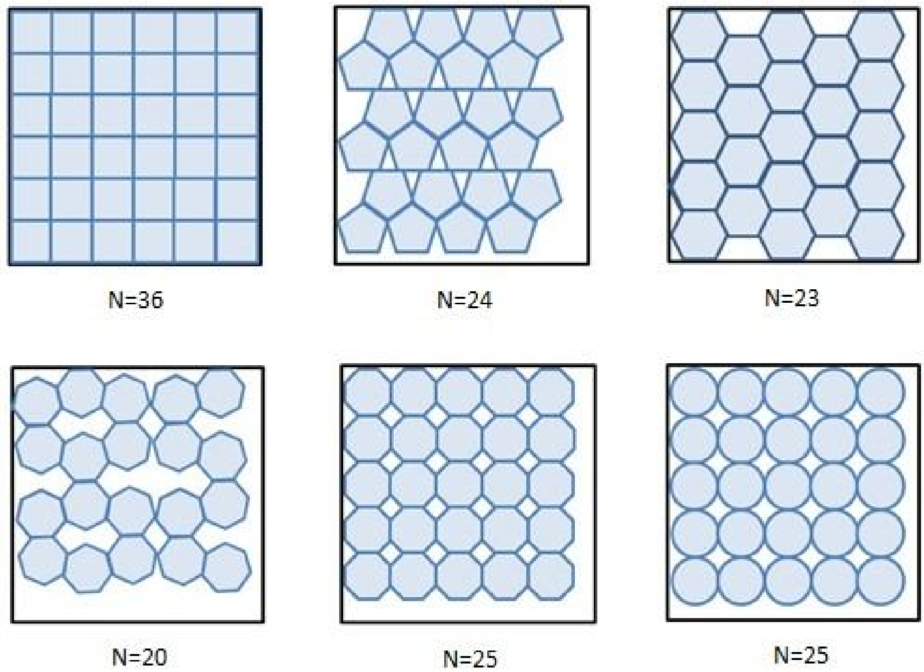

Fig. 13: Maximize number of pans in different shapes that can fit in the square oven

Calculate the number under the other two conditions, we get the table below.

Table 3: Maximize number of pans in different shapes that can fit in the rectangularovens

condition	L/W	area of oven	number		
			square	regular pentagon	regular hexagon
1	1	$36A^2$	36	24	23
2	2	$72A^2$	72	54	45
3	3	$108A^2$	108	84	72
condition	L/W	area of oven	number		
			regular heptagon	regular octagon	circle
1	1	$36A^2$	20	25	25
2	2	$72A^2$	40	50	50
3	3	$108A^2$	60	80	79

2.3 Analyze the results

Due to the area of the oven is different, it is hard to see the relationship among different values of *L/W*. We define a new concept: area occupying ratio*M as*

$$M = \frac{NA^2}{S_{oven}} \quad (2\text{-}7)$$

We can transfer Table 3 to Table 4 as follows.

Table 4: Maximize area occupying ratios of pans in different shapes.

condition	L/W	area of oven	ratio(M)		
			square	regular pentagon	regular hexagon
1	1	$36A^2$	1.000	0.667	0.639
2	2	$72A^2$	1.000	0.750	0.625
3	3	$108A^2$	1.000	0.778	0.667
condition	L/W	area of oven	ratio(M)		
			regular heptagon	regular octagon	circle
1	1	$36A^2$	0.556	0.694	0.694
2	2	$72A^2$	0.556	0.694	0.694
3	3	$108A^2$	0.556	0.741	0.731

From Table 4, we see that the area occupying ratio of square is maximal. With the increasing of *L/W*, most of the area occupying ratios of pans in different shapes are relatively steady. However, when the value of *L/W* reaches a certain quantity, the area occupying ratios may have a value jump. For example, before the value of *L/W* reaches 3, the area occupying ratio of regular octagon is steady. However, when the value reaches 3, then the ratio jumps from 0.694 to 0.741. We also get that the ratio does not decrease or increase progressively as the values of *n*(the number of edges) increase.

3 Combine Model One with Model Two

From Table 2 and Table 4, we get

Table 5: Comprehensive index of pans shaped in different shapes and heated in different ovens

W/L=1				
	weight	square	regular pentagon	regular hexagon
M (area occupying ratio)	p	1.000	0.667	0.639
Q (Homogeneity degree of heat)	1-p	0.310	0.210	0.160
Mp-Q(1-p)		1.31p-0.31	0.877p-0.21	0.799p-0.16
	weight	regular heptagon	regular octagon	circle
M (area occupying ratio)	p	0.556	0.694	0.694
Q (Homogeneity degree of heat)	1-p	0.115	0.088	0.000
Mp-Q(1-p)		0.671p-0.115	0.782p-0.088	0.694p

W/L=2				
	weight	square	regular pentagon	regular hexagon
M (area occupying ratio)	p	1.000	0.750	0.625
Q (Homogeneity degree of heat)	1-p	0.310	0.210	0.160
Mp-Q(1-p)		1.31p-0.31	0.96p-0.21	0.785p-0.16
	weight	regular heptagon	regular octagon	circle
M (area occupying ratio)	p	0.556	0.694	0.694
Q (Homogeneity degree of heat)	1-p	0.115	0.088	0.000
Mp-Q(1-p)		0.671p-0.115	0.782p-0.088	0.694p

W/L=3				
	weight	square	regular pentagon	regular hexagon
M (area occupying ratio)	p	1.000	0.778	0.667
Q (Homogeneity degree of heat)	1-p	0.310	0.210	0.160
Mp-Q(1-p)		1.31p-0.31	0.988p-0.21	0.827p-0.16
	weight	regular heptagon	regular octagon	circle
M (area occupying ratio)	p	0.556	0.741	0.731
Q (Homogeneity degree of heat)	1-p	0.115	0.088	0.000
Mp-Q(1-p)		0.671p-0.115	0.829p-0.088	0.731p

Comparing the comprehensive index (Mp-Q (1-p)) of one polygon to the others, we finally get that

When the value of *L/W* is 1 or 2, if p>0.503, the comprehensive index of pans shaped in square is higher than that of circular pans. Otherwise, the comprehensive index of circular pans is higher than that of pans shaped in square.

When the value of *L/W* is 3, if p>0.535, the comprehensive index of pans shaped in square is higher than that of circular pans. Otherwise, the comprehensive index of circular pans is higher than that of pans shaped in square.

All in all, we see that the comprehensive index of pans shaped in different shapes decrease at first and then increase with the increasing of the number of edges.

Conclusion

In the view of regular polygon and circle, square and circle have the highest comprehensive index. That is to say, considering both sides of area occupying ratio and homogeneity degree of heat, square and circle are outstanding than otherregular polygon. Therefore, we can suppose that if corners of a square are round rather than hard-edged, it may both save the space and are heated evenly. Due to the limitation of time and knowledge, we do not make further exploration here. But the guess is really worthy of further study.

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