International Journal of Mathematics Research.
ISSN 0976-5840 Volume 5, Number 2 (2013), pp. 223-232

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# Some New Families of Harmonic Mean Graphs

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**Abstract**

A Graph G= (V, E) with p vertices and q edges is called a Harmonic Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from 1,2….q+1 in such way that when each edge e=uv is labeled with $f(e=uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic mean labeling of G.

In this paper we investigate the Harmonic mean labeling behaviour for Some New Families of Graphs.

**Keywords**: Graph, Harmonic mean graph, Path, Cycle, Prism graph, Ladder graph, Step ladder.

## 1. Introduction

The graph considered here will be finite undirected and simple. The vertex set is denoted by V(G) and the edge set is denoted by E(G). A cycle of length n is $C_n$ and a path of length n is denoted by $P_n$. The union of two graphs $G_1 = (V_1,E_1)$ and $G_2 = (V_2 E_2)$ is a graph $G=G_1 \cup G_2$ with vertex of $V=V_1 \cup V_2$ and edge set $E= E_1 \cup E_2$. The Cartesian product of two graphs $G_1 = (V_1 E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G= (V,E) = G_1 x G_2$ with $V = V_1 x V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 x G_2$ whenever $u_1 = v_1$ and $u_2$ is adjacent to $v_2$ or $u_2 = v_2$ or $u_1$ is adjacent to $v_1$. The corona of two graphs $G_1$ and $G_2$ is the graph $G= G_1 o G_2$ formed by one copy of $G_1$ and

$|V(G_1)|$ copies of $G_2$ where the $i^{th}$ vertex of $G_1$ is adjacent to every vertex in the $i^{th}$ copy of $G_2$. The product $P_m x P_n$ is called planar grid and $P_2 x P_n$ is called a ladder. For all other standard terminology and notations we refer Harary [1].

S. Somasundaram and S.S.Sandhya introduced Harmonic mean labeling of graphs in [3] and studied their in [4], [5] and [6]. In this paper, we investigate Some New Families of Harmonic mean graphs.

The definitions and other informations which are useful for the present investigation are given below.

**Definition 1.1**

A Graph G with p vertices and q edges called to Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f($x$) from 1,2….q+1 in such a way that when each edge e = uv is labeled with f(e = uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are distinct. In this case f is called a Harmonic Mean labeling of G.

**Definition 1.2**

$P_n$. $A(K_1)$ is a graph obtained by attaching a pendant vertex alternatively to the vertices of $P_n$.

**Definition 1.3**

Let $P_n$ be a path on n vertices denoted by (1,1), (1,2)….(1,n) and with n-1 edges denoted by $e_1$, $e_2$…..$e_{n-1}$ where $e_i$ is the edge joining the vertices (1, $i$) and (1, $i$+1). On each edge $e_i$ = $i$ =1,2….n-1. We erect a ladder with n-($i$-1) steps including the edge $e_i$. The graph obtained is called a step ladder graph and is denoted by $S(T_n)$, where n denotes the number of vertices in the base.

**Definition 1.4**

The prism $D_n$, n ≥3 is a trivalent graph which can be defined as the cartesian product $P_2 x C_n$ of a path on two vertices with a cycle on n vertices.

We shall make frequent reference to the following results.

**Theorem 1.5 [3]**

Any path is a Harmonic mean graph

**Theorem 1.6 [3]**

Any cycle is a Harmonic mean graph.

**Theorem 1.7 [3]**

Combs are Harmonic mean graphs.

**Theorem 1.8 [3]**

Ladders are Harmonic mean graphs.

## 2. Main Results

**Theorem 2.1**

$P_n$. $A(K_1)$ is a Harmonic mean graph.

**Proof**

Let $P_n$. $A(K_1)$ be the given graph. Let $u_1,u_2$……………$u_n$ be the vertices of $P_n$ and $v_1,v_2$……………..$v_m$ be the vertices which are joined alternatively in $P_n$.

Here we consider the two different cases.

Case (i) If the pendant vertex starts from $u_2$, then define a function.

$f: V(P_n. A(K_1) \to \{1,2….q+1\}$ by

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_i) = f(u_{i-2})+3$ for all $i$ =3,4,5,6,7….n and $f(v_i) = 3_i$, for all $i$ = 1,2…m.

Then the edges labels are all distinct.

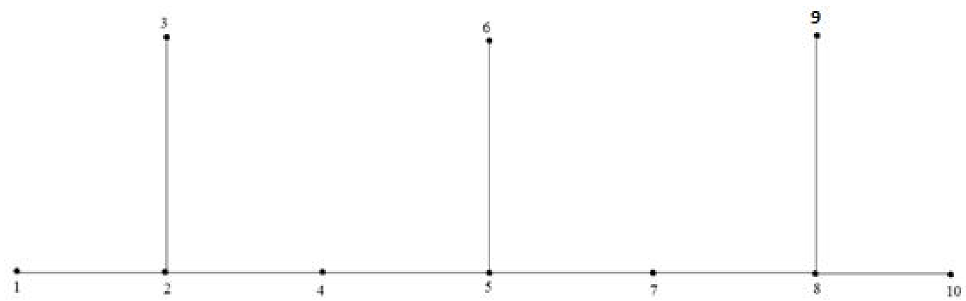


**Figure 1**

**Case (ii)**

If the pendant vertex starts from $u_1$ then define a function $f: V(P_nA(K_1)) \to \{1,2……….q+1\}$

by $f(u_1) = 1$, $f(u_2) = 3$

$f(u_i) = f(u_{i-2})+3$, $V_i$ = 3, 4,5,6….n

$f(v_1) = 2$

$f(v_i) = f(v_{i-1})+3$, $i$ = 2,3…………..m

In this case also we get distinct edge labels.

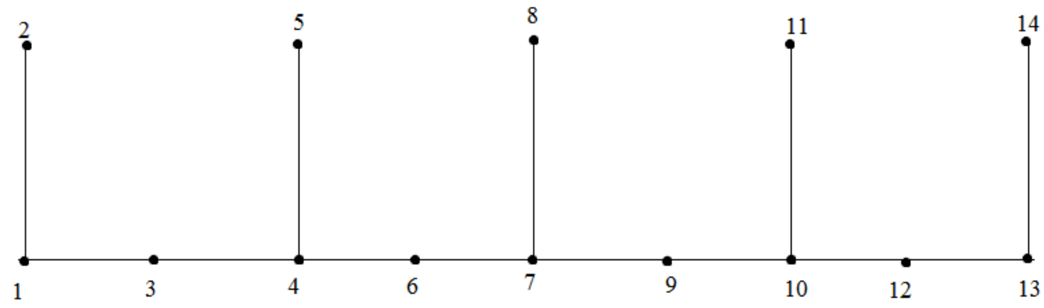


**Figure 2**

In the view of the above defined labeling pattern f is a Harmonic mean labeling of G.

**Theorem 2.2**
The step Ladder $S(T_n)$ is a Harmonic mean graph

**Proof**
Let $S(T_n)$ be the given step ladder. Let $P_n$ be a path on n vertices denoted by (1,1), (1,2)…..(1,n) and with n-1 edges denoted by $e_1, e_2 \ldots e_{n-1}$ where $e_i$ is the edge joining the vertices (1,*i*) and (1, *i*+1).

The step Ladder graph $S(T_n)$ has vertices denoted by (1,1), (1,2)……(1,n), (2,1), (2,2)………(2,n), (3,1), (3,2)…..(3,n-1)………..(n,1), (n,2).

In the ordered pair (*i*, j) *i* denotes the row (counted from bottom to top) ladj denote the column (from left to right) in which the vertex occurs.

Define a function f: $V(S(T_n)) \rightarrow \{1,2\ldots.q+1\}$

by $f(i,1) = n^2+i-1,\ 1 \le i \le n$

$f(1,j) = (n-j+1)^2,\ 2 \le j \le n$

$f(i,j) = (n-j+1)^2 + i-1,\ 2 \le j \le n,\ 2 \le j \le n-i+2$

Hence $S(T_n)$ is a Harmonic mean graph

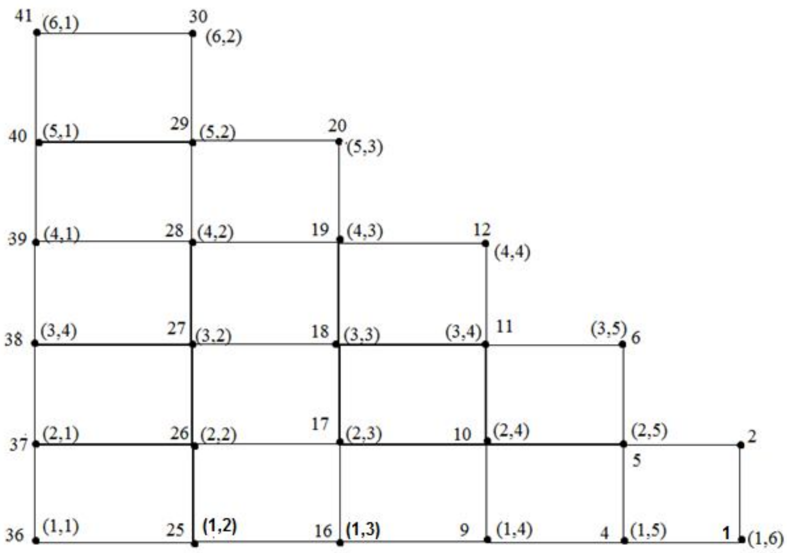


**Figure 3**

**Theorem 2.3**

$D_n . K_2$ is a Harmonic mean graph

**Proof**

Let $D_n . K_2$ be the given graph and let $t_i$, $u_i$, $v_i$, $w_i$, $1 \leq i \leq n$ be the vertices of $D_n . K_2$.

Define a function $f: V(D_n . K_2) \rightarrow \{1,2\ldots.q+1\}$

by $f(t_i) = 6(i\text{-}1)+1,\ 1 \leq i \leq 2$

$f(t_i) = 5i\text{-}3,\ 3 \leq i \leq n$

$f(u_i) = 6(i\text{-}1)+2,\ 1 \leq i \leq 2$

$f(u_i) = 5i\text{-}2,\ 3 \leq i \leq n$

$f(v_i) = 6\ (i\text{-}1)+3,\ 1 \leq i \leq 2$

$f(v_i) = 5i\text{-}1,\ 3 \leq i \leq n$

$f(w_i) = 6(i\text{-}1)+4,\ 1 \leq i \leq 2$

$f(w_i) = 5i,\ 3 \leq i \leq n$

Edges are labeled with

$f(t_iv_i) = 6(i\text{-}1)+1,\ 1 \leq i \leq 2$

$f(t_iu_i) = 5i-2, 3 \leq i \leq n$

$f(u_iv_i) = 2, 1 \leq i \leq 2$

$f(u_iv_i) = 5i-1, 2 \leq i \leq n$

$f(v_1w_1) = 3$

$f(v_iw_i) = 5i, 2 \leq i \leq n$

$f(v_1v_2) = 4$

$f(v_i v_{i+1}) = 5i+1, 2 \leq i \leq n-1$

$f(v_nv_1) = 6$

$f(w_1w_2) = 5$

$f(w_iw_{i+1}) = 5i+2, 2 \leq i \leq n-1$

$f(w_nw_1) = 8$

Hence f provides a Harmonic mean labeling for $D_n . K_2$

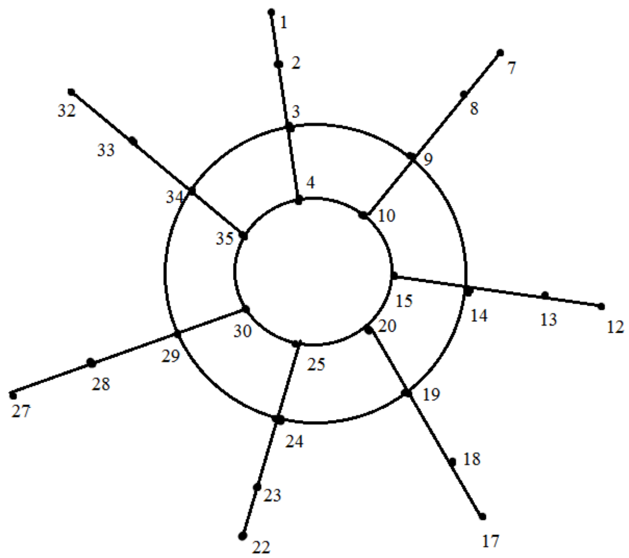


**Figure 4**

**Theorem 2.4**

Let G be a graph obtained by attaching paths of length 0, 1,2…..n-1 on both sides of each vertex of $P_n$, then G is a Harmonic mean graph.

**Proof**

Let G be a graph obtained by attaching paths of length o,1,2….n-1 on both sides of each vertex of $P_n$.

Let $u_{11}$, $u_{22}$….$u_{nn}$ are the vertices of $P_n$.

Define a function f: V(G) → {1,2….q+1} by $f(u_{ij}) = (i\text{-}1)^2 + j$, $1 \leq i \leq n$ $1 \leq j \leq 2i\text{-}1$.

The above labeling pattern f is a Harmonic mean labeling of $G_n$

Consequently we have the following

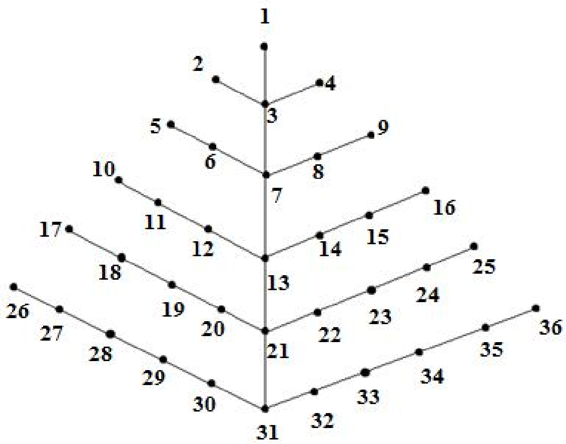


**Figure 5**

**Theorem 2.5**

Let G be a graph obtained by attaching pendant edges to both sides of each vertex of a path $P_n$. The G is a Harmonic mean graph.

**Proof**

Consider a graph G obtained by attaching pendant edges to both sides of each vertex of a path $P_n$. Then G is a Harmonic mean graph.

**Proof**

Consider a graph G obtained by attaching pendant edges to both sides of each vertex of a path $P_n$.

Let $u_i$, $v_i$, $w_i$, $1 \leq i \leq n$ be the vertices of G.

Define a function f:V(G) → {1,2…..q+1}

by $f(u_i) = 3i$, $1 \leq i \leq n$

$f(v_i) = 3i-1, 1 \le i \le n$

$f(w_i) = 3i-2, 1 \le i \le n$

Edges are labeled with

$f(u_i v_i) = 3i-1, 1 \le i \le n$

$f(u_i u_{i+1}) = 3i, 1 \le i \le n-1$

$f(u_i w_i) = 3i-2, 1 \le i \le n$

Hence G is a Harmonic mean graph.

**Example 2.6**
The labeling pattern is shown in the following figure.

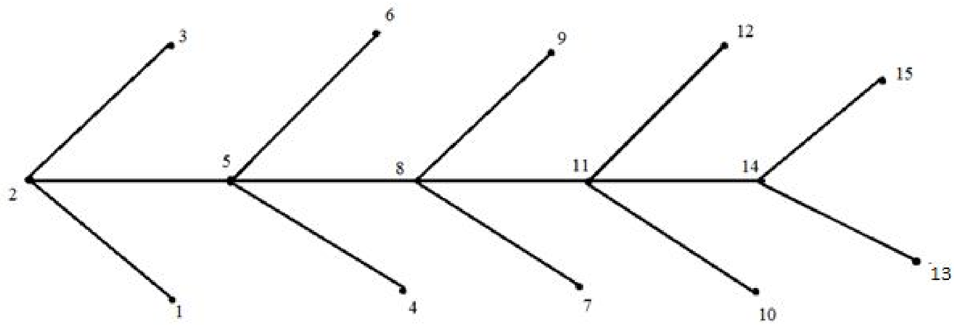


**Figure 6**

**Theorem 2.8**
$P_nAK_3$ is a Harmonic mean graph
Proof: Consider the graph $P_nAK_3$ with vertices $u_i$ $v_i$, $1 \le i \le n$
Now we define $f: V(P_nAK_3) \to \{1,2\ldots.q+1\}$

by $f(u_1) = 3$

$f(u_i) = 4i-2, 2 \le i \le n.$

$f(v_i) = 4i-3, 1 \le i \le n$

$f(w_1) = 2$

$f(w_i) = 4i-1, 2 \le i \le n$

Edges are labeled with

$f(u_iu_{i+1}) = 4i$, $1 \leq i \leq n-1$

$f(u_1v_1) = 2$

$f(u_iv_i) = 4i-3$, $2 \leq i \leq n$

$f(u_iw_i) = 4i-1$, $1 \leq i \leq n$

$f(v_1w_1) = 1$

$f(v_iw_i) = 4i-2$, $2 \leq i \leq n$

Hence $P_nAK_3$ is a Harmonic Mean graph

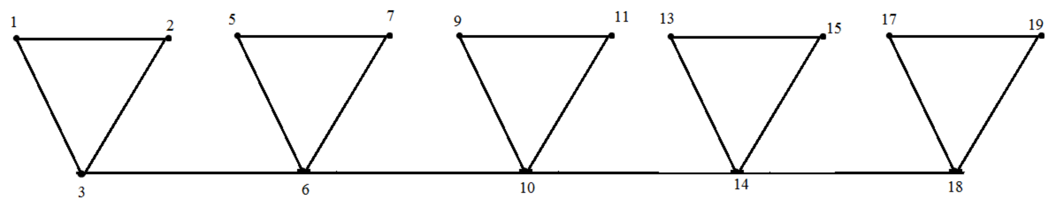


**Figure 7**

**Theorem 2.9**
$C_nAK_3$ is a harmonic mean graph.

**Proof**
Let G be a graph $C_nAK_3$ with vertices $u_i$, $v_i$, $w_i$, $1 \leq i \leq n$.
Define a function

$f: V(G) \rightarrow \{1,2\ldots.q\}$ by

$f(u_i) = 4i-1$, $1 \leq i \leq n$, $f(v_i) = 4i-3$, $1 \leq i \leq n$, $f(w_i) = 4i$, $1 \leq i \leq n$.

Edges are labeled with $f(u_1u_2) = 3$

$f(u_i\, u_{i+1}) = 4i+1$, $2 \leq i \leq n-1$

$f(u_iv_i) = 4i-3$, $1 \leq i \leq 2$

$f(u_iv_i) = 4i-2$, $3 \leq i \leq n$

$f(u_1w_1) = 3$ $f(u_iw_i) = 4i$, $2 \leq i \leq n$, $f(v_1w_1) = 2$

$f(v_iw_i) = 4i-1$, $2 \leq i \leq n$

Hence $C_nAK_2$ is a harmonic mean graph.

**Example 2.10**
A Harmonic mean labeling of $C_5AK_3$ given below

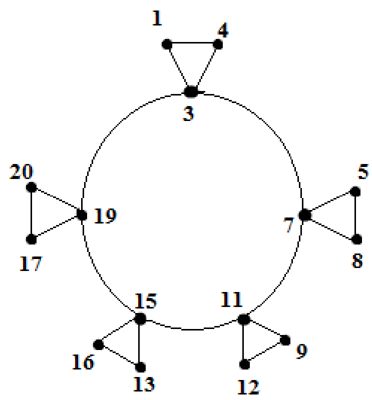


**Figure 8**

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