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Harmonic Mean Labeling on Double Triangular Snakes

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Abstract

A graph G= (V,E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,….,q+1 in such a way that when each edge e=uv is labeled with f(uv)= $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then the edge labels are distinct. In this case, f is called Harmonic mean labeling of G. In this paper we prove that Double Triangular snake and Alternate Double Triangular snake graphs are Harmonic graphs.

Keywords: Graph, Harmonic mean graph, Double Triangular snake, Alternative Double Triangular snake.

1.Introduction

All graph in this paper are finite, simple and undirected graph G=(V,E) with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harry [2]. We will provide brief summary of definitions and other information which are prerequisites for the present investigation.

Definition 1.1: A graph G = (V,E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label vertices x∈V with distinct labels f(x) 1,2……….., q+1 in such a way that when each edge e=uv is labeled with f(uv)= $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil$ (or) $\left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2: Triangular snake Tn, is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} to new vertex v_i.

Definition 1.3: An Alternate Triangular snake $A(T_n)$ is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i.

Definition 1.4: A Double triangular snake $D(T_n)$ is the graph obtained from the path $u_1u_2 \ldots u_n$ by joining u_i, u_{i+1} with two new vertices v_i and w_i, $1 \le i \le n-1$.

Definition 1.5: Alternate Double triangular snake $A(DT_n)$ is the graph obtained from the path $u_1u_2 \ldots u_n$ by joining u_i, u_{i+1} (Alternatively) with two new vertices v_i and w_i $1 \le i \le n-1$.

S. Somasundaram and S.S.Sandhya introduced Harmonic mean labeling of a Graph in [4] and studied their behaviour in [5] and [6] . In this paper we prove that Double Triangular snakes and Alternate Double Triangular snakes are Harmonic mean graphs.

2. Main Results

Theorem 2.1: A Double Triangular snake $D(T_n)$ is a harmonic mean graph.

Proof, Consider a path $u_1u_2 \ldots u_n$. Join u_i, u_{i+1} with two new vertices v_i, w_i $1 \le i \le n-1$.

Define a function f: $V(D(T_n)) \to \{1,2 \ldots q+1\}$ by

$f(u_1)=3$; $f(u_i)=5i-4$, $2 \le i \le n$;

$f(v_1)=1$; $f(v_i)=5i-3$, $2 \le i \le n-1$;

$f(w_i) = 5i-1$, $1 \le i \le n-1$;

The edges are labeled with

$f(u_1u_2) =4$; $f(u_iu_{i+1})=5i-2$, $2 \le i \le n-1$;

$f(u_iv_i)=5i-4$, $1 \le i \le n-1$;

$f(u_2u_1) =2$; $f(u_{i+1}v_i)=5i-1$, $2 \le i \le n-1$;

$f(u_1w_1)=3$; $f(u_iw_i)=5i-3$, $2 \le i \le n-1$;

$f(u_{i+1}w_i)=5i$, $1 \le i \le n-1$;

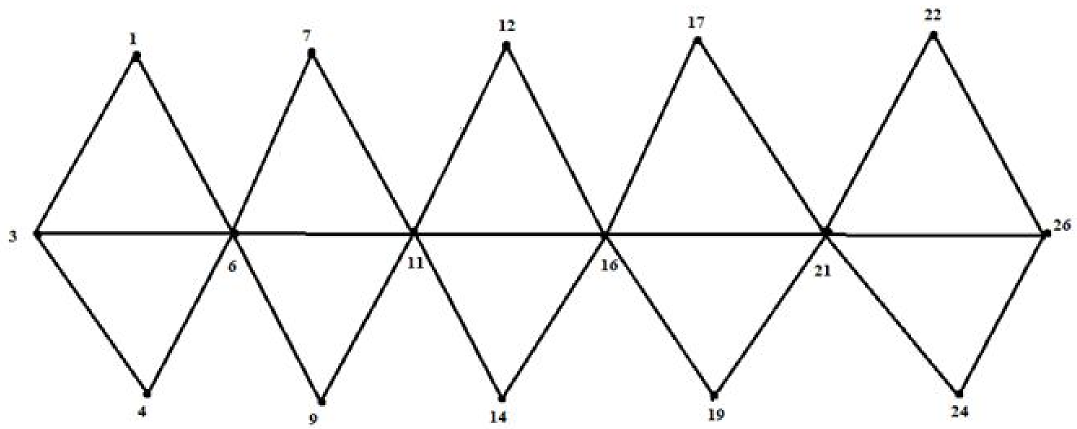

Figure : 1

In the view of the above labeling, f provides Harmonic mean labeling for the graph $D(T_n)$.

Theorem 2.2: Alternative Double Triangular snake $A(D(T_n))$ is a Harmonic mean graph.

Proof: Let G be the graph $A(D(T_n))$. consider the path $u_1u_2\ldots u_n$. To construct G, join u_i, u_{i+1} (alternatively) with two new vertices v_iw_i, $1\le i\le n-1$. There are two different cases to be considered.

Case 1: If the Double Triangle starts from u_1 we need to considered two subcases.

Subcase1(a): If n is odd, then
Define a function $f:V(G)\rightarrow\{1,2\ldots\ldots q+1\}$ by

$f(v_1)=1$; $f(v_i) = 6(i-1)$, $2\le i\le\frac{n-1}{2}$;
$f(u_i) =3$; $f(u_i) = 3i-1$, $2\le i\le n-1$; $f(u_n)=3(n-1)$;
$f(w_i) = 6i-2$, $1\le i\le\frac{n-1}{2}$;

The edges are labeled with
$f(u_iu_{i+1}) = 3i+1$ for all $i = 1,3\ldots,n-2$;
$f(u_iu_{i+1}) = 3i$ for all $i = 1,3\ldots,n-3$;
$f(u_{2i-1}v) = 6i-5$ for all $i=1,2\ldots\ldots,\frac{n-1}{2}$;
$f(u_{2i}v_i) = 6i-4$ for all $i = 1,2\ldots\ldots,\frac{n-1}{2}$;
$f(u_{2i-1}w_i) = 6i-3$ for all $i=1,2\ldots,\frac{n-1}{2}$;
$f(u_{2i}w_i) = 6i-1$ for all $i=1,2\ldots\ldots,\frac{n-1}{2}$;

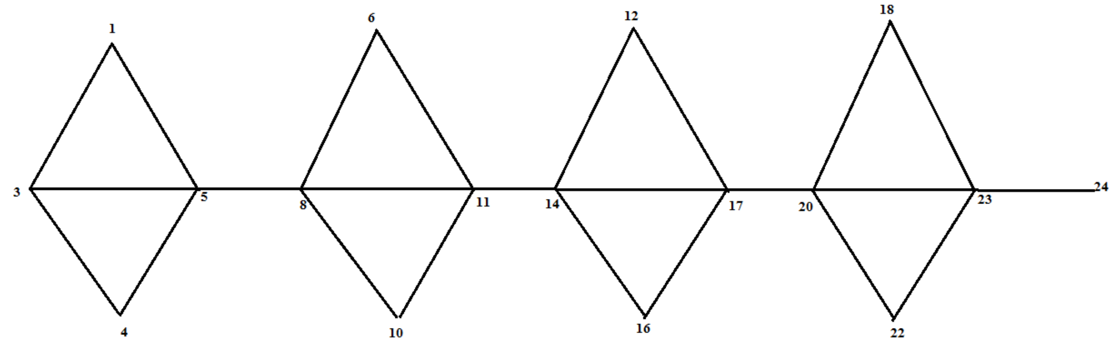

Figure : 2

In this case f is a harmonic mean labeling.

Subcase (1) (b) : If n is even then

Define a function f: V(G) $\rightarrow$\{1,2…..q+1\} by

$f(v_1)$ =1; $f(v_i)$ = 6i-1, $2\leq i\leq\frac{n}{2}$;

$f(u_1)$ = 3; $f(u_i)$ = 3i-1, $1\leq i\leq\frac{n}{2}$;

The edges are labeled with

$f(u_iu_{i+1})$ = 3i-1 for all i= 1,3,……,n-1;

$f(u_iu_{i+1})$ = 3i for all i = 2,4…….,n-2;

$f(_{2i-1}v_i)$ = 6i-5, for all I = 1,2,….$\frac{n}{2}$;

$f(u_{2i}v_i)$ = 6i-4, for all i=1,2.., $\frac{n}{2}$;

$f(u_{2i-1}w_i)$=6i-3, for all i=1,2…., $\frac{n}{2}$;

$f(w_iw_i)$ = 6i-1, for all i=1,2….,$\frac{n}{2}$;

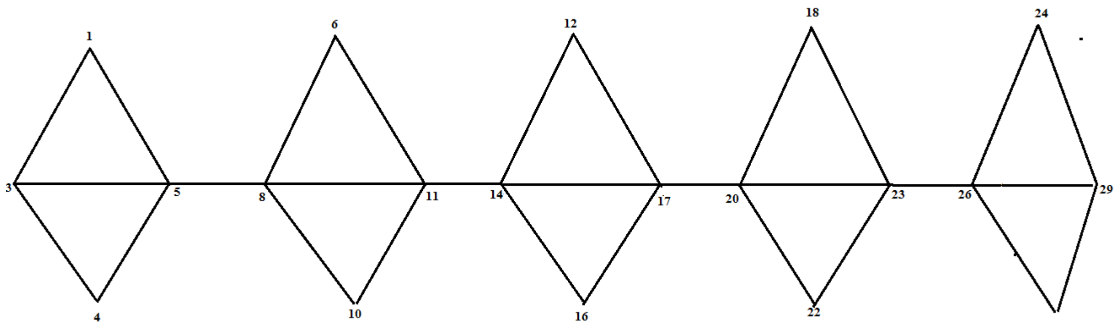

Figure: 3

In this case f is Harmonic mean labeling

Case 2(a): If the triangle starts from u_2, we have to consider two sub cases.

Subcase 2(a): If n is odd then.

Define a function f: V(G) →{1,2………q+1} by

$f(u_iu_{i+1})$ =3i-2 for all i=1,3……..n-2;

$f(u_iu_{i+1})$ = 3i-1 for all i=2,4……n-1;

$f(u_{2i}v_i)$ = 6i-4 to for all i=1,2……, $\frac{n-1}{2}$;

$f(u_{2i}w_i)$ = 6i-3 for all i=1,2……,$\frac{n-1}{2}$;

$f(u_{2i+1}w_i)$ =6i for all i=1,2…..$\frac{n-1}{2}$;

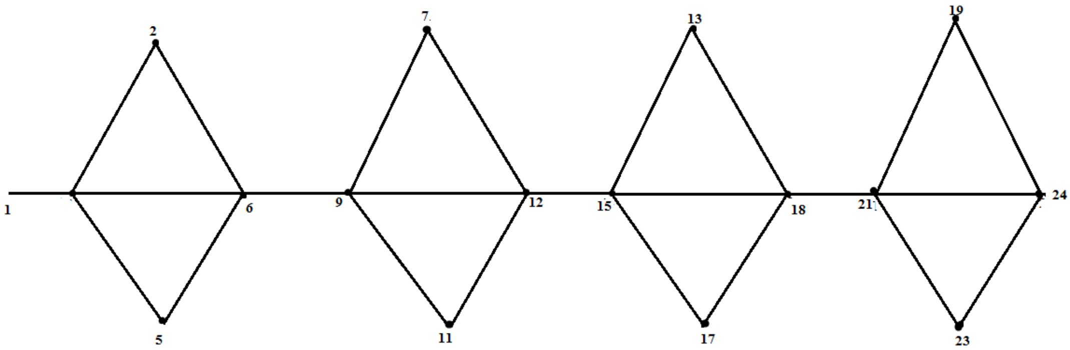

Figure: 4

In this case f provides Harmonic mean labeling of G.

Subcase 2(b): If n is even

Define a function f:V(G) →{1,2…q+1} by

$f(u_1)$=1; $f(u_2)$=4; $f(u_i)$=3(i-1), 3≤i≤n-1; $f(u_n)$ = 3n-5;

$f(v_1)$ = 2; $f(v_i)$=6i-5, 2≤i≤ $\frac{n-2}{2}$;

$f(w_i)$ = 6i-1, 2≤i≤ $\frac{n-2}{2}$;

Then the edges are labeled with

$f(u_iu_{i+1})$ = 3i-2 for all i=1,3…,n-1;

$f(u_iu_{i+1})$ = 3i-1 for all i = 2,4….n-2;

$f(u_{2i}v_i)$ = 6i-4 for all i = 1,2…, $\frac{n-2}{2}$;

$f(u_{2i+1}v_i)$ = 6i-3 for all i = 1,2,…, $\frac{n-2}{2}$;

$f(u_{2i}w_i)$ = 6i-2 for all i=1,2…, $\frac{n-2}{2}$

$f(u_{2i+1}w_i)$ = 6i for all i=1,2……, $\frac{n-2}{2}$

In this case, f provides Harmonic mean labeling of G.

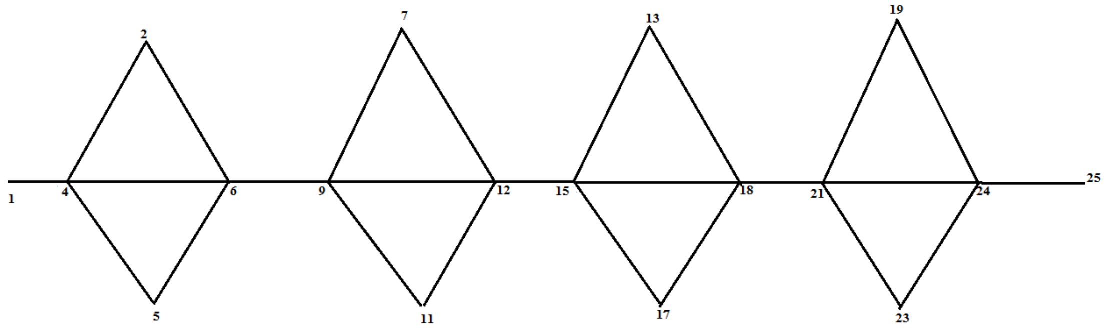

Figure: 5

From all the above cases, we conclude that Alternate Double Triangular Snake $A(D(T_n))$ is a Harmonic mean graph.

References

[1] J.A.Gallian, 2010, A dynamic survey of graph labeling. The electronic Journal of Combinatories 17#DS6.

[2] Harry F, 1988, Graph Theory, Narosa publishing House Reading, New Delhi.

[3] Somasundaram.,S., and Ponraj R., 2003, Mean labeling of graphs, National Academy of Science Letters Vol.26, p.210-213.

[4] Somsundaram S., and Ponraj R., and Sandhya S.S., Harmonic mean labeling of Graphs Communicated.

[5] Sandhya S.S., Somasundaram S., and Ponraj R., Some Results on Harmonic Mean Graphs, International journal of Contemporary Mathematical Sciences 7(4) (2012), 97-208.

[6] Sandhya S.S., Somasundaram S., and Ponraj R., Harmonic Mean labeling of Some Cycle Related Graphs, International Journal of Mathematical Analysis Vol.6 No.40 , (2012) 1997-2005.